CS/Math 240: Intro to Discrete Math 4/21/2011

Homework 10

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Guidelines:

This assignment covers finite automata and regular expressions. It is due on 5/3 at the beginning of class. This is the last assignment you need to turn in. Congratz on finishing the course and good luck!

Questions:

1. Give regular expressions for each of the languages from problem 4 of HW 9.

2. Show that if L is a regular language over the alphabet Σ, then so is the set of all strings in L that have no nontrivial extension in L, i.e., all strings $x \in L$ for which there is no string $y \neq \epsilon$ such that $xy \in L$.

3. For a given language L over an alphabet Σ, consider the equivalence relation R_L from problem 3 of HW 7.

   (a) Show how to construct a (not necessarily finite) automaton M_L that accepts L where the states of M_L are the equivalence classes of R_L.

   (b) Show that for any finite automaton $M = (S, \Sigma, \nu, s_0, A)$ and any two strings $x, y \in \Sigma^*$, if $\nu(s_0, x) = \nu(s_0, y)$ then x and y belong to the same equivalence class of $R_{L(M)}$.

   (c) Conclude that L is regular iff the number of equivalence classes of R_L is finite, and that in the latter case the minimum number of states of any finite automaton accepting L equals the number of equivalence classes of R_L.

   (d) Construct the equivalence classes of R_{D_6}, where D_6 denotes the language over $\{0, 1, 2\}$ defined in problem 4 of HW 9. Also construct a finite automation for D_6 with 6 states. How do these states relate to the states of the automaton M_6 in the model solutions for problem 4 of HW 9?

4. In class we showed how to transform a nondeterministic finite automaton with k states into an equivalent deterministic finite automaton with no more than 2^k states. The goal of this problem is to show that in some cases the exponential blowup in the number of states is inherent.

   Let k be a positive integer. Consider the language L_k consisting of all strings over the alphabet $\{1, 2, \ldots, k\}$ that do not contain every symbol of the alphabet. Prove that any deterministic finite automaton that accepts L_k has at least 2^k states but that there exists a nondeterministic finite automaton with $k + 1$ states that accepts L_k.

5. Given a regular expression R, is it always possible to rewrite it in such a way that all unions are disjoint? In other words, can you always construct a regular expression R' such that $L(R) = L(R')$ and such that for any subexpression of the form $R'_1 \cup R'_2$ in R', the languages $L(R'_1)$ and $L(R'_2)$ are disjoint?

Extra Credit:

Determine the minimum number of states of a finite automaton accepting the language D_k from problem 4 of HW 9 as a function of k.