**887 Spring '03 HOMEWORK assignment 6, due 14 apr 03**

**(1)** Prove that the splines $E_n$ of a previous homework satisfy the recurrence

$$E_{n+1} = \int_0^1 E_n(\cdot + t)\,\mathrm{d}t/(-2).$$

For the record, the function

$$\mathcal{E}_n := E(\cdot - (n+1)/2)/E(-(n+1)/2)$$

is called the **Euler spline** of degree $n$. It has simple knots, at $(n/2) + \mathbb{Z}$, and satisfies $\mathcal{E}_n(m) = (-1)^m$, all $m \in \mathbb{Z}$.

**(2)** A **monospline** of degree $k$ with knot sequence $\mathbf{t}$ is, by definition, any element of $\Pi_k + S_{k,\mathbf{t}}$.

Show that monosplines with simple knots occur naturally as the Peano kernel in a quadrature rule based on function values.

**(3)** With $\mu := \int_0^1$, show that the 1-periodic function $\mathcal{B}_k$ that agrees with $p_k^\mu$ on $[0\,.\,.\,1)$ is a monospline of degree $k$. For the record, $\mathcal{B}_k$ is the **Bernoulli spline** of degree $k$.

**(4)** Following Glaeser (and Schoenberg), a (polynomial) spline is called **perfect** if its highest nontrivial derivative is absolutely constant. For example, the Euler spline $\mathcal{E}_n$ of a previous homework is perfect.

Prove that $M(\cdot|t_0, \ldots, t_k)$ is perfect if $t_j = \cos(k-j)\pi/k$, all $j$.