

Dynamic Programming

Thursday, October 5, 2017 5:03 PM

Problem A: Train Sorting

- Algorithm: Longest Increasing Subsequence
- Example

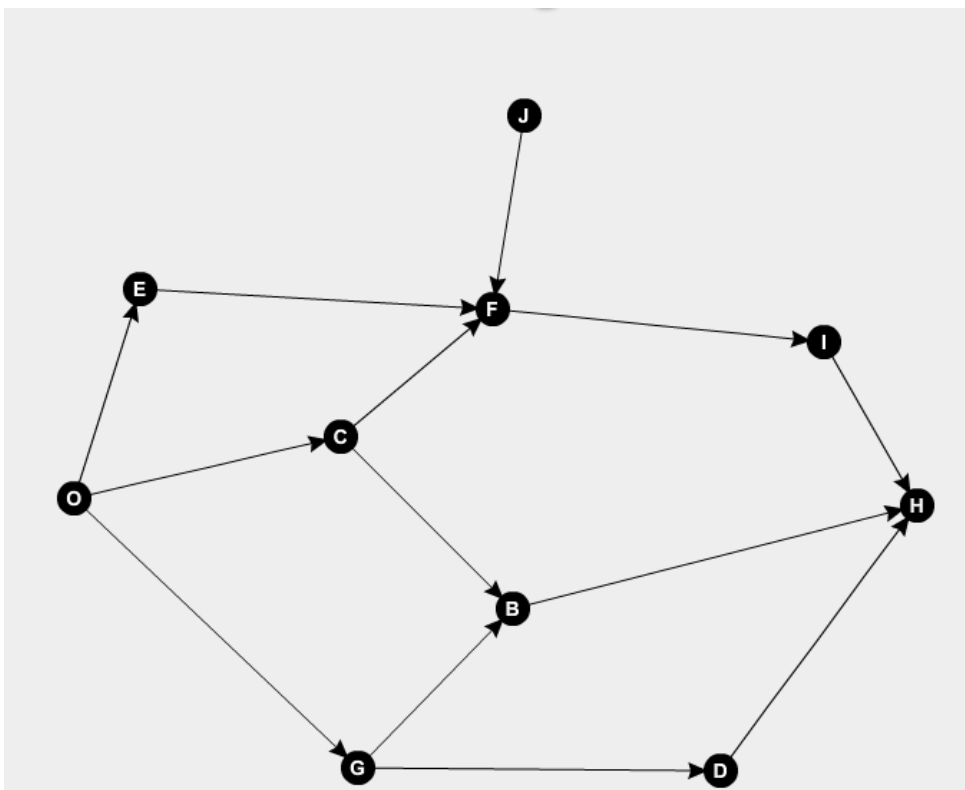
Sequence	1	7	5	4	8	3	9
Longest Count	1	2	2	2	3	2	4

- Steps
 - Calculate the LIS and LDS
 - We want to find $\min(\text{LIS}[i] + \text{LDS}[i] - 1)$ where i is the index of the middle train.

LIS	1	1	1	2	3	4	1
LDS	1	2	2	3	3	3	4
index	0	1	2	3	4	5(i)	6

- In the example above, $i=5$.
- Therefore the answer to this question will be 6
- Reference
 - <http://www.geeksforgeeks.org/longest-increasing-subsequence/>
 - https://en.wikipedia.org/wiki/Longest_increasing_subsequence

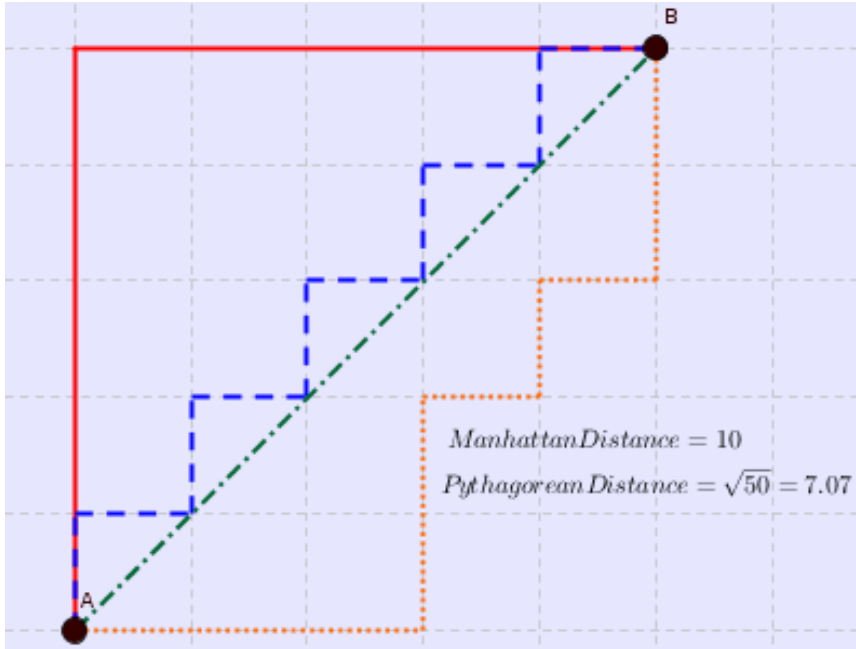
Problem B: A Walk Through The Forest



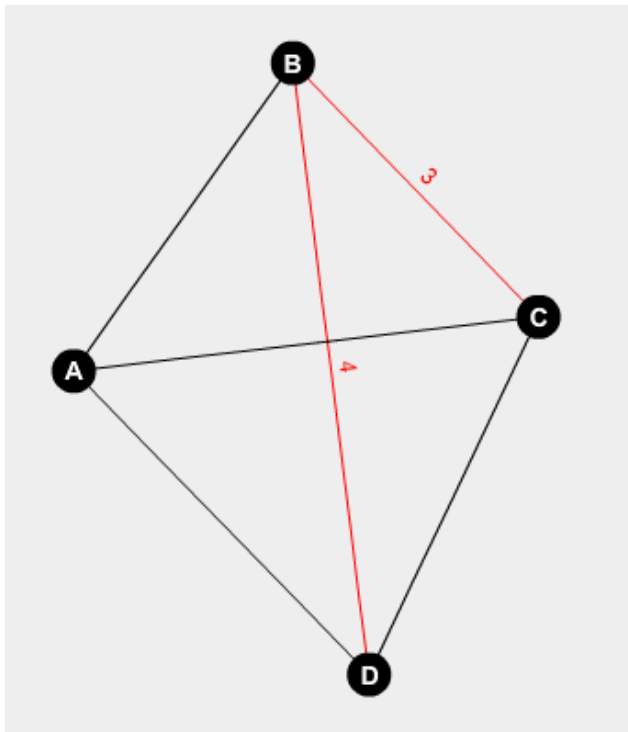
- Optimization: Memoize the recursion

Question C: Collecting Beepers

- Manhattan distance



- Algorithm



	A	B	C	D
{A,B}	4	5		
{A,B,C}			8	
{A,B,D}				9
...				

- Graph (V, E)

- $U \subseteq V, u, v \in U$
- $f(U, u) = \min(f(U \setminus \{v\}, v) + d(v, u))$
- The number of state you can have: $2^{|U|} \cdot |V|$
- Time complexity: $\Theta(2^{|U|} \cdot |V|^2) \ll \Theta(n!)$

• Reference

- https://en.wiktionary.org/wiki/Manhattan_distance

Problem D: Prince and Princess

- Example: (when $n = 3$)

index	0	1	2	3	4	5	6	7
	1	7	5	4	8	3	9	
	1	4	3	5	6	2	8	9

- Let's make an array of size 9

index	0	1	2	3	4	5	6	7	8
	0	6	5	3	2	4	1	4	6

- Transferred array:

0	3	5	2	N/A	N/A	4	6
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• Reference

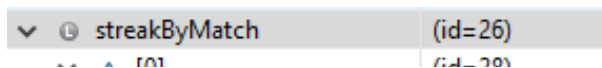
- <http://www.programering.com/a/MjMwMTMwATU.html>

Problem E: Winning Streak

- Random Variable: X_i
- $F_{X_i}(j) = \mathbb{P}(x_i \leq j)$
- We want to see how $f(i, j)$ and $f(i - 1, j)$ is related
- Consider the sequence: $\underbrace{G_1 G_2 \dots}_{i-j-2} \underbrace{L W W W W}_{j+1}$
- $f(i, j) = f(i - 1, j) - (1 - p)p^{j+1}f(i - j - 2, j)$

$i \setminus j$	0	1	2	3
0	1	1	1	1
1	$1 - p$	1	1	1
2		$1 - p^2$	1	1
3			$1 - p^3$	1

- Take $n = 3, p = 0.4$ as example



▼ ▲ [0]	(id=20)
▲ [0]	1.0
▲ [1]	1.0
▲ [2]	1.0
▲ [3]	1.0
▼ ▲ [1]	(id=29)
▲ [0]	0.6
▲ [1]	1.0
▲ [2]	1.0
▲ [3]	1.0
▼ ▲ [2]	(id=30)
▲ [0]	0.36
▲ [1]	0.84
▲ [2]	1.0
▲ [3]	1.0
▼ ▲ [3]	(id=31)
▲ [0]	0.216
▲ [1]	0.744
▲ [2]	0.936
▲ [3]	1.0

- Reference
 - <http://lbv-pc.blogspot.com/2012/06/winning-streak.html>