

CRITICAL GAPS AND FOLLOW-UP HEADWAYS AT CONGESTED ROUNDBABOUTS

Dongxi Zheng

Research Assistant

TOPS Laboratory, Department of Civil and Environment Engineering

University of Wisconsin-Madison

1415 Engineering Drive, Room 1241, Madison, WI, 53706

Phone number: (608)335-0889, E-mail: dzheng3@wisc.edu

Madhav V. Chitturi, PhD

Assistant Researcher

TOPS Laboratory, Department of Civil and Environment Engineering

University of Wisconsin-Madison

B243 Engineering Hall, 1415 Engineering Drive, Madison, WI 53706

Phone number: (608) 890-2439, Fax: (608)262-5199, E-mail: mchitturi@wisc.edu

Andrea R. Bill

Associate Researcher

TOPS Laboratory, Department of Civil and Environment Engineering

University of Wisconsin-Madison

B243 Engineering Hall, 1415 Engineering Drive, Madison, WI 53706

Phone number: (608) 890-3524, Fax: (608)262-5199, E-mail: bill@wisc.edu

David A. Noyce, PhD, PE

Associate Professor

Director, Traffic Operations and Safety (TOPS) Laboratory

University of Wisconsin-Madison

Department of Civil and Environment Engineering

1204 Engineering Hall, 1415 Engineering Drive

Madison, WI 53706

Phone number: (608) 265-1882, Fax: (608)262-5199, E-mail: noyce@engr.wisc.edu

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1 **ABSTRACT**

2 Roundabouts are relatively new to the United States but are being constructed rapidly. Critical
3 gap and follow-up headway are two important factors in determining capacity of roundabouts. .
4 Data were collected at four approaches of three congested roundabouts in Wisconsin to estimate
5 critical gaps and follow-up headways. Congestion was determined by presence of queued
6 vehicles on the approach. . Primary analyses on critical gap and follow-up headway include
7 value comparison 1) between current study and previous findings, 2) between vehicle types, and
8 3) between whether or not exiting vehicles are considered. The analyses results show that 1)
9 critical gap and follow-up headways were typically lower at congested roundabouts, 2) larger
10 vehicles had higher critical gap and follow-up headway values, 3) by considering exiting
11 vehicles, critical gap and follow-up headway estimates decreased significantly,. Critical gap and
12 follow-up headway estimates were slightly more consistent across sites as queue length increased,
13 and the variability of gap acceptance was reduced. The reduced variability in gap acceptance
14 provides a better approximate for the upper bound of driver randomness.

1 INTRODUCTION

2 A modern roundabout is an unsignalized intersection where entering traffic is required to yield to
3 the circulating traffic. The yield-to-circulating rule is one of the main differences between
4 modern roundabouts and traditional traffic circles. While modern roundabouts were first
5 designed in the United Kingdom in the 1960s, their prevalence in the United States (U.S.) did not
6 begin until 1990 (1). A number of research studies have shown that roundabouts are successful
7 in not only reducing the frequency of crashes but also the injury severity of crashes (1-7). Studies
8 have shown that roundabouts operate more efficiently than signalized intersections and
9 conventional unsignalized intersections (two-way stop and four-way stop) (8).

10 According to the 2010 Highway Capacity Manual roundabout capacity depends on
11 critical gap and follow-up headway (ref?). However, limited studies have been performed on
12 operational characteristics of congested roundabouts in the US. (1,9). Additionally, critical gaps
13 and follow-up headways were not reported for different vehicle types. Heavy vehicles are
14 expected to have significantly larger critical gaps and follow-up headways at roundabouts than
15 passenger cars as is the case at other unsignalized intersections (9). Previous research reported
16 that considering exiting vehicles significantly decreased critical gaps at single-lane roundabouts
17 (10). However, no research has studied effect of exiting vehicles on follow-up headways or at
18 multi-lane roundabouts.

19 The primary objective of this research is to study critical gaps and follow-up headways at
20 congested roundabouts. Data were collected at three congested roundabouts in Wisconsin.
21 Results are compared with previously reported values and effects of vehicle type and exiting
22 vehicles on critical gaps and follow-up headways are determined.

23 LITERATURE REVIEW

24 Critical gap and follow-up headway (also termed critical headway and follow-up time in
25 literature, respectively) are key parameters for most of the capacity models of roundabouts. A
26 gap is the time difference between two successive circulating vehicles passing the conflicting
27 line. A critical gap is the minimum gap that an entering driver would use to enter the roundabout.
28 A follow-up headway is the time difference between two successive vehicles entering the
29 roundabout using a same gap, under saturated condition.

30 Many methods have been proposed to estimate critical gap from gap data (11-19). Miller
31 and Brilon, et al. evaluated several critical gap estimation methods and both reported that the
32 maximum likelihood method gives the best results (20, 21). Maximum likelihood method was
33 originally introduced by Miller and Pretty in calculating the critical gap of overtaking behavior
34 (19). Due to the similarity of gap acceptance mode, maximum likelihood method can be applied
35 in many scenarios such as two-way-stop controlled intersections, permissive left turns at
36 signalized intersections, roundabouts, etc. Troutbeck further specified how to use maximum
37 likelihood method to determine critical gaps from traffic movements (22). The maximum
38 likelihood method was adopted by NCHRP 572 as the method of estimating critical gaps (1).

39 NCHRP Report 572 reported critical gaps and follow-up headways based on data
40 collected from 25 approaches of 15 roundabouts (11 single-lane roundabouts and 4 multi-lane
41 roundabouts) (1). Most queues at the 15 roundabouts were one or two minutes in duration,
42 although the maximum continuous queue recorded was 31 minutes. Default values of critical
43 gaps and follow-up headways were suggested in Highway Capacity Manual (HCM) (9). Some

1 recent studies also studied critical gaps and follow-up headway (23, 24). Unfortunately, neither
2 critical gaps nor follow-up headways have been reported by vehicle type.

3 Mereszczak et al. were the first to incorporate the effect of exiting vehicles on critical
4 gaps at single-lane roundabouts (10). When the effect of exiting vehicles is considered, critical
5 gap was found to be smaller than that when the effect of exiting vehicles is not considered.
6 Capacity predictions improved when the effect of exiting vehicles was considered (10). However
7 the effects of exiting vehicles on critical gaps at multi-lane roundabouts and on follow-up
8 headways (single/multi-lane roundabouts) have not been studied.

9 A number of studies have been done to identify factors that influence driver's gap
10 acceptance. Polus, et al. examined the critical gap as a function of average waiting time at
11 roundabouts and found critical gap reduces with increasing waiting time (25). This finding was
12 similar to what has been discovered in scenarios other than roundabouts (26). Mensah, et al. also
13 reported that as people got accustomed to roundabouts, critical gaps decreased (27). Wang, et al.
14 modeled the inconsistency of drivers' gap acceptance behaviors at roundabouts using Cellular
15 Automata and reproduced many features of traffic flow using the model (28). The above findings,
16 together with the randomness of driver behavior and observation errors, could explain why a
17 driver's largest rejected gap was sometimes larger than the accepted gap. Additionally, whether
18 the variability of gap acceptance would change by including exiting vehicles is worth studying.
19

1 DATA COLLECTION

2 Data were collected for four congested approaches at three roundabouts located in the southwest,
3 southeast, and northeast regions of Wisconsin. The driver populations in the three regions are
4 different. Two of the approaches were either single-lane-entering against multi-lane-circulating
5 or multi-lane-entering against single-lane-circulating , and here after referred to as combined
6 approaches; the other two approaches were two lanes for both entering and circulating , named
7 multi-lane approaches. Detailed data collection information is included in Table 1. Data
8 collection periods were chosen on weekdays in the afternoon including the PM peak. During the
9 data collection period there were no incidents or inclement weather at the roundabouts.

10 A typical field setup is illustrated in Figure 1. Video cameras were installed at an
11 upstream location and the intersection corner. The two different camera views were designed to
12 capture four vehicle events: 1) arriving, 2) entering, 3) conflicting, and 4) exiting. The time
13 stamp notations and definitions of the four events are also shown in Figure 1. For an arriving
14 event, the situation could fall into either of the two: 1) the entering vehicle anticipated no
15 conflicts and entered the roundabout with little deceleration or yielding; 2) the entering vehicle
16 either slowed down to a very low speed or stopped in front of the yield bar before the driver
17 found an acceptable gap to enter. In the first situation, the arrival event was identical to the
18 entering event, so were their time stamps. In the second situation, the two events were distinct.

19

20 Congestion times at roundabouts used in this study were identified based on observed
21 queuing. Table 1 shows the number of fully queued minutes for the four approaches and
22 illustrates that the chosen approaches were congested for significant durations. For the multilane
23 approach M1 left lane was fully queued for 268 minutes, right lane for 77 minutes and both lanes
24 for 76 minutes. Similarly for approach M2, left, right and both lanes were fully queued for 107,
25 150 and 91 minutes respectively. At C1 79 fully queued minutes were observed. At C2, only the
26 left lane was fully queued for 33 minutes, while the right lane was not fully queued.

27

28

29 .

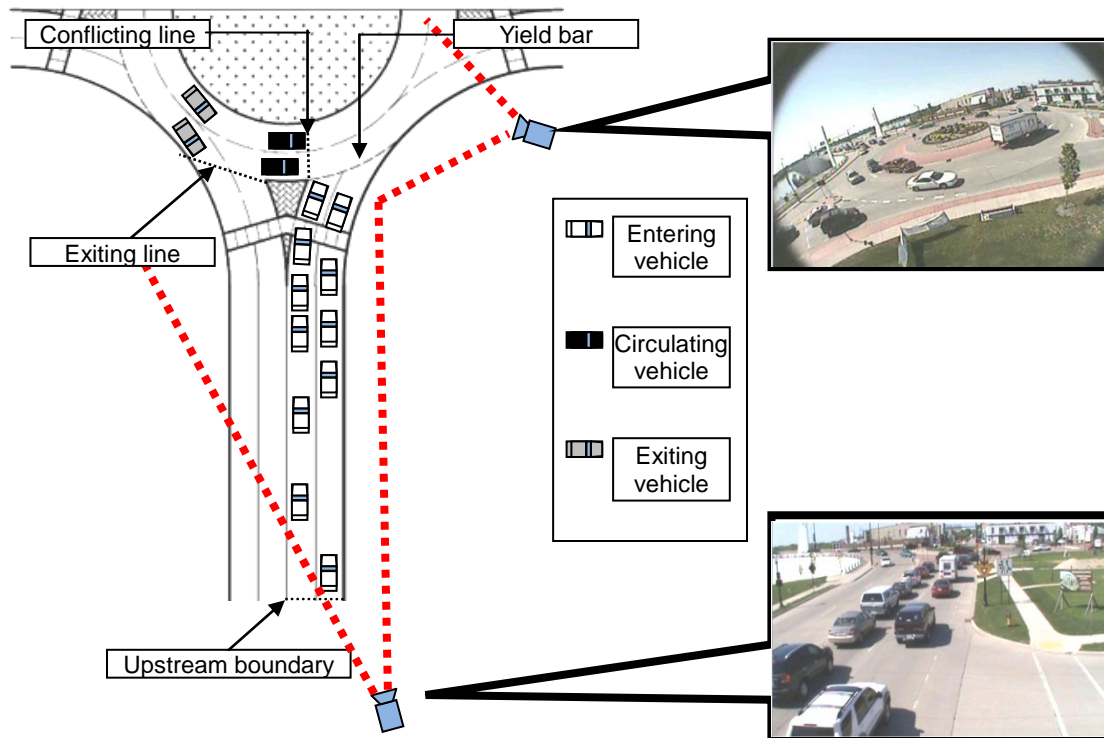
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1 **TABLE 1 Summary of Data Collection Information**

Approach No.	C1	C2	M1	M2
Direction	SB	WB	NB	EB
Roundabout	Canal Street at 25 th Street	STH 78 at CTH ID	STH 32 at STH 57	STH 32 at STH 57
Geometry	1 lane entering 2 lanes circulating	2 lanes entering 1 lane circulating	2 lanes entering 2 lanes circulating	2 lanes entering 2 lanes circulating
Date	April 15 th , 2010 Thursday	April 8 th , 2010 Thursday	May 19 th , 2010 Wednesday	May 19 th , 2010 Wednesday
Time Span	1:50 PM to 6:00 PM	1:20 PM to 5:50 PM	1:20 PM to 6:20 PM	2:50 PM to 6:00 PM
Peak Hour Entering Volume (veh/hr/lane)	554	635	325	1045
Sum of Fully Queuing Minutes	79	33 (left lane**) 0 (right lane**) 0 (both lanes**)	268 (left lane) 77 (right lane) 76 (both lanes)	107 (left lane) 150 (right lane) 91 (both lanes)
Equivalent Travel Time (s) *	1.3 (inner lane) 2.2 (outer lane)	1.7	1.1 (inner lane) 1.9 (outer lane)	1.2 (inner lane) 2.6 (outer lane)

* *Equivalent Travel Time: a parameter for including the exiting vehicles into gap measuring. The equivalent travel time will be explained and defined later in the "Two Gap Measuring Techniques" section.*

** *left lane: either only queued in the left lane or queued in both lanes; right lane: either only queued in the right lane or queued in both lanes; both: queued in both lanes.*



Time Stamp Notation	Event Name	Event Definition
T_{arr}	Arrival	An entering vehicle comes to a stop or reaches the lowest speed in front of the yield bar before it enters the roundabout. If the entering vehicle does not stop, the event refers to the vehicle's front reaches the yield bar and is the same as T_{ent} .
T_{ent}	Entering	The front of an entering vehicle reaches the yield bar, followed instantly by the vehicle getting into the circulating roadway.
T_{con}	Conflicting	The front of a circulating vehicle reaches the conflicting line.
T_{ext}	Exiting	The front of an exiting vehicle reaches the exiting line.

FIGURE 1 A typical field setup.

1
2
3

1 METHODOLOGY

2 Using the videos collected at the roundabouts, time stamps for the four vehicle events were
 3 manually extracted. Also, every entering vehicle was classified into one of three categories:
 4 passenger cars (including sedans, sport/utility vehicles, minivans, vans, and pick-up trucks),
 5 trucks (single-unit trucks, truck tractor-semitrailer combinations, and truck tractors with
 6 semitrailers in combination with full trailers), and motorcycles (29). Additionally, for every
 7 entering vehicle, the number of vehicles waiting behind it in the same lane (queue length) was
 8 also recorded. The queue length information was used to ensure that the vehicles were queued as
 9 well as to study the effect of queue length on critical gap and follow-up headway. For follow-up
 10 headway, the samples were collected only when the vehicles were in queue. Gap and headway
 11 data were computed using the time stamps. Critical gaps and follow-up headways were estimated
 12 based on the gap and headway data, respectively, and categorized by vehicle type.

13 Two Gap Measuring Techniques

14 A gap is defined as the time difference between two consecutive conflicting events (Equation 1).
 15 For the two lane roundabouts, gaps were measured across both lanes, as was done in the NCHRP
 16 572 study.

$$t_g = T_{con}^{(i+1)} - T_{con}^{(i)} \quad (1)$$

Where

t_g = A gap between circulating vehicle i and circulating vehicle $i+1$, second;

$T_{con}^{(i)}$ = Time stamp of the conflicting event of circulating vehicle i , second;

$T_{con}^{(i+1)}$ = Time stamp of the conflicting event of circulating vehicle $i+1$, second.

17

18 Another term usually associated with gap is a lag. A lag is defined as the time difference
 19 between an arrival event and next conflicting event (Equation 2).

20

$$t_l = T_{con} - T_{arr} \quad (2)$$

where

t_l = A lag, second;

T_{con} = Time stamp of the conflicting event of the first circulating vehicle faced by the
 entering vehicle, second;

T_{arr} = Time stamp of the arrival event of the entering vehicle, second.

1 Above definitions of gap and lag do not consider the effect of exiting vehicles. The
 2 technique used by Mereszczak et al. to measure gaps and lags considering the effect of exiting
 3 vehicles is summarized in Equation 3 (10):

4

$$t = (T_{event\ 2} + \Delta t) - T_{event\ 1} \quad (3)$$

where

t = A gap when event 1 is a conflicting event or an exiting event; or a lag when event 1 is an arrival event, seconds;

$T_{event\ 1}$ = The time stamp of a conflicting event, an exiting event, or an arrival event, seconds;

$T_{event\ 2}$ = The time stamp of a conflicting event or an exiting event, seconds;

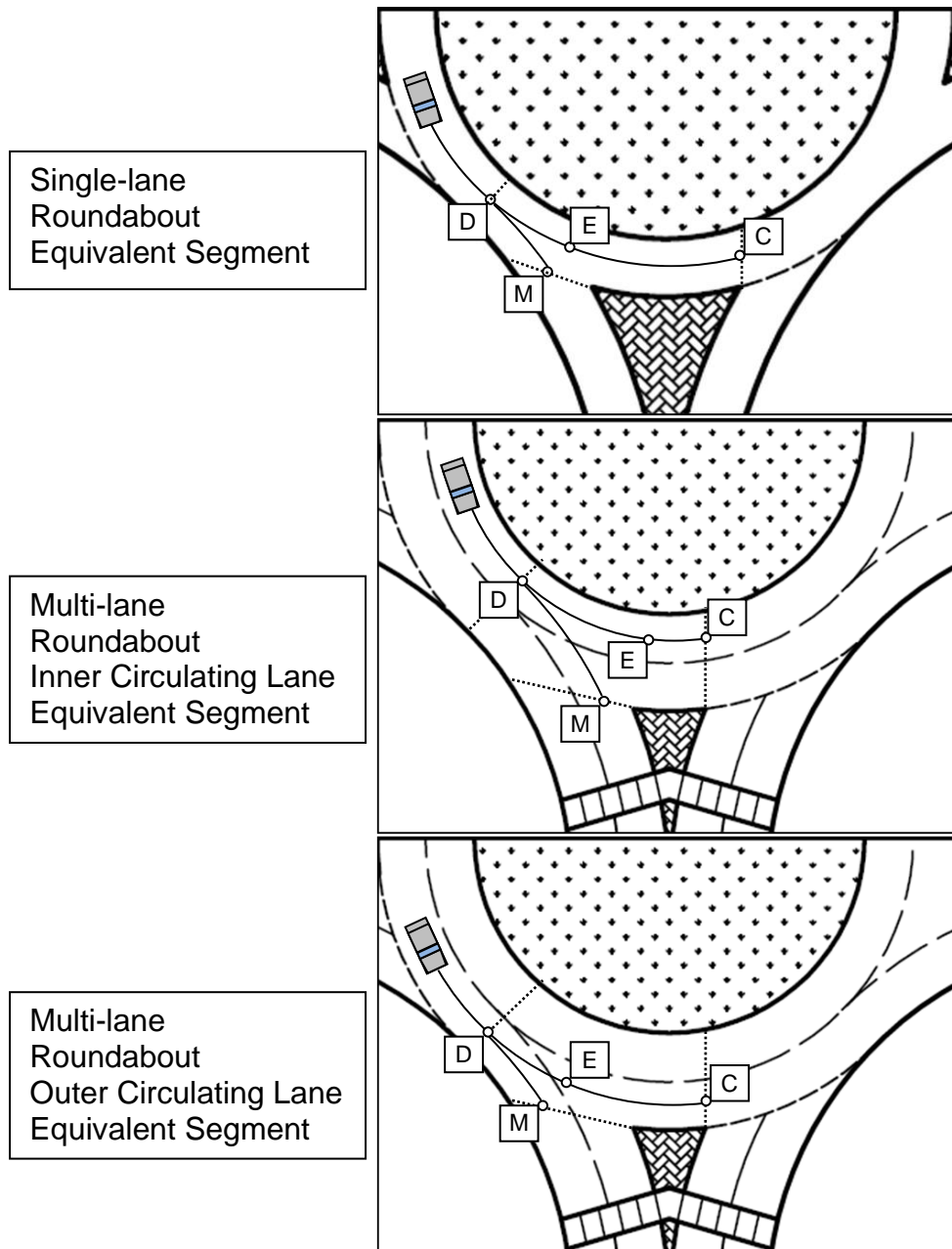
Δt = 0 second, when event 2 is a conflicting event;
 Equivalent travel time (ETT), when event 2 is an exiting event.

5

6 In Equation 3, $T_{event\ 2}$ comes successively after $T_{event\ 1}$. The equivalent travel time (ETT)
 7 assigned to Δt when event 2 is an exiting event was first proposed by Mereszczak et al. as “the
 8 travel time that would have occurred between the exit point and conflict point had the exiting
 9 vehicle remained in the circulating flow”(10). Mereszczak et al. calculated the ETT by dividing
 10 the distance from the middle point of the exiting line to the conflicting line with the average
 11 circulating speed. For the present study, a modified approach was taken to measure ETTs for
 12 both single-lane roundabouts and multi-lane roundabouts. The approach is to equal the ETT to
 13 the sample average travel time of circulating vehicles running through an “equivalent segment”.
 14 The concept of equivalent segment is illustrated in Figure 2 for both single-lane and multi-lane
 15 roundabouts. In Figure 2, Point D is the point from which the circulating center path and the
 16 exiting center path diverge; Point M is the intersection between the exiting line and the exiting
 17 center path; Point E is located such that the length of segment DE (of the circulating center path)
 18 equals the length of segment DM (of the exiting center path); and Point C is the intersection
 19 between the circulating center path and the conflicting line. Since segment DE equals segment
 20 DM, Point E is approximately where an exiting vehicle would have reached if it had continued
 21 circulating. Therefore, segment EC (of the circulating center path) is the equivalent segment of
 22 which the travel time matches the idea of ETT. About 20 vehicles per hour per lane were
 23 sampled to measure ETTs. The ETTs used for the studied approaches are summarized in Table 1.

24 For the current study, two techniques were used for measuring a gap (or a lag). The first
 25 technique did not consider the exiting vehicles, and thus only counts gaps between conflicting
 26 vehicles. The second technique considered the exiting vehicles as virtual conflicting vehicles,

1 and counts gaps between both circulating vehicles and exiting vehicles, by applying an
 2 equivalent travel time when an exiting vehicle is involved.



3
 4 **FIGURE 2 Illustrations of equivalent segments.**
 5

6 **Estimation Methods of Critical Gap and Follow-up Headway**

7 Maximum Likelihood Method (MLM) is the state-of-the-practice to estimate critical gaps (20,
 8 21). In the current study, MLM was used to estimate the mean and the standard deviation of
 9 critical gap, assuming a log-normal distribution for critical gap. Only entering vehicles that
 10 rejected at least one gap (or lag) were used as samples for MLM. Procedures of finding the MLM

1 solutions were based on Troutbeck (22). For follow-up headway, averages and standard
 2 deviations were taken from sample follow-up headways. Only those vehicles that were in queue
 3 were used for computing follow-up headway.

4 RESULTS AND ANALYSES

5 Table 2 summaries the sample sizes, means, and standard deviations of critical gaps (n_c , t_c , and
 6 sd_c , respectively) and of follow-up headways (n_f , t_f , and sd_f , respectively). The sample size n_c for
 7 critical gaps only included vehicles that rejected a lag. The estimates were categorized by vehicle
 8 type for both scenarios of considering and not considering the exiting vehicles. When the sample
 9 sizes of the three vehicle types did not sum up to that of all vehicle types, one bike or one bus
 10 could have appeared and resulted in the minor difference.

11

12 **TABLE 2 Critical Gaps and Follow-Up Headways**

	Not considering exiting vehicles						Considering exiting vehicles					
	n_c	t_c (s)	sd_c (s)	n_f	t_f (s)	sd_f (s)	n_c	t_c (s)	sd_c (s)	n_f	t_f (s)	sd_f (s)
Approach C1												
<i>Passenger Cars</i>	536	5.5	1.9	1203	2.6	1.4	575	4.6	1.2	814	2.3	1.0
<i>Trucks</i>	10	6.3	0.1	5	3.9	2.1	10	5.1	1.5	2	1.9	0.1
<i>Motorcycles</i>	2	4.9	5.5	15	1.6	0.9	2	3.2	2.3	12	1.5	1.0
<i>All</i>	548	5.5	1.9	1223	2.6	1.4	587	4.6	1.2	828	2.3	1.0
Approach C2												
Left lane												
<i>Passenger Cars</i>	236	4.9	1.5	765	4.4	2.5	263	3.8	0.9	459	3.7	2.1
<i>Trucks</i>	3	9.3	1.5	9	6.3	3.6	4	7.2	2.0	5	5.4	3.3
<i>Motorcycles</i>	- ¹	- ¹	- ¹	- ¹	- ¹	- ¹	- ¹	- ¹	- ¹	- ¹	- ¹	- ¹
<i>All</i>	239	5.0	1.7	774	4.4	2.5	267	3.9	1.0	464	3.7	2.1
Right lane												
<i>Passenger Cars</i>	58	4.4	0.4	121	4.9	2.7	64	4.0	0.7	60	4.0	2.4
<i>Trucks</i>	2	2.8	0.4	1	2.36	- ²	2	4.1	0.1	1	2.4	- ²
<i>Motorcycles</i>	- ¹	- ¹	- ¹	1	4.67	- ²	- ¹	- ¹	- ¹	1	4.7	- ²
<i>All</i>	60	4.4	0.4	123	4.9	2.7	66	4.0	0.7	62	4.0	2.3
Approach												
<i>Passenger Cars</i>	294	4.9	1.4	886	4.4	2.5	327	3.9	0.9	519	3.7	2.1
<i>Trucks</i>	5	7.8	3.9	10	5.9	3.7	6	6.0	2.2	6	4.9	3.2
<i>Motorcycles</i>	- ¹	- ¹	- ¹	1	4.67	- ²	- ¹	- ¹	- ¹	1	4.7	- ²
<i>All</i>	299	4.9	1.5	897	4.4	2.5	333	3.9	1.0	526	3.7	2.1
Approach M1												
Left lane												
<i>Passenger Cars</i>	870	4.1	0.9	638	3.1	1.2	787	3.3	0.6	214	2.5	0.9
<i>Trucks</i>	76	5.0	1.1	36	3.7	1.2	73	3.6	0.7	6	3.1	0.9
<i>Motorcycles</i>	20	3.7	0.6	23	2.0	1.4	15	2.9	0.6	12	1.5	1.1
<i>All</i>	966	4.2	1.0	698	3.1	1.3	875	3.3	0.6	233	2.5	1.0
Right lane												
<i>Passenger Cars</i>	638	3.3	1.0	406	3.0	1.2	610	3.0	0.6	125	2.6	1.2
<i>Trucks</i>	20	4.7	2.1	15	3.4	0.8	18	3.6	1.0	2	2.3	0.7
<i>Motorcycles</i>	12	3.4	0.3	4	2.5	1.2	11	3.0	0.3	1	0.7	-
<i>All</i>	670	3.4	1.0	425	3.0	1.2	639	3.1	0.6	128	2.6	1.2
Approach												

	Not considering exiting vehicles						Considering exiting vehicles					
	n_c	$t_c(s)$	$sd_c(s)$	n_f	$t_f(s)$	$sd_f(s)$	n_c	$t_c(s)$	$sd_c(s)$	n_f	$t_f(s)$	$sd_f(s)$
<i>Passenger Cars</i>	1508	3.8	1.1	1044	3.0	1.2	1397	3.2	0.6	339	2.6	1.0
<i>Trucks</i>	96	4.9	1.4	51	3.6	1.1	91	3.6	0.8	8	2.9	0.9
<i>Motorcycles</i>	32	3.6	0.5	27	2.1	1.4	26	3.0	0.5	13	1.4	1.1
<i>All</i>	1636	3.8	1.1	1123	3.0	1.2	1514	3.2	0.6	361	2.5	1.1
Approach M2												
Left lane												
<i>Passenger Cars</i>	332	4.2	1.2	1700	2.9	1.1	402	3.7	0.7	450	2.2	0.7
<i>Trucks</i>	4	7.4	1.0	23	3.1	1.0	5	3.8	0.9	6	2.6	0.6
<i>Motorcycles</i>	7	3.6	0.8	45	2.1	1.3	7	3.4	0.6	19	1.6	0.8
<i>All</i>	343	4.2	1.2	1768	2.8	1.2	414	3.7	0.7	475	2.2	0.7
Right lane												
<i>Passenger Cars</i>	255	3.8	1.1	2087	2.8	1.1	306	3.5	0.8	560	2.3	0.8
<i>Trucks</i>	10	4.8	0.9	61	3.8	1.7	9	3.8	0.5	14	2.5	0.8
<i>Motorcycles</i>	3	5.1	0.4	57	1.9	0.9	4	3.5	1.1	26	1.5	1.0
<i>All</i>	268	3.8	1.2	2206	2.8	1.1	319	3.5	0.8	600	2.2	0.8
Approach												
<i>Passenger Cars</i>	587	4.0	1.2	3787	2.8	1.1	708	3.6	0.8	1010	2.3	0.7
<i>Trucks</i>	14	4.9	0.9	84	3.6	1.5	14	3.8	0.6	20	2.5	0.7
<i>Motorcycles</i>	10	3.6	0.8	103	2.0	1.0	11	3.5	0.6	46	1.6	0.9
<i>All</i>	611	4.0	1.2	3975	2.8	1.1	733	3.6	0.8	1076	2.2	0.8

¹ -: missing data

² -: value not applicable

1 Comparison with Previous Studies

2 A number of U.S. studies have been conducted to estimate critical gaps and follow-up headways
 3 at roundabouts. Among them, NCHRP Report 572 is one of the most comprehensive studies
 4 based on 18 single-lane roundabout approaches and seven multi-lane roundabout approaches (1).
 5 Also, some default critical gap and follow-up headway values are suggested in the HCM (9). In
 6 2011, Schroeder, et al and Wei, et al. presented estimates of critical gaps and follow-up
 7 headways for two congested triple-lane roundabout approaches in Michigan and three single-lane
 8 roundabout approaches in Carmel, IN, respectively (23, 24). Except for the study by Schroeder,
 9 et al in Michigan, all the above findings were not solely focused on congested roundabouts.
 10 Table 3 compares the critical gaps and follow-up headways between the current study and the
 11 above studies, with particular anticipation of the current congested values being lower than
 12 general estimates. Standard deviations are given, if available, inside parentheses.

13 Approaches M1 and M2 (two lanes entering and circulating) had significant congestion
 14 as reflected in the number of fully queued minutes (shown in Table 1). Critical gaps observed at
 15 M1 and M2 are lower than the average critical gaps reported in NCHRP 572 report, but similar
 16 to the minimum critical gaps observed in the NCHRP 572 study. The critical gaps found in this
 17 research are lower than the critical gaps reported from the two Michigan roundabouts, possibly
 18 because the roundabouts in the Michigan study were not congested. Follow-up headways at M1
 19 and M2 are similar to values reported in the NCHRP 572 study and slightly smaller than values
 20 reported in Michigan study.

21 Critical gaps at approach C2 (two lanes entering and one lane circulating) were slightly
 22 higher than the average but within the range of values reported in the NCHRP 572 study. Follow-
 23 up headways at C2 were considerably higher than the average values reported in NCHRP 572
 24 study. The higher critical gaps and follow-up headways could be due to limited congestion as
 25 well as an upgrade of four percent on the approach to the roundabout.

26 Critical gap observed at C1 was higher than the average but within the range of critical
 27 gap values reported in NCHRP 572 study for a single lane approach. Also the critical gap at C1
 28 (5.5 s) is considerably higher than the critical gaps reported in the Carmel study (3.79 and 3.39 s).
 29 Possible reasons could be higher congestion as well as better driver familiarity with roundabouts
 30 in Carmel which has over 60 roundabouts currently. The follow-up headway at C1 was same as
 31 the lower bound reported in NCHRP 572 study but higher than the values reported in Carmel
 32 study.

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34

35

36

37 Table 3 COMPARISON WITH PREVIOUS FINDINGS

	Single entering lane		Multiple entering lanes	
Critical gap (s)				
<i>Current study**</i>	<i>C1 (1e-2c*)</i>	5.5 (1.9)	<i>C2 Left</i>	5.0 (1.7)
			<i>(2e-1c*)Right</i>	4.4 (0.4)
			<i>Both</i>	4.9 (1.5)

			<i>M1 Left</i>	4.2 (1.0)
			<i>(2e-2c*)Right</i>	3.4 (1.0)
			<i>Both</i>	3.8 (1.1)
			<i>M2 Left</i>	4.2 (1.2)
			<i>(2e-2c*)Right</i>	3.8 (1.2)
			<i>Both</i>	4.0 (1.2)
<i>NCHRP 572</i>	<i>Average</i>	5.0 (1.2)	<i>Left Average</i>	4.8 (2.1)
<i>(Method 2)</i>	<i>Range</i>	4.2 – 5.9	<i>Range</i>	4.2 – 5.5
			<i>Right Average</i>	4.3 (1.5)
			<i>Range</i>	3.4 – 4.9
			<i>Both Average</i>	4.5 (1.7)
<i>HCM 2010</i>	<i>1e-1c*</i>	5.19	<i>2e-1c*</i>	5.19
	<i>1e-2c*</i>	4.11	<i>2e-2c*</i>	4.29
<i>Carmel study</i>	<i>Site A - WB</i>	3.79 (0.59)		
	<i>Site B - SB</i>	-		
	<i>Site B - EB</i>	3.39 (0.49)		
<i>Michigan study**</i>			<i>Site 1</i>	4.58
			<i>Site 2</i>	5.41
			<i>Combined</i>	4.66
Follow-up headways (s)				
<i>Current study**</i>	<i>C1(1e-2c*)</i>	2.6 (1.4)	<i>C2 Left</i>	4.4 (2.5)
			<i>(2e-1c*)Right</i>	4.9 (2.7)
			<i>Both</i>	4.4 (2.5)
			<i>M1 Left</i>	3.1 (1.3)
			<i>(2e-2c*)Right</i>	3.0 (1.2)
			<i>Both</i>	3.0 (1.2)
			<i>M2 Left</i>	2.8 (1.2)
			<i>(2e-2c*)Right</i>	2.8 (1.1)
			<i>Both</i>	2.8 (1.1)
<i>NCHRP 572</i>	<i>Average</i>	3.4 (1.2)	<i>Left Average</i>	3.1 (1.4)
<i>(Method 2)</i>	<i>Range</i>	2.6 – 3.7	<i>Range</i>	2.9 – 5.0
			<i>Right Average</i>	3.0 (1.2)
			<i>Range</i>	2.8 – 4.4
			<i>Both Average</i>	3.1 (1.3)
<i>HCM 2010</i>	<i>Default</i>	3.2	<i>Default</i>	3.2
<i>Carmel study</i>	<i>Site A - WB</i>	2.43 (0.63)		
	<i>Site B - SB</i>	2.10 (0.66)		
	<i>Site B - EB</i>	-		
<i>Michigan study**</i>			<i>Site 1</i>	3.37 (1.36)
			<i>Site 2</i>	3.27 (1.40)
			<i>Combined</i>	3.34 (1.37)

* *1e (2e) - 1c (2c): one (or two) entering lane(s) against one (or two) circulating lane(s);*

** : *studies on congested roundabouts*

- : *data not available*

1 Differences among Vehicle Types

- 2 In HCM, for a minor stream of a two-way stop-control intersection, the critical gap and the
- 3 follow-up headway are calculated in the form of a base value plus one or more adjustments (9).
- 4 The proportion of heavy vehicles is considered one of the adjustments for both critical gap and
- 5 follow-up headway (9). However, for roundabouts, the differences in critical gaps and follow-up

1 headways among vehicle types have not been examined yet. In the current study, critical gaps
 2 and follow-up headways were estimated for three vehicle types. The comparisons between
 3 vehicle types are summarized in Table 4, where PC stands for passenger car, TK stands for truck,
 4 and MC stands for motorcycle.

5 For M1 and M2, a consistent decreasing trend was observed from trucks to motorcycles.
 6 When the exiting vehicles were not considered, the trucks were 0.9 – 1.1 seconds higher than
 7 passenger cars in critical gap and 0.6 – 0.8 seconds higher in follow-up headway. The passenger
 8 cars were 0.2 – 0.4 seconds higher than motorcycles in critical gap and 0.8 – 0.9 seconds higher
 9 in follow-up headway. When the exiting vehicles were considered, the differences generally
 10 reduced. The trucks were 0.2 – 0.4 seconds higher than passenger cars in critical gap and 0.2 –
 11 0.3 seconds higher in follow-up headway. The passenger cars were 0.1 – 0.2 seconds higher than
 12 motorcycles in critical gap and 0.7 – 1.2 seconds higher in follow-up headway.

13 For C1 and C2, the sample sizes were relatively small and the trends from trucks to
 14 motorcycles were less consistent. However, without considering the estimates based on small
 15 samples (* values in Table 4), the trucks still had highest critical gaps and follow-up headways
 16 than passenger cars, and the passenger cars were higher than motorcycles in follow-up headway.

17

18 **TABLE 4 Comparison of Critical Gaps and Follow-up Headways among Vehicle Types**

Approach	Actual Values (s)					
	Not Considering Exiting Vehicles			Considering Exiting Vehicles		
	TK	PC	MC	TK	PC	MC
Approach based critical gap (s)						
<i>C1</i>	6.3	5.5	4.9*	5.1	4.6	3.2*
<i>C2</i>	7.8*	4.8	-	6.0*	3.9	-
<i>M1</i>	4.9	3.8	3.6	3.6	3.2	3.0
<i>M2</i>	4.9	4.0	3.6	3.8	3.6	3.5
Approach based follow-up headway (s)						
<i>C1</i>	3.9*	2.6	1.6	1.9*	2.3	1.5
<i>C2</i>	5.9	4.4	4.7*	4.9*	3.7	4.7*
<i>M1</i>	3.6	3.0	2.1	2.9	2.6	1.4
<i>M2</i>	3.6	2.8	2.0	2.5	2.3	1.6

- : data not available

* : estimates based on no more than ten samples

19

1 **Effects of Considering Exiting Vehicles**

2 Consideration of exiting vehicles changes the samples for gaps. With a new sample of gaps, the
3 estimated critical gap is subject to change as well. By considering the influence of exiting
4 vehicles, two successively entering vehicles that used the same circulating gap might be using
5 different gaps. As a result, the headway between two such successively entering vehicles is no
6 longer a follow-up headway sample. Such change in follow-up headway samples could change
7 the average follow-up headway. Further, since critical gap and follow-up headway might change
8 with considering exiting vehicles, so might their trends with certain factors, such as queue length.
9 More fundamentally, the change in gaps could also affect the observation of drivers' gap
10 acceptance behavior.

11 ***Reduction of Estimates***

12 In the research by Mereszczak, et al., a significant average reduction of 1.0 seconds in critical
13 gap was found at single-lane roundabouts when exiting vehicles were considered (10). In the
14 current study, effects of exiting vehicles was considered on critical gap and follow-up headway
15 and at single-lane and multi-lane entry roundabout approaches.

16 Comparison results are shown in Table 2. Standard t-test was used for testing significance.
17 For critical gaps, since log-normal distribution is assumed, a transformation to normal
18 distribution is needed before applying t-test. All reductions were found to be significant at the
19 0.05 level. For single entry lane approach C1, the reduction of critical gap is 0.9 seconds, close to
20 what Mereszczak et al. found for single-lane roundabouts (10). Additionally, the follow-up
21 headway was reduced by 0.3 seconds. For multi entry lane approaches (C2, M1, and M2), the
22 reductions of critical gap range from 0.5 seconds to 0.9 seconds in left lanes, 0.3 seconds to 0.4
23 seconds in right lanes, and 0.4 seconds to 1.0 seconds in both lanes. The reductions of follow-up
24 headway range from 0.6 seconds to 0.7 seconds in the left lanes, 0.4 seconds to 1.0 seconds in
25 right lanes, and 0.5 seconds to 0.7 seconds in both lanes.

1 **TABLE 5 Primary Effects of Considering the Exiting Vehicles**

Approach	Not Considering Exiting Vehicles			Considering Exiting Vehicles			Reduction(s)	p	Significant Difference? $\alpha = 0.05$	
	Samples	Mean (s)	Std. Dev (s)	Samples	Mean (s)	Std. Dev (s)				
Critical gap										
C1	548	5.5	1.9	587	4.6	1.2	0.9	$< 10^{-67}$	Yes	
C2	<i>Left</i>	239	5.0	1.7	267	3.9	1.0	1.1	$< 10^{-48}$	Yes
	<i>Right</i>	60	4.4	0.4	66	4.0	0.7	0.4	$< 10^{-5}$	Yes
	<i>Both</i>	299	4.9	1.5	333	3.9	1.0	1.0	$< 10^{-55}$	Yes
M1	<i>Left</i>	966	4.2	1.0	875	3.3	0.6	0.9	$< 10^{-191}$	Yes
	<i>Right</i>	670	3.4	1.0	639	3.1	0.6	0.3	$< 10^{-15}$	Yes
	<i>Both</i>	1636	3.8	1.1	1514	3.2	0.6	0.6	$< 10^{-145}$	Yes
M2	<i>Left</i>	343	4.2	1.2	414	3.7	0.7	0.5	$< 10^{-22}$	Yes
	<i>Right</i>	268	3.8	1.2	319	3.5	0.8	0.3	$< 10^{-6}$	Yes
	<i>Both</i>	611	4.0	1.2	733	3.6	0.8	0.4	$< 10^{-25}$	Yes
Follow-up headway										
C1	1223	2.6	1.4	828	2.3	1.0	0.3	$< 10^{-18}$	Yes	
C2	<i>Left</i>	774	4.4	2.5	464	3.7	2.1	0.7	$< 10^{-6}$	Yes
	<i>Right</i>	123	4.9	2.7	62	3.9	2.3	1.0	$< 10^{-1}$	Yes
	<i>Both</i>	897	4.4	2.5	526	3.7	2.1	0.7	$< 10^{-7}$	Yes
M1	<i>Left</i>	698	3.1	1.3	233	2.5	1.0	0.6	$< 10^{-32}$	Yes
	<i>Right</i>	425	3.0	1.2	128	2.6	1.2	0.4	$< 10^{-10}$	Yes
	<i>Both</i>	1123	3.0	1.2	361	2.5	1.1	0.5	$< 10^{-39}$	Yes
M2	<i>Left</i>	1768	2.8	1.2	475	2.2	0.7	0.6	$< 10^{-71}$	Yes
	<i>Right</i>	2206	2.8	1.1	600	2.2	0.8	0.6	$< 10^{-102}$	Yes
	<i>Both</i>	3975	2.8	1.1	1076	2.2	0.8	0.6	$< 10^{-184}$	Yes

2

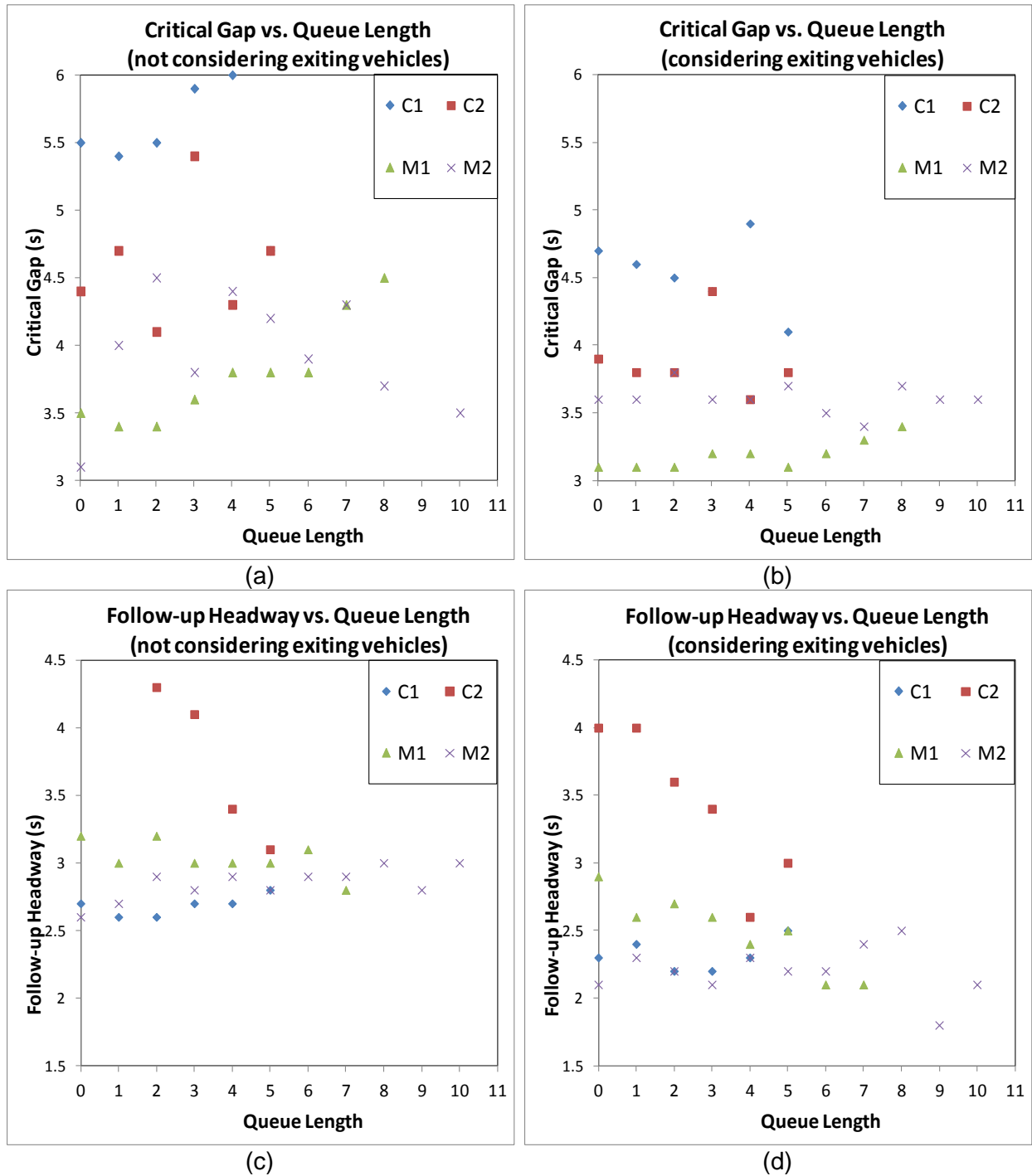
3 ***Effect of Queue Length***

4 Previous studies have shown that longer waiting times result in shorter critical gaps (25, 26).
5 Whether or not critical gaps and follow-up headways would change with the queue length behind
6 the vehicle is worth studying. As a result, critical gaps and follow-up headways from the four
7 studied approaches were plotted against queue length behind the vehicle, under both scenarios of
8 considering and not considering exiting vehicles (Figure 3). For critical gaps, the queue lengths
9 refer to the numbers of vehicles behind the entering vehicles. For follow-up headways, the queue
10 lengths refer to the numbers of vehicles behind the following vehicles. A minimum sample size
11 requirement of six was chosen for estimating each data point in Figure 3. In fact, most of the
12 sample sizes ranged from 10 to 300. Only eight data points (out of the entire 123 data points) had
13 sample sizes below ten.

14 In Figure 3a, the critical gaps are randomly distributed across the range of 3 to 6 seconds,
15 no particular trend could be found as the queue length increases. When the exiting vehicles were
16 considered (Figure 3b), the bandwidth of critical gaps narrows between 3 and 5 seconds, and as
17 the queue length increases, the critical gaps converge slightly better. In Figure 3c, the follow-up
18 headway of C2 decreases dramatically as the queue length increases, while the follow-up
19 headways of other approaches remain relatively constant within a 1-second bandwidth. When the
20 exiting vehicles were considered (Figure 3d), the decreasing trend of S2 is alleviated, and the
21 follow-up headways of other approaches decrease further but still remain approximately in a 1-
22 second bandwidth, with only a slight decreasing trend. In summary, without considering exiting

1 vehicles, no consistent trend was found for critical gap or follow-up headway as queue length
 2 changed; when exiting vehicles were considered, both critical gap and follow-up headways
 3 tended to be slightly more consistent across roundabout sites as queue length increased.

4



5 **FIGURE 3** Effects of queue length: (a) critical gap vs. queue length without exiting vehicles; (b)
 6 critical gap vs. queue length with exiting vehicles; (c) follow-up headway vs. queue length without
 7 exiting vehicles; (d) follow-up headway vs. queue length with exiting vehicles.

1 **Reducing Variability of Gap Acceptance**

2 In the current study, when exiting vehicles were not considered, 187 drivers were observed with
 3 smaller accepted gaps compared with the largest rejected gaps, which violated the assumption of
 4 MLM and have been called “inconsistent drivers” (22). Several reasons could have resulted in
 5 these observations. First, with certain factors, such as increased waiting time, drivers might tend
 6 to accept smaller gaps and become inconsistent in critical gaps (25-27, 30). Second, randomness
 7 might exist when drivers evaluated gaps. Third, random errors could occur during the data
 8 collection (e.g., time stamp extraction) and lead to imprecise gap measuring. Last, without
 9 considering exiting vehicles, two or more small gaps involving exiting vehicles might be counted
 10 as a large gap but still be rejected. Thus, the below analysis aims to answer to what extent the
 11 variability of gap acceptance could be affected by including exiting vehicles.

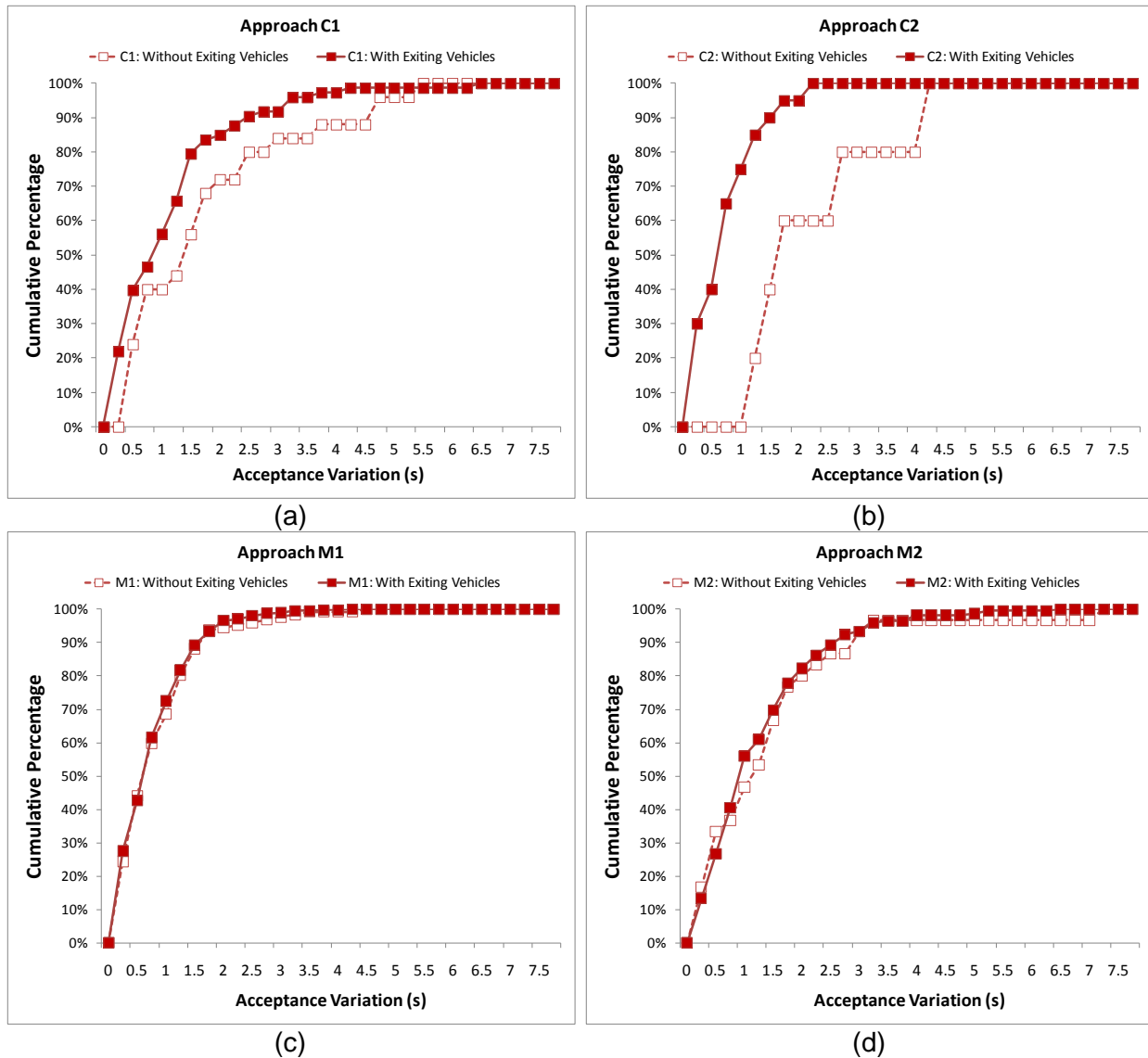
12 For quantitative measurement, acceptance variation is defined as the difference between
 13 accepted gap and largest rejected gap, when the former is smaller than the later. Table 6
 14 summarizes the statistics of acceptance variations. The cumulative distributions of acceptance
 15 variations are shown in Figure 4. The results indicate that when the exiting vehicles were
 16 considered, 1) the range of average acceptance variations dropped from 0.73 – 2.15 seconds to
 17 0.66 – 1.20 seconds and 2) the whole cumulative distribution of acceptance variations shifted to
 18 the smaller side. However, shown by Figure 4, the reduction shift is much larger at approach C2
 19 than at M1 and M2, while that of C1 is in between. The potential explanation is, since M1 and
 20 M2 are multi-lane approaches, gaps were measured across circulating lanes and were already
 21 small. By considering the exiting vehicles, the sample gaps had little to decrease in length; while
 22 on the other hand, C2 is multi-entry against single circulating lane, large gaps were measured
 23 within lane and could be reduced considerably when exiting vehicles were considered. C1,
 24 although has two circulating lane, only showed heavy volume in one circulating lane for most of
 25 the data collection period, thus, resulting in a reduction between C2 and multi-lane approaches.

26 For practical purposes, the average reduced acceptance variation of 0.92 seconds
 27 (considering exiting vehicles) is recommended as an upper bound of driver randomness. Drivers
 28 with acceptance variation smaller than 0.92 seconds could be excluded from “inconsistent
 29 drivers”. Further study is needed to determine if such drivers can be included in the calculation
 30 of critical gap and how the estimate will be affected.

31

32 **TABLE 6 Statistics of Acceptance Variation (in second)**

	Sample Size	Maximum (s)	3 rd Quartile (s)	Median (s)	1 st Quartile (s)	Minimum (s)	Average (s)	Standard Deviation (s)
Not Considering Exiting Vehicles								
<i>C1</i>	25	5.37	2.43	1.30	0.52	0.26	1.69	1.49
<i>C2</i>	5	4.13	2.56	1.53	1.31	1.22	2.15	1.23
<i>M1</i>	127	4.31	1.18	0.60	0.23	0.01	0.73	0.64
<i>M2</i>	30	7.15	1.72	1.17	0.44	0.04	1.37	1.39
Considering Exiting Vehicles								
<i>C1</i>	73	6.43	1.33	0.84	0.28	0.03	1.08	1.12
<i>C2</i>	20	2.08	0.94	0.57	0.16	0.02	0.66	0.55
<i>M1</i>	411	4.24	1.10	0.60	0.23	0.01	0.73	0.64
<i>M2</i>	225	6.42	1.66	0.91	0.48	0.01	1.20	1.04



1 **FIGURE 4 Cumulative distributions of acceptance variations: (a) approach S1; (b) approach S2; (c)**
 2 **approach M1; (d) approach M2.**

3 **CONCLUSIONS AND RECOMMENDATIONS**

4 With the roundabouts being relatively new to the U.S., there is limited knowledge on the
 5 operational characteristics of congested roundabouts. Field data were collected at four
 6 approaches of three congested roundabout in Wisconsin. A roundabout was defined as congested
 7 if at least one approach had persistent queues. The objectives included investigating critical gap
 8 and follow-up headway and how vehicle type, queue length, and exiting vehicles influence those
 9 values needed for modeling capacity. In addition driver inconsistency was also studied.

10 Critical gaps were estimated using the maximum likelihood method assuming a log-
 11 normal distribution. The average critical gaps (for all vehicles) computed at the two approaches
 12 (M1 and M2) that were most congested, were similar to the lower bound values reported in
 13 NCHRP 572, although the standard deviations were considerably lower. Critical gaps observed
 14 at C1 and C2 were higher than the average but within the range of values reported by NCHRP

1 572. Follow-up headways at three approaches (C1, M1, and M2) were similar to values reported
2 in NCHRP 572 study. Follow-up headway at C2 was considerably higher possibly due to limited
3 congestion and presence of an upgrade of four percent on the approach.

4 Vehicle type was found to have considerable effect on average critical gap and follow-up
5 headway. Similar to other intersection types, trucks had the highest values for both critical gap
6 and follow-up headway followed by passenger cars and motorcycles. Results from two
7 approaches with substantial trucks and motorcycles suggest that average critical gap (when
8 exiting vehicles were not considered) of trucks was about 1.0 seconds higher than that of cars
9 while cars were about 0.3 seconds higher than motorcycles. Average follow-up headway (without
10 considering exiting vehicles) followed a similar trend with trucks greater than cars by 0.7
11 seconds and cars greater than motorcycles by 0.8 seconds. The differences in critical gaps and
12 follow-up headways between vehicle types reduced when exiting vehicles were considered.
13 Considering the significant difference in average critical gap and follow-up headway between
14 vehicle types, it is strongly recommended that a weighted average of critical gaps/follow-up
15 headways be used in capacity analysis, especially at locations with significant trucks. Data
16 would need to be collected at more sites to validate critical gap and follow-up headway for
17 specific vehicle types.

18 Considering exiting vehicles was found to reduce average critical gap and follow-up
19 headway significantly. Data showed a decreasing trend in critical gap and follow-up headway
20 (when exiting vehicles are considered) as the number of vehicles queued behind a vehicle
21 increase. The average and range of difference between accepted gap and largest rejected gap for
22 inconsistent drives reduced considerably when exiting vehicles were considered. Further analysis
23 should examine at what level drivers can perceive differences in gaps so that useful data are not
24 lost.

25 This study illustrates the effects of vehicle type, queue length, and exiting vehicles on
26 critical gap and follow-up headway values. Since critical gap and follow-up headway are the
27 main variables in the capacity model, care should be used in understanding what values should
28 be used.

29

30

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