

Homework 8

Instructor: Dieter van Melkebeek

Guidelines:

This assignment covers graphs. It is due on 4/14 at the beginning of class. Good luck!

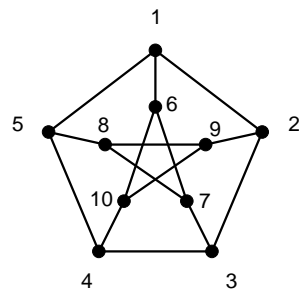
Questions:

1. Show that a graph is bipartite iff it does not contain any cycles of odd length.
2. Let d and n be positive integers. Consider the graph $G = (V, E)$ where V consists of n points equally spaced on a circle and E consists of those pairs of vertices that span an angle of $\frac{d}{n}2\pi$ with the center of the circle.

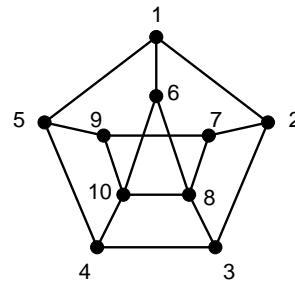
Determine for which values of d and n this graph is connected.

Hint: Recall the property from problem 4(c) of HW 4.

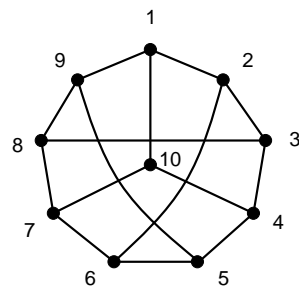
3. Consider the following graphs.



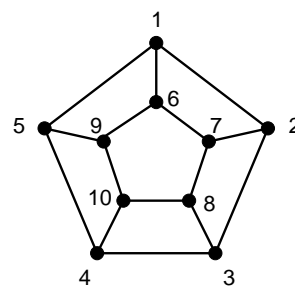
(a) G_1



(b) G_2



(c) G_3



(d) G_4

- (a) Determine which pairs of graphs (G_i, G_j) are isomorphic. Either give an isomorphism, or exhibit a property that is preserved under isomorphisms and that G_i has but G_j does not. As always, you need to argue all your claims.
- (b) Determine which graphs G_i are planar.

4. Show the following properties of simple planar graphs without self-loops.
 - (a) They contain a vertex of degree at most 5.
 - (b) They can be colored with 6 colors.

You can only use results we fully proved in class. In particular, you cannot rely on the four-color theorem.

5. Alice and Bob attend a party with three other married couples. At this party a good deal of handshaking took place, but noone shook hands with their spouse, and noone shook hands with anyone more than once. Before leaving the party, Alice asks the other seven people how many hands they shook. She received a different answer from each of the seven. How many times did Alice shake hands at the party? How about Bob?