spline curves

A planar **curve** is, by definition, the continuous image of some interval $[a \mathinner{..} b]$ in the plane, i.e., a planar set of the form

$$\{f(t) : a \le t \le b\}$$

with f a continuous function from some interval $[a..b]$ into the plane $\mathbb{R}^2$. As the argument t moves from a to b, the point $f(t)$ in the plane traces out the curve. Many different functions f give rise to the same curve. E.g., the functions

$$g : [a \mathinner{..} b] \to \mathbb{R}^2 : t \mapsto f((a+b)-t)$$

and

$$h : [0 \mathinner{..} 1] \to \mathbb{R}^2 : t \mapsto f(a + t(b-a))$$

trace out the same curve as f. Each such continuous is said to be, or to provide, a **parametrization** for the curve it traces.

Given some *sequence* $o_1, \ldots, o_n$ of points in the plane, we can certainly use polynomial interpolation or our various spline interpolation techniques to construct a curve that passes through these points. For example,

$$P_1(t) := o_1 + t(o_2 - o_1)$$

traces out the straight line that passes through the two points o_1 and o_2. In effect, P_1 is a polynomial with *vector* coefficients, i.e., a vector-valued polynomial.

If we put our plane point sequence $o_1, \ldots, o_n$ into the $2 \times n$ matrix `o`, then the `MATLAB` command `curve = spline(1:n,o);` provides a spline curve through these points, and we could plot that curve then as follows

```
points = ppval(curve,linspace(1,n));
plot(points(1,:),points(2,:))
```

If we had some feeling about the tangents at the ends of that curve, we could use complete spline interpolation. For example, to get a pretty good circle, we could use the following:

```
x = linspace(0,2*pi,5);
circle = spline(x,[0 1 0 -1  0 1 0; ...
                   1 0 1  0 -1 0 1], linspace(0,2*pi));
plot(circle(1,:),circle(2,:))
axis equal
```

Here, we have specified the vector `[0;1]` as the tangent vector at the two endpoints which is the correct direction and length, given that we are presumably running once around the unit circle as the parameter moves from 0 to 2π, hence the speed should be 1. If we had used $x = 0:4$ instead, the speed should have been $(2\pi)/4 = \pi/2$, i.e., the tangent vector should have been `[0; pi/2]`.