B-splines and Eulerian numbers

Check `pages005.pdf`: "cardinal B-splines" for the fact that the so-called **(shifted) Eulerian number**

$$A(k-1,i) := \sum_{j=0}^{k} (-1)^j \binom{k}{j} (i-j)_+^{k-1}, \quad i = 1,\ldots,k-1,$$

equals $(k-1)!B(i|0,\ldots,k)$, i.e., the value at knot i of the appropriately scaled cardinal B-spline of order k with support $[0\,..\,k]$.

$A(n,i)$ counts the number of permutations of length n with exactly i ascending runs. The **Eulerian number**

$$\left\langle {n \atop i-1} \right\rangle := A(n,i)$$

correspondingly counts the number of permutations of length n with exactly $i-1$ ascents, i.e., reversals in order.

Application of the 'Euler map' $f \mapsto ()^1 Df$ carries $S_r := \sum_{j=1} j^r ()^j$ to S_{r+1}. This can be used to work out that

$$S_r(x) = \frac{x}{(1-x)^{r+1}} \sum_{j=0}^{r-1} ()^j A(r,j+1).$$

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