CS412 **Assignment 3** (as of 11:41 28sep) Fall 2000

due date: 10 October 2000

STAPLE your homework. MARK your homework clearly with your NAME. In addition, write the first letter of your LAST NAME boldly into the upper left corner of the first page of your homework.

1. (30 points) (a) Use Calculus (and the discussion in class and in the additional course material available on the web concerning catastrophic cancellation) to generate a version of `MyExp4.m` (on page 53) that would return not only the value but also the first derivative of the function whose values this M-file computes. The first line should read `function [y,dy] = MyExp4(x)` and the function should work properly even when `x` is a matrix.

(b) Explain why it would be a fair test of your version to check whether `y` equals `dy`.

(c) Try out your version by applying it to a matrix `x` of size 33-by-34 whose entries are random numbers uniformly distributed in the interval $[-2\,..\,2]$, and report (in the form of a properly edited diary) on the average of the absolute and the relative errors you encounter if you treat the computed `dy` as an approximation to the computed `y`.

2.(25 points) (a) Suppose given three data points (x_1, y_1), (x_2, y_2), and (x_3, y_3) with the property that $x_1 < x_2 < x_3$ and that y_1 and y_3 are of opposite sign. Write a function `root = inverseq(x,y)` that returns the inverse quadratic interpolant at 0, i.e., the value at 0 of the unique polynomial q of degree ≤ 2 that satisfies $q(y_i) = x_i$ for $i = 1{:}3$.

(b) Try out your `function inverseq` on the data $(x_i, f(x_i))$ with $(x_1, x_2, x_3) = 4/3 : 1/3 : 2$ and $f(x) = x^3 - 3$, being sure to compute the error of the resulting approximation to $3^{1/3}$. (If you use (`InterpN2` and `HornerN` from the book, the body of your `inverseq` could be a one-line statement.)

3. (10 points) Use the error formula for polynomial interpolation to determine as large an h as possible with the property that, at any point x between $-h$ and h, the absolute error at x in the polynomial interpolant to $f(x) = \cos(2x)$ at $-h, 0, h$ is no bigger than 10^{-4}.

4. (2*5 points) For each of the following, state whether it is true or false, and give a brief reason (or evidence) for your answer.

1. There is exactly one cubic polynomial that agrees with the function $\sin(x)$ at the three points $1, 2, 3$.
2. The smallest positive fl.# number that `matlab` recognizes as being different from zero is 2^{-1025}.
3. If $f(x) = 4 + 3(x-1) + 2(x-2)^2 + (x-3)^3$, then the divided difference of f at x_1, x_2, x_3, x_4 is 1.
4. The two-command sequence `[x,y] = sort(rand(1,10)); find(diff(x)<0)` returns the empty matrix, `[]`, as an answer.
5. Except for 0 and some non-numbers like ±`inf` or `NaN`, `matlab` only works with *normalized* fl.#s.