Publications by Helmut Werner
22.3.1931–22.11.1985

A list compiled and commented by R. Schaback, Göttingen, March 9, 2013, with help from Paul Janßen and Dietrich Braess.

1. The dissertation \[28\] of Helmut Werner in Göttingen 1954 concerned Plateau’s problem of constructing \(n\)-connected surfaces of constant mean curvature \(H\) for a given set of \(n\) rectifiable Jordan boundary curves placed on the unit sphere. Erhard Heinz had considered \(n = 1\) and \(|H| < (\sqrt{17} - 1)/8\), and Helmut Werner generalized this to arbitrary \(n\) and \(|H| < 1/2\). In \[30\], he went back to \(n = 1\) and proved existence without assuming rectifiability, but there is no uniqueness any more.

Throughout the time in Münster and Bonn, Real Analysis and Partial Differential Equations were never out of focus for Helmut Werner, and there was quite some emphasis on these things in the daily life of the institute, in particular after 1970 when Willi Jäger came to Münster. However, besides the report \[32\] and the conference talk \[31\] there are no other publications along this line.

2. After his doctorate, Helmut Werner went into industry and to the Nuclear Research Center in Karlsruhe. The related publications in Physics are \[120\] and \[26\], while \[29\] \[101\] \[113\] \[117\] \[116\] \[114\] \[115\] \[119\] \[118\] are concerned with the efficient and reliable calculation of elementary functions on computers. This was when Approximation Theory came into Helmut Werner’s life, and it came in an extremely application- and computer- oriented way that determined the whole future. Note that there are no papers on orthogonal or Bernstein polynomials, no papers on abstract approximation or Harmonic Analysis, and nothing along the lines of what could be called “Russian” Approximation Theory before 1960. These things were taught to students, but there was absolutely no incentive in Münster or Bonn to do research in these directions.

3. While \[29\] and \[101\] still used polynomials for approximation, Helmut Werner turned to rational approximation very soon, focusing on Chebyshev approximation \[33\] \[35\] \[36\] \[34\] \[37\] \[38\] \[40\] \[41\] \[43\] \[44\] \[48\] \[123\] \[50\] \[49\] \[99\], and starting from the goal of providing efficient approximations to elementary functions as in the papers cited in the previous section. There are no papers on other norms, since they would not yield strict and pointwise reliable error bounds, and many papers are concerned with fast and stable algorithms for computations of rational approximations. This line of research ended around 1968, with nothing more to do.

4. Since rational functions suffer from possible instabilities due to either close–by zeros of the numerator and the denominator or double near–real zeros of the denominator, stability questions are a serious issue, and
they come up in a particularly nasty way when considering interpolation. In a series of papers on interpolation and Padé approximation [54, 73, 77, 81, 84, 85, 88, 127, 18, 89, 93, 95], Helmut Werner investigated these things repeatedly and carefully, his goal being to find an algorithm that handles degeneracies more or less automatically and in a stable way. This subject fascinated him from around 1971 until his early death in 1985, and there were plenty of opportunities to cooperate with the Belgian school which was (and still is) the center of Padé approximation worldwide (e.g. Luc Wuytack and Annie Cuyt).

5. Besides rational functions, other nonlinear families like exponential sums or splines with free knots came into focus in the late sixties. Helmut Werner contributed a rigorous existence proof for best Chebyshev approximations by exponential sums [51, 53] using Ordinary Differential Equations for the crucial compactness argument. Dietrich Braess, who got his Habilitation in Münster in 1968 before he went to Bochum in 1970, pushed this research much further, finalizing it with his book “Nonlinear Approximation Theory” in 1986. Unfortunately, the term “Nonlinear Approximation” later changed its meaning into something related to n-term approximation, sparse representations and dictionaries. These things should better be called “Selective Approximation”.

6. Since Helmut Werner started teaching in Münster, he made sure to supply his students with useful lecture notes. His first lecture “Vorlesung über Approximationstheorie” [45] appeared in 1966 as number 14 of the series “Lecture Notes in Mathematics” of Springer that became very popular later, reaching more than 2000 volumes as of 2013. In the institute, it was called “Telefonbuch” because of its similarity to the Yellow Pages telephone directory and its frequent use. Since it focused on what Helmut Werner considered helpful for research within his group, it provided a perfect starting point for most of his early doctoral students. But also his standard courses were turned into textbooks sooner or later, usually with some help of a doctoral student. The basic second-year course on Numerical Analysis led to a series of books “Praktische Mathematik” published by Springer in two volumes and three editions [52, 121, 122, 83], and a fourth edition in one volume “Numerische Mathematik” [27] that appeared after Helmut Werner’s death in 1985. This series was continued by R. Schaback and H. Wendland to a fifth edition in 2005.

Another lecture note series concerned numerical methods for ordinary differential equations [97, 98]. Helmut Werner’s wife Ingrid, a high school teacher in mathematics, joined him for producing a text intended for students heading for a teacher’s exam. [124, 108, 125, 109]. All of these texts were usually first tested as handouts before they were finalized to be ready for print.

The standard Numerical Analysis lecture for second–year students in the late sixties or early seventies started with a one–week crash course in AL–
GOL or (later) FORTRAN, and there were regular exercises that required programming. But only at a few places, programs could be incorporated into the final texts, due to restrictions by publishers. In this respect, the texts do not reflect the computer-oriented spirit of the actual teaching environment. Helmut Werner would have enjoyed modern hands-on classroom demonstrations of numerical methods with sophisticated graphical output on beamers.

7. Inspired by a bright idea of Helmut Werner, a dissertation in 1969 introduced rational splines as a new research subject into the institute at Münster. This led to many follow-up publications [55, 57, 60, 61, 62, 63, 65, 111, 66, 129, 68, 126, 71, 70, 80, 73, 81, 87, 107] by Helmut Werner and various collaborators. In their simplest form, rational splines generalize cubic splines to $C^2$ piecewise rational functions with linear denominators and quadratic numerators, where the zero of the denominator can vary freely. When applied to generate numerical methods for solving ordinary differential equations, one gets techniques that predict singularities with good accuracy. This is why many of the above papers deal with ordinary differential equations.

8. Univariate integration formulas have error bounds whose Peano kernels are “monosplines”. This research area was very popular in the seventies, and Helmut Werner contributed a series of papers [1, 2, 25, 4, 3] in close cooperation with his friend Henry Loeb of Oregon State University.

9. Surprisingly, Helmut Werner’s publications do not contain papers on multivariate approximation, with the exception of [79]. This is particularly strange since he was very much interested in partial differential equations and finite elements, having frequent guests from these areas giving talks at the institute.

10. Helmut Werner organized many conferences, and consequently there is quite a number of proceedings. Between 1974 and 1981, the standard Oberwolfach conferences on Approximation Theory and Applications usually were organized by Lothar Collatz, Günter Meinardus, and Helmut Werner [9, 13, 14, 15, 16, 10].

Later, stimulated by Helmut Werner’s increasing interest in rational interpolation and Padé approximation, there were two other proceedings [128, 100].

11. Besides teaching at the University, Helmut Werner also cared about curricula and content of high school mathematics [11, 47, 12, 56, 80], in particular in connection with promoting Applied Mathematics in these areas. The majority of teachers in the sixties and seventies usually had no idea about Numerical Analysis beyond the 1D Newton method for root finding, and no idea about the influence of computers on modern mathematics [8]. Therefore Helmut Werner also organized courses for teachers, long before PCs came into general use.
12. From here on, this commented list moves from mathematics to applications. Helmut Werner always understood Mathematics and Computer Science as something that should help mankind directly, and he preferred those parts of mathematics that at least helped to develop useful computer programs.

Now, as of 2013, we have plenty of “Computational” sciences and associated journals, starting from Computational Mathematics and going via Computational Physics to Computational Biology and Bioinformatics. In the sixties, there was no such thing, and there were no centers for “Scientific Computing” or “Zentren für Wissenschaftliches Rechnen”. But Helmut Werner saw his Computing Center in Münster as exactly this.

The following years degraded Computing Centers worldwide to be just service institutions that kept computers running. It took two or more decades until the science administration realized that the influence of computation on the classical branches of science is so strong that one has to build up new centers for computational sciences. It would have been much more innovative if Computing Centers would have been run in Helmut Werner’s way right from their start in the early sixties.

13. In Helmut Werner’s Computing Center in Münster, various cooperations with colleagues in medicine were started. In papers on ophthalmology [20, 105, 106, 23, 22, 112, 21], Helmut Werner developed formulas for artificial lenses that could be corrected after surgery by contact lenses instead of standard glasses. He supplied nomograms that could be easily used by any ophthalmologist worldwide. Other applications concerned an identification problem for injured knees [91], and maturation processes of red blood particles [114]. On the more mathematical side, there is a paper [78] on the Waltman–Hoppenstaedt model for epidemics.

Finally, Helmut Werner published summaries [58, 5, 72, 90, 96] of his contributions to mathematical models in medicine, and there also is a paper on applications in the Humanities [64].

14. The commented list closes with Helmut Werner’s favorite application, the automatic translation of text into Braille. This prize–winning project started already in Hamburg and moved with him to Münster and Bonn. Rolls of punched paper tapes, provided by standard Linotype machines of newspapers or book publishers, came into the Computing Center and were directly fed into a machine that punched a Braille cliché that was then used for printing final Braille output. The rules for translation needed some changes to make automatic translation easier and more reliable. There are many papers and surveys [39, 42, 46, 113, 102, 67, 24, 112, 21, 67, 69, 11] on this in Helmut Werner’s publication list.

15. But there are a few things to be mentioned that did not find their way into publications.
First, Helmut Werner never added his name at the end of a list of co-authors just because he was Head of the institute or the Computing Center. He only appeared in the list if he had made a serious contribution to the publication. Results of PhD theses written under his supervision were usually published as single-authored by the PhD student. But since Helmut Werner readily gave advice to everybody asking for it, the impact of his support is much larger than seen on the surface. His PhD students partially worked on subjects that do not appear in Helmut Werner’s publication list, e.g. Ordinary Differential Equations, matrix eigenvalue problems, and Potential Theory.

Second, Helmut Werner was an early supporter of \TeX{} and \LaTeX{}. For example, he let two students write a \TeX{} extension that could typeset music scores, and he made sure that this was accepted as a thesis.

More generally, he supported everything that was new and promised to be useful, without caring much whether it was Mathematics or Computer Science or a practically useful tool for, say, a medical application. This attitude was rare and it still is. One can only hope that many others will follow Helmut Werner’s standards.

Publications by Helmut Werner


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[113] H. Werner and G. Raymann. An approximation to the Fermi integral 

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