Liber III

PRINCIPIA MATHEMATICA.

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jacent ad partes puncti S versus A, & sigنا affirmativa terminis SK, SJ, &c. qui jacent ad alteas partes puncti S. Et findis probe observatis, erit RS = a + b + c + d + e + f + g, &c.

Cas. 2. Quod si punctorum H, I, K, L, &c. inaequalia sint inter-

vallia HI, IK, &c. colliga perpendicularorum AH, BI, CK, &c. differentias primas per intervallum perpendicularorum divisas a, b, c, d, &c. divisiones secundas per intervallum bina divisas a, b, c, d, &c. ter-

tias per intervallum terna divisas a, b, c, d, &c. &c. quattuor per intervallum quaterna divisas a, b, c, &c. &c. deinque ad eff, ita ut sit k =

\[ \frac{AH - BI}{HI} : \frac{BI - CK}{IK} : \frac{CK - DL}{KL} \]

et c.

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Inveniis differentiae, dic AH = a, -HS = p, p in -IS = q, q into +SK = r, r into +SM = t; erit ordinatim applicata R =

[Notes: 3 M E E, but E₀ = E₁.

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NEWTON'S MATHEMATICAL PRINCIPLES

Case 2. But if HI, IK, &c., the intervals of the points H, I, K, L, &c., are unequal, take b, 2b, 3b, 4b, 5b, &c., the first differences of the perpendiculars AH, BI, CK, &c., divided by the intervals between those perpendiculars; c, 2c, 3c, 4c, &c., their second differences, divided by the intervals between every two; d, 2d, 3d, &c., their third differences, divided by the intervals between every three; e, 2e, &c., their fourth differences, divided by the intervals between every four, and so forth; that is, in such manner, that b may be =

\[ \frac{AH - BI}{HI} , \frac{1}{2}(b - c) = \frac{BI - CK}{IK} , \frac{1}{3}b = \frac{CK - DL}{KL} , \frac{1}{4}c = \frac{DL - EM}{ML} , \frac{1}{5}d = \frac{EM - FN}{NL} , \frac{1}{6}e = \frac{FN - GO}{OL} \]

&c. And those differences being found, let AH be = a, -HS = p, p into -IS = q, q into +SK = r, r into +SM = t; proceeding in this manner to ME, the last perpendicular but one; and the ordinate RS will be = a + b + c + d + e + f + g, &c.

Cor. Hence the areas of all curves may be nearly found; for if some number of points of the curve to be squared are found, and a parabola be supposed to be drawn through those points, the area of this parabola will be nearly the same with the area of the curvilinear figure proposed to be squared; but the parabola can be always squared geometrically by methods generally known.

LEMMA V

Certain observed places of a comet being given, to find the place of the same at any intermediate given time.

Let HI, IK, KL, LM (in the preceding fig.) represent the times between the observations; HA, IB, KC, LD, ME, five observed longitudes of the comet; and HS the given time between the first observation and the longitude required. Then if a regular curve ABCDE is supposed to be drawn through the points A, B, C, D, E, and the ordinate RS is found out by the preceding Lemma, RS will be the longitude required.

By the same method, from five observed latitudes, we may find the latitude at a given time.

If the differences of the observed longitudes are small, let us say 4 or 5 degrees, then three or four observations will be sufficient to find a new longitude and latitude; but if the differences are greater, as of 10 or 20 degrees, five observations ought to be used.