CS/Math 240: Intro to Discrete Math 1/25/2011

Homework 1

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Guidelines:

- This assignment covers propositional and predicate logic.
- Read the problems, and start thinking about them right away. You can discuss the assignment with the instructor, the TAs, and your fellow students in this course. Besides those, the scribe notes, and your own lecture notes, no other sources are allowed.
- You should spell out the solutions on your own. Write your name and student ID on every sheet of paper you turn in.
- The assignment is due on 2/3 at the beginning of class. Model solutions will be made available at that time, so there is no way late assignments can be accepted.
- Good luck!

Questions:

1. Consider the following proposition about integers a, b, and c.

   If a divides bc, then a divides b or a divides c.

   For each proposition below, state (i) whether it is the converse or the contrapositive of the proposition above, or neither, and (ii) whether it is equivalent to the proposition above.

   (a) If a divides b or a divides c, then a divides bc.
   (b) If a does not divide b or a does not divide c, then a does not divide bc.
   (c) a divides bc and a does not divide b and a does not divide c.
   (d) If a does not divide b and a does not divide c, then a does not divide bc.
   (e) If a divides bc and a does not divide c, then a divides b.

   Start by writing each of the compound propositions symbolically in terms of three propositional variables representing elementary propositions. Argue your answers for (ii).

2. Write each of the propositional formulas below into an equivalent form that uses as few variable occurrences as possible.

   (a) $P \wedge (Q \vee P)$
   (b) $(P \Rightarrow Q) \Rightarrow (P \Rightarrow R)$
   (c) $(P \wedge Q \wedge R) \vee (\neg P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge Q \wedge \neg R)$

   Prove the equivalence in each case, and argue minimality for cases (a) and (b).

3. Consider the following propositions about sequences of positive integers $a_0, a_1, a_2, \ldots$.

   P_1: $(\forall i \in \mathbb{N})\,(\exists j \in \mathbb{N})\, j > i \wedge (a_j > a_i \vee a_j < a_i)$

   P_2: $(\forall i \in \mathbb{N})\,((\exists j \in \mathbb{N})\,(j > i \wedge a_j > a_i) \wedge (\exists j \in \mathbb{N})\,(j > i \wedge a_j < a_i))$

   State for each proposition and each of the following sequences whether the proposition holds for the sequence, and justify briefly:

   (a) 1, 2, 3, 2, 3, 4, 3, 4, 5, 4, 5, 6, ...

   (b) 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 1, 1, ...

4. Let D denote the domain consisting of all faculty members and students at the University of Wisconsin, along with the students' parents. Alice and Bob are two individuals in D. Consider the following predicates.

   $F(x)$: x is a faculty member
   $S(x)$: x is a student
   $A(x)$: x attended/will attend the meeting
   $K(x, y)$: x knows y
   $P(x, y)$: x is a parent of y

   Using the above constants and predicates, translate each statement below, i.e., give a natural English sentence that corresponds to each symbolic proposition, and vice versa.

   (a) If all students attended the meeting, then all faculty members attended the meeting.

   (b) $(\exists x)\,(\exists y)\,(S(x) \wedge P(y, x) \wedge \neg A(y))$

   (c) Some student knows everyone who attended the meeting.

   (d) $(\forall x)\ (S(x) \wedge K(\text{Bob}, x) \Rightarrow \neg A(x))$

   (e) No student who attended the meeting knows a faculty member.

   (f) $(\forall x)\ ((F(x) \wedge A(x)) \Rightarrow K(\text{Bob}, x))$

   (g) Alice and Bob attended the meeting, but neither of them knows any student.

   (h) $(\forall x)\,(\forall y)\ ((\exists z)\,(\exists w)\,(S(z) \wedge A(z) \wedge P(x, z) \wedge S(w) \wedge A(w) \wedge P(y, w)) \Rightarrow K(x, y))$

   (i) Alice will attend the meeting, unless she knows no student who will attend the meeting.

   (j) $(\exists x)\ (F(x) \wedge A(x) \wedge K(x, \text{Alice}) \wedge (\forall y)\,(P(y, \text{Alice}) \Rightarrow K(x, y)))$

5. Four witnesses at a trial make the following statements.

   Alice: Either Bob is guilty and Carol innocent, or Bob is innocent and Carol guilty.

   Bob: If Alice or David is innocent, then so is Carol.

   Carol: Either Alice or Bob is guilty, but I am innocent.

   David: Bob is innocent or Carol is guilty if and only if Bob is innocent and Carol is guilty.

   Assume that each person is either innocent or guilty.

   (a) Is it possible that everyone is telling the truth? Explain why or why not.

   (b) If each innocent person always tells the truth, and each guilty person always lies, is it possible to determine who is guilty and who is innocent? If so, how?