887 Spring '03 HOMEWORK assignment 1, due Monday, 10 Feb 03

(1) Prove: If $(p_n : n \in \mathbb{N})$ is a sequence of polynomials that converges uniformly on $[a\,..\,b]$ to some $f \notin \Pi$, then $\sup_n \deg p_n = \infty$.

(2) Prove: *If U is a linear positive operator on $C[a\,..\,b]$ for which $U()^j = ()^j$ for $j = 0, 1, 2$, then $U = \mathrm{id}$.*

(3) *Prove: If $f \in C[0\,..\,1]$ vanishes at 0 and 1, then the sequence*

$$B_n^{\lfloor\rfloor} f : t \mapsto \sum_j \lfloor \binom{n}{j} f(j/n) \rfloor \gamma_{j,n-j}, \quad n = 1, 2, \ldots$$

consists of polynomials with integer *coefficients and converges uniformly to f. Here, $\gamma_{r,s} : t \mapsto t^r(1-t)^s$ are the polynomials familiar from Bernstein's polynomial operator B_n, and*

$$\mathbb{R} \to \mathbb{R} : t \mapsto \lfloor t \rfloor,$$

the **floor function**, *associates t with the largest integer no bigger than t.*

(4) *Prove: An $f \in C[0\,..\,1]$ is approximable by polynomials with* integer *coefficients if and only if $f(0), f(1) \in \mathbb{Z}$. (Feel free to use the result of the previous problem even if you haven't managed to prove it.)*