Extra Credit Opportunity & FREE PIZZA

- Attend a talk today and write a short report to earn up to 2 pts of extra credit toward your homework total points.
- Event: Professors & Pizza - Debugging: Art or Science or Both?
  - Date: Tuesday, 4/10/12
  - Time: 5:00-6:00 pm
  - Room: 1240 CS
  - What: Some say debugging is an art and can't be taught. This has been the fallback explanation for why debugging isn't taught since the dawn of programming. In this talk I will show that the art of debugging is based on science. Science can be taught. Using the concepts presented in this talk should greatly accelerate the maturation of your art.
- URL: http://www.cs.wisc.edu/event/professors-pizza-debugging
Lec 11: Solving Systems of Equations

- Exam 2 Returned Today
  - High Score = 60 (100%)
  - Mean = 49/60 (82%)
  - Low Score = 21/60 (34%)

- Symbolic Computing Today:
  - Maple Shortcuts
  - Solving Equations as a single System (sets)
  - Numerically Solving Systems in Maple
  - Explicit plotting [lists of expressions] versus Implicit plotting [lists of equations]

- Next Lecture: Differentiation and Integration
Homework 6

- Due before 9am next week Thursday, 4/19/12
- Problem 1 is available (though, the actual constant values are subject to change)
- Problem 2&3 will be available by Thursday morning
- Complete examples and exercises for this week's module and practice using greek letters, subscripted variable names, built-in functions, and explicit and implicit plotting.
Maple Shortcuts

- Ctrl-K  *adds an execution group*
- Ctrl-Shift-K  *adds a paragraph (text) group*
- Ctrl-Delete  *removes an execution or paragraph group*
- Execute entire worksheet
- Execute selected execution groups
- Interrupt current operation
Find Symbolic Solutions

- Use the `solve` command to symbolically (algebraically) solve an equation for any variable in the equation.

- When one equation or a set of equations is solved for one or more variables, Maple returns one or more solutions expressions for those variables in terms of the other variables in the equation.

- Not all equation systems can be solved symbolically by Maple.
RootOf is an “intermediate solution”

- If `solve` produces a result that includes a `RootOf` term, try an additional step to find a symbolic solution.

- The `allvalues` command is used to find solutions from `RootOf` results.

```
solset := solve((...) 
solall := allvalues( solset )
```

- If `allvalues` returns `RootOf`, try solving numerically.
Find Numeric Solution

- When Maple is unable to find a symbolic solution, use Maple to find a solution *numerically*.

- To solve numerically:
  - Substitute in values for all known variables.
  - Use `fsolve` to numerically solve the set of equations for the remaining variables.

- Note: Only one solution set is found. Use the third argument to direct Maple to the specific solution you seek.
Maple Demo

\[
\begin{align*}
> \quad e1 & := 3x - 4y^2 + \frac{z^3}{7} = 14 \\
\end{align*}
\]

\[
\begin{align*}
e1 & := 3x - 4y^2 + \frac{1}{7}z^3 = 14 \\
\end{align*}
\]

\[
\begin{align*}
> \quad solForX & := solve(e1, x) \\
\end{align*}
\]

\[
\begin{align*}
solForX & := \frac{4}{3}y^2 - \frac{1}{21}z^3 + \frac{14}{3} \\
\end{align*}
\]

\[
\begin{align*}
> \quad solForY & := solve(e1, y) \\
\end{align*}
\]

\[
\begin{align*}
solForY & := \frac{1}{14}\sqrt{147x + 7z^3 - 686}, \quad -\frac{1}{14}\sqrt{147x + 7z^3 - 686} \\
\end{align*}
\]

\[
\begin{align*}
> \quad solForZ & := solve(e1, z) \\
\end{align*}
\]

\[
\begin{align*}
solForZ & := (98 - 21x + 28y^2)^{1/3}, \quad -\frac{1}{2}(98 - 21x + 28y^2)^{1/3} - \frac{1}{2}1\sqrt[3]{(98 - 21x + 28y^2)^{1/3}}, \quad -\frac{1}{2}(98 - 21x + 28y^2)^{1/3} \\
& + 28y^2)^{1/3} + \frac{1}{2}1\sqrt[3]{(98 - 21x + 28y^2)^{1/3}} \\
\end{align*}
\]
Solving A System (set) of Equations

- Use the `solve` command, a **set** of equations, and a **set** of variables.
- A *symbolic solution result* will be one or more **set**(s) of equations with the variables on the left hand side and the expression for that variable on the right hand side.
  - \{ x=..., y=..., z=... \}, \{ x=..., y=..., z=... \}, ...
- Multiple solution sets are possible and occur when solving non-linear systems of equations.
- Maple can symbolically solve many (not all) linear and non-linear systems of equations.
Solve Two Equations for Two Unknowns

\[ eq1 := y + 10 = -2x^2 + 4x \]
\[ eq2 := y + 2x = 10x^3 + 5 \]

\[ sol := solve\left\{eq1, eq2\right\}, \{x, y\} \]

- In this case, \texttt{solve} returns a solution with terms that include \texttt{RootOf}
Maple Demo

\[
\begin{align*}
> eq1 := & y + 10 = -2 \cdot x^2 + 4 x \\
> eq2 := & y + 2 \cdot x = 10 \cdot x^3 + 5 \\
> sol := & solve(\{eq1, eq2\}, \{x, y\}) \\
> sol & = \{x = RootOf(15 + 2 \cdot Z^2 - 6 \cdot Z + 10 \cdot Z^3, label = _L4), y = -10 - 2 \cdot RootOf(15 + 2 \cdot Z^2 - 6 \cdot Z + 10 \cdot Z^3, label = _L4)\} \\
> solall := & allvalues(sol) \\
> solall & = \begin{cases} 
\frac{x}{60} & = \left(\frac{20798 + 90 \sqrt{52633}}{60}\right)^{1/3} + \frac{46}{15 \left(20798 + 90 \sqrt{52633}\right)^{1/3}} - \frac{1}{15} - \frac{1}{2} \cdot \sqrt{3} \left(\frac{1}{30} \left(20798 + 90 \sqrt{52633}\right)^{1/3}\right) \\
\frac{y}{15} & = \frac{92}{15 \left(20798 + 90 \sqrt{52633}\right)^{1/3}} - \frac{1}{15} - \frac{1}{2} \cdot \sqrt{3} \left(\frac{1}{30} \left(20798 + 90 \sqrt{52633}\right)^{1/3}\right) - 2\left(\frac{1}{60} \left(20798 + 90 \sqrt{52633}\right)^{1/3}\right) \\
\end{cases}
\end{align*}
\]
implicitplot vs (explicit) plot

➢ eq1 := y+10 = -2*x^2→ + 4*x
➢ eq2 := y+2*x = 10*x^3→ + 5

➢ with(plots):
➢ implicitplot([eq1,eq2], x=-2..4, y=-30..10, color=[red,green])

➢ ex1 := solve(eq1,y)
➢ ex2 := solve(eq2,y)
➢ plot([ex1,ex2], x=-2..4, y=-30..10, color=[red,green])
implicitplot vs (explicit) plot
Solve Two Equations for Two Unknowns

\[
eq 1 \quad : \quad y + 10 = -2x^2 + 4x
\]

\[
eq 2 \quad : \quad y + 2x = 10x^3 + 5
\]

\[
sol \quad := \quad \text{solve}\{\eq1,\eq2\},\{x,y\}
\]

\[
solall \quad := \quad \text{allvalues}\(sol\)
\]

\[
\text{evalf}\(\text{solall}\)
\]

\[
> \text{evalf}\(\text{solall}\)
\]

\[
\{x = 0.5987260647 + 0.8455229900 \, \text{i}, \quad y = -6.892223292 + 1.357145350 \, \text{i}, \quad \}, \quad \{x = -1.397452129, \quad y = -19.4955343, \quad \}
\]

\[
\{x = 0.5987260647 - 0.8455229900 \, \text{i}, \quad y = -6.892223292 - 1.357145350 \, \text{i}\}
\]
Accessing parts of a set or list

- Use square brackets: \texttt{sol[i][j][k]}

- Use comma separated list: \texttt{sol[i,j,k]}

- Use comma separated subscript: \texttt{sol_{i,j,k}}
If Maple can't find a symbolic solution, confirm the existence of a solution (by plotting to look for points of intersection), and then find the solution numerically using `fsolve`.

```plaintext
p1 := fsolve({eq1,eq2},{x,y}, x=0..1)
p2 := fsolve({eq1,eq2},{x,y}, x=0..2)
p3 := fsolve({eq1,eq2},{x,y}, x=4..6)
```
Named Sequences

- to be able to refer to more than one of something, create a *sequence* and assign a name.
- do not use ( ), { }, or [ ] notation
- separate items with commas
- use the named sequence in command argument lists, sets, and lists

```plaintext
constants := g = 9.8, L = 2
ranges := x = -1..1, y = -2..4
plot(x^3-4*cos(x), ranges)
```
create sets to treat more than one expression, equation, or range to be used as a single entity when the order does not matter. NO DUPS

- use curly braces { }
- separate items with commas
- use sets with
  - `solve`
  - `fsolve`
  - `subs`
  - `evalf`
create lists to treat more than one expression, equation, or range as a single entity when the order does matter

use square brackets [ ]

separate items with commas

use lists with

- plot
- implicitplot
- plot3d
- implicitplot3d
Multiple variables, exprs, eqns

- Group multiple items as a:
  - sequence
  - \{ set \}
  - \[ list \]

\begin{align*}
eq s & := \text{eqn1, eqn2, eqn3} \\
eq Set & := \{ \text{eqs} \} \\
eq List & := \[ \text{eqs} \]
\end{align*}
Variables with Subscripts: $x_1$

- Use the underscore to indicate that a variable is subscripted: $x_0$ to get $x_0$, $case_i$ to get $case_i$, $theta_0$ to get $theta_0$.

- If any variable with the same name has a subscript, then all variables with that name must also have a subscript value.

- To remove a subscript from a variable name, delete the subscript's value and then also delete the “underscore” that put into subscript mode.
Enter equations
solve

symbolic solution found?

RootOf?

yes
allvalues

no
symbolic solution found?

yes
Use symbolic solution

subs
evalf

no

must solve numerically

or

there is no solution
Find Numeric Solution

Enter equations

subs

fsolve

solution found?

yes

Use numeric solution

yes

are other solutions needed?

no

done

no

with different sub vals?

yes

no
Next Week

- Symbolic Differentiation
- Symbolic Integration