Euclid's Algorithm

Horner's method is a special case of **Euclid's Algorithm** which constructs, for given polynomials p and $h \neq 0$, (unique) polynomials q and r with $\deg r < \deg h$ so that

$$p = hq + r.$$

For variety, here is a nonstandard discussion of this algorithm, in terms of elimination.

Assume that

$$h(t) = a_0 + a_1 t + \cdots + a_d t^d, \quad a_d \neq 0,$$

and

$$p(t) = b_0 + b_1 t + \cdots + b_n t^n.$$

Then we seek a polynomial

$$q(t) = c_0 + c_1 t + \cdots + c_{n-d} t^{n-d}$$

for which

$$r := p - hq$$

has degree $< d$. This amounts to the square upper triangular linear system

$$\begin{aligned} a_d c_0 + a_{d-1} c_1 + \cdots + a_0 c_d &= b_d \\ a_d c_1 + a_{d-1} c_2 + \cdots + a_0 c_{d+1} &= b_{d+1} \\ &\vdots \\ a_d c_{n-d-1} + a_{d-1} c_{n-d} &= b_{n-1} \\ a_d c_{n-d} &= b_n \end{aligned}$$

for the unknown coefficients $c_0, \ldots, c_{n-d}$ which can be uniquely solved by back substitution since its diagonal entries all equal $a_d \neq 0$.

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