887 Spring '03 HOMEWORK assignment 2, due Monday, 3 Mar 03

(1) Determine two distinct best approximations to $()^2$ from $\Pi_{0,2}[-1\,..\,1] \subset L_2[-1\,..\,1]$ and prove that there are no others.

(2) Let $X = c_0 := \{f : \mathbb{N} \to \mathbb{R} : \lim_{n\to\infty} f(n) = 0\}$ be the ls of real sequences that converge to 0. As a closed linear subspace of ℓ_∞, it is a Banach space wrto the uniform norm. For M, take

$$M := \{f_0, f_1, \ldots\},$$

(in particular, M is a countable set), with

$$f_j := (\underbrace{1, \ldots, 1}_{j \text{ terms}}, 0, \ldots), \quad j = 0, 1, 2, \ldots.$$

Prove that M is a bounded existence set, yet fails to be nearly compact.

(3) Prove that compactness of T is essential for the characterization theorem, 30, by considering best approximation from $M = \Pi_0 \subset C((-1\,..\,1))$ to the function $g = ()^1$. Specifically, show that, with $m \in \mathcal{P}_M(g)$, no linear functional $\lambda_{U,w} \perp M$ with $\#U \le 2$ takes on its norm on $g - m$.

(4) Recall that $\mathbb{T}$ is the *circle*. Show that any linear Haar subspace of $C(\mathbb{T})$ necessarily has odd dimension.