Answers to all problems in chapter 7

7.1 If $X = \{0\}$ and $A \in L(X, Y)$ is invertible, then, necessarily, also $\dim Y = 0$, and also A^{-1} has a trivial space as its domain, hence $\|A^{-1}\| = 0$, by (7.6).

7.2 By (7.12), $\|\operatorname{id}_X\| = \|VV^{-1}\| \le \|V\|\|V^{-1}\|$, while, for $\dim X > 0$, $\|\operatorname{id}_X\| = 1$.

7.3 Both $[\,]$ and its inverse have the trivial space as their domain, hence have norm 0, therefore $\kappa([\,]) = 0$.

7.4 Since $\|M_\alpha x\| = \|\alpha x\| = |\alpha|\|x\|$, get $\|M_\alpha\| = \max_{x\ne 0} \|M_\alpha x\|/\|x\| = \max_{x\ne 0} |\alpha| = |\alpha|$. (For $X = \{0\}$, would have $M_\alpha = 0$, hence $\|M_\alpha\| = 0$.)

7.5 $p = 1$: $\|D\|_1 = \max_j \|D(:, j)\|_1 = \max_j |D(j, j)|$.

$p = \infty$: $\|D\|_\infty = \|D^{\rm t}\|_1 = \max_j |D^{\rm t}(j, j)| = \max_j |D(j, j)|$.

$p = 2$: $\|Dx\|_2^2 = \sum_j |D(j, j)x_j|^2 \le \max_j |D(j, j)|^2\|x\|_2$, with equality if $x = e_{j^*}$ with $j^* = \operatorname{argmax}_j |D(j, j)|$. Hence $\|Dx\|_2 \le \max_j |D(j, j)|\|x\|_2$ with equality if $x = e_{j^*}$, therefore $\|D\|_2 = \max_j |D(j, j)|$.