CS/Math 240: Intro to Discrete Math 2/23/2011

First Midterm Exam

Instructor: Dieter van Melkebeek

Guidelines:

- **Do NOT turn this page until you have received the signal to start. In the mean time read the instructions below carefully.**
- Fill in your name, student ID, and circle your discussion section below.
- This booklet consists of 6 sheets of paper, containing guidelines, 4 questions, and one extra credit question. When you receive the signal to start, please make sure that your copy of the test is complete.
- Do not separate the pages of this booklet.
- Answer each question directly on this booklet, in the space provided, and use the reverse side of the pages for rough work. If you need more space for one of your solutions, use the reverse side of a page and indicate clearly the part of your work that should be marked.
- In your answers, you may use without proof any result or theorem covered in lectures, lecture notes, discussion sections, or homework, as long as you give a clear statement of the result(s)/theorem(s) you are using. You must justify all other facts required for your solutions.
- Write up your solutions carefully! In particular, use notation and terminology correctly and explain what you are trying to do.
- Good luck!

Identifying Information:

Name:

Student ID:

Section:

301	302	303	304	305
Nilay	Nilay	Roger	Roger	Dalibor
M 9:55-10:45	W 9:55-10:45	M 12:05-12:55	W 12:05-12:55	W 1:20-2:10

For TA use only:

Score: / 20

Grade:

Question 1: [5 points]

Consider the following predicates over the domain of all people.

$A(x)$: x is an artist
$G(x)$: x goes mad
$P(x)$: x is a poet
$M(x)$: x is a mathematician
$F(x, y)$: x is a friend of y

Using the above predicates and d to denote Leonardo da Vinci, translate each statement below, i.e., give a natural English sentence that corresponds to each symbolic statement, and vice versa.

1. Poets and artists do not go mad but some mathematicians do.

2. $(\forall x)\,(P(x) \Rightarrow A(x))$

3. Leonardo da Vinci is an artist and a mathematician, not a poet, but he is the friend of a poet.

4. $(\forall x, y)\,(\neg M(x) \vee \neg F(y, x))$

5. Without friends one goes mad.

Question 2: [5 points]

Consider the domain of the positive integers. Let M_a denote the set of multiples of a, i.e., $M_a = \{a, 2a, 3a, \ldots\}$.

Prove that $(\forall a, b)\ \big(M_a \cap M_b = M_{\gcd(a,b)} \Leftrightarrow a = b\big)$.

Question 3: [5 points]

Show that for every integer $n \geq 1$,

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}.$$

Question 4: [5 points]

You are given a chocolate bar with $m \times n$ squares of chocolate, and your task is to divide it into mn individual squares. You are only allowed to split one piece of chocolate at a time using a vertical or a horizontal break.

For example, suppose that the chocolate bar is 2×2. A horizontal split makes two 1×2 pieces. Each of these pieces requires a vertical split to form single squares. This gives a total of three splits.

Prove that the number of times you split the bar does not depend on the sequence of splits you make.

Extra Credit Question: [3 points]

In a TV show you are presented with two boxes, one of which contains a prize. The host explains that two stagehands Alice and Bob pack the boxes. Alice always puts a true statement on the box, and Bob always puts a false statement on the box. You don't know who packed the boxes, or even if they were both packed by the same person or by different people. The boxes say:

The prize is not here.	Exactly one box was packed by Bob.

Which box do you choose? Why?