CS412 **Assignment 6 (as of 14nov00)** Fall 2000

due date: 28 November 2000

STAPLE your homework. MARK your homework clearly with your NAME. In addition, write the first letter of your LAST NAME boldly into the upper left corner of the first page of your homework . For this to be of real help, the vertical edge of the upper left corner should be longer than the horizontal edge.

1. (15 points) For the matrix A given on page 222 of the textbook, construct the LU factorization (for PA for some suitable permutation matrix P) as obtained by (a) *scaled partial pivoting*, as well as by (b) *partial pivoting*. For each, hand in the two factors, L and U, as well as the permutation matrix P. Also compute $PA - LU$ (e.g., by `Matlab`) to be sure you got it right. (Feel free to do all the calculations with the aid of `Matlab`, in which case it would be fine just to hand in the (cleaned-up) diary).

2. (10 points) Determine the least-squares solution to the linear system $[a]x = b$, given that $a = (1, 1, 1)$, and $b = (b_1, b_2, b_3)$ is some 3-vector. (Your answer should be an expression involving the entries of b. Be sure to justify your answer.)

3. (35 points) Do P7.3.1 (on page 266 of the text), but don't bother with the particular sample problem proposed there. Rather, apply it to the problem $-D(xDu(x)) + xu(x) = x^3 - x^2 - 4x + 1$ on $[0\,..\,1]$ with $u(0) = u(1) = 0$ whose solution is $u(x) = x(x-1)$. Show that your function performs satisfactorily by solving this problem with $n = 50$ and comparing the approximate solution to the exact values; you should get an error of less than 10^{-14}.

The theory for this is given on pages 255–256. Be aware that, in particular, there, $p_i = p(x_i + h/2)$, while $q_i := q(x_i)$, and $r_i := r(x_i)$. In effect, your function need to do nothing more than generate the main diagonal and the first subdiagonal of the relevant tridiagonal linear system, then use the book's `CholTrid` to factor, and `CholTridSol` to solve, using the appropriate right side.

Hand in a well-documented listing of the function you wrote, as well as a listing of the script that makes use of the function and a listing of the three functions needed. For uniformity's sake, do call those three functions p, q, r.

4. (10 points) What can go wrong with bisection as coded in the book's `function root = Bisection(fname,a,b,delta)` (on pages 280-281) if the statement `if fa*fmid<=0` is replaced by `if fa*fmid<0`? (Be sure to give a specific example.)

5. (3*5 points) For each of the following, state whether it is true or false, and give a brief reason (or evidence) for your answer.

1. Whatever the intent of the bisection method may be, its effect is to find a small interval $[a\,..\,b]$ across which the given function changes sign.
2. `Matlab`'s `fzero` function uses Newton's method.
3. If inverse interpolation is based on just two points, it is just one step of the Secant method.
4. The iteration discussed in Section 8.1.1, for finding $\sqrt{A}$, is a particular example of Newton's method.
5. A symmetric positive definite matrix is diagonally dominant.