| CS/Math 240: Intro to Discrete Math | 2/19/2011 |
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## Sample Midterm Exam

Instructor: Dieter van Melkebeek

**Guidelines:**

- This sample gives you an idea what to expect for the first midterm exam. On the actual exam, the questions will be spaced so you can answer them on the sheets you are given. The actual exam will also spell out more guidelines and will include an extra credit problem.

- The solutions to the sample exam will be discussed during the review sessions of 2/21 and 2/22.

**Questions:**

1. Consider the following predicates over the domain of all programs.

    $J(x)$:    $x$ is written in Java
    $P(x)$:    $x$ is written in Python
    $C(x)$:    $x$ is correct
    $S(x, y)$:    $x$ is a subprogram of $y$

    Using the above predicates, translate each statement below, i.e., give a natural English sentence that corresponds to each symbolic statement, and vice versa.

    (a) Some incorrect program is written in Java.

    (b) $\neg(\forall x)\, C(x) \Rightarrow J(x)$

    (c) No Java program is correct.

    (d) $(\forall x)\, \neg J(x) \Leftrightarrow C(x)$

    (e) Only programs written in Python produce incorrect output.

    (f) $((\forall x)\, J(x) \Rightarrow C(x)) \vee ((\forall x)\, J(x) \Rightarrow \neg C(x))$

    (g) If some Java program is correct, then all Java programs are correct.

    (h) $(\forall x)\, (((\forall y)\, S(y, x) \Rightarrow C(y)) \Rightarrow C(x))$

    (i) Subprograms of Python programs aren't written in Java.

2. Show that there do not exist positive integers $a$, $b$, and $c$, such that $4a^3 + 2b^3 = c^3$.

3. The Fibonacci sequence satisfies $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for every integer $n \geq 3$. Show that for every integer $n \geq 1$

$$\sum_{i=1}^{n} iF_i = nF_{n+2} - F_{n+3} + 2.$$

4. Show that $8^n - 3^n$ is divisible by 5 for every integer $n \geq 0$.