CS/Math 240: Intro to Discrete Math 3/6/2011

Grading Key for Midterm 1

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Problem 1 (5 points)

- 1 point per subproblem

Problem 2 (5 points)

- 1 point for correctly understanding the statement
- 1 point for the structure of the proof
- 1 point for proving if condition
- 2 points for proving only if condition

Problem 3 (5 points)

- 1 point for the structure of the proof
- 1 point for the base case
- 3 points for the induction step

Problem 4 (5 points)

- 1 point for the correct expression: mn-1
- 1 point for the structure of the proof (strong induction)
- 1 point for the base case
- 2 points for the induction step

Extra Credit Problem (3 points)

- 1 point for the correct answer
- 2 points for the argument

Letter Grades scale

Letter grades for this assignment have been given based on the following scale

A	$\geq$	18
AB	$\geq$	17
B	$\geq$	15
BC	$\geq$	14
C	$\geq$	10
D	$\geq$	7

Common Mistakes

- For problem 1e, several students have given $\forall x \forall y(\neg F(y, x) \Rightarrow G(x))$, which is incorrect because it would mean x would go mad if any person is not friends with him which is not the same as x would go mad if nobody is friends with x.

- Be careful when you place parentheses while converting into symbolic statements. For instance, in problem 1e, $(\forall x)\neg(\exists y)(F(y, x) \Rightarrow G(x))$ is not equivalent to $(\forall x)([\neg(\exists y)(F(y, x)] \Rightarrow G(x))$.

- Although many people correctly identified the number of cuts required in problem 4, several had difficulties getting the induction proof to work.

  Always clearly state what the property $P(N)$ is you're trying to prove by induction and, in particular, what parameter N you're using. Some students took N to mean either the number of rows alone or the number of columns. However, you run into trouble because those parameters individually do not need to decrease after one cut. For example, if you take the number of rows as the parameters, then a vertical cut does not decrease the induction parameter. Instead, one can choose as the parameter the sum of the number of rows and columns (N=m+n), or the number of squares (N=mn). Each of those parameters decreases after a cut.

  When you pick one of those parameters, a cut does decrease it but can decrease it by more than 1. This means you need to use strong induction rather than regular induction on that parameter.

  For the induction step, some students only considered the case where an $m \times n$ bar is split into an $(m-1) \times n$ piece and a $1 \times n$ piece (as well as the corresponding vertical case). Instead, you need to consider all possible cuts for the first step, i.e., breaking the bar horizontally into a $k \times n$ and an $(m-k) \times n$ bar for every possible k in the range $1 \leq k < m$, as well as the corresponding vertical case.

  On the other hand, some people considered every possible *sequence* of cuts for the induction step. This is not needed – it suffices to consider all possibilities for the first step and then use induction. It is a good idea to try all possible sequences for some small examples to get a sense of what is going on, but not for the actual proof by induction, as this wastes the power of the induction scheme.