Problem 1 – SVM Learning by the SMO Algorithm (20 points)

You have the small dataset below that involves three features (A, B and C) and the Category (-1 for negative and +1 for positive). Show one update step of the SMO algorithm on this data, assuming the initial alphas and b are 0. Use a Gaussian kernel. Assume \( \gamma = 1 \) and \( C = 10 \). Show all your work.

<table>
<thead>
<tr>
<th>( A )</th>
<th>( B )</th>
<th>( C )</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>+1</td>
</tr>
</tbody>
</table>

\[
K_{ij} = e^{-\frac{-||x_i - x_j||^2}{\gamma}}
\]

\[
K_{11} = K_{22} = e^{-0} = 1
\]

\[
K_{12} = e^{-3} = 0.05
\]

\[ u_1 = 0 \quad u_2 = 0 \]

\[ E_1 = 0 - (-1) = +1 \quad E_2 = 0 - 1 = -1 \]

\[
\alpha_2^{\text{new}} = 0 + \frac{1(1 - (-1))}{1 + 1 - 2(0.05)} = \frac{2}{1.9} = 1.05
\]

\[
\alpha_1^{\text{new}} = 1.05 \quad 0 < 1.05 < 10 \text{ so no clipping.}
\]

\[
b^{\text{new}} = 1 + (-1)(1.05)(1) + (1)(1.05)(0.05) + 0
\]

\[ = 0.0025 \]
Problem 2 – Bayesian network parameter learning and EM (20 points)

Given the Bayes net below, show the result of one cycle of the EM algorithm to update the CPTs using a single data point with A=true, B=true, C=missing, D=false. (Just cross out old values and write in new ones.)

\[
\begin{array}{cccc}
\text{A} & \text{B} & \text{C} & \text{D} \\
\hline
\text{F} & \text{T} & \text{T} & \text{F} \\
\end{array}
\]

\[
P(A) \frac{3}{0.27} \quad <2.8> \\
P(C) \begin{array}{ll}
\text{T} & 0.7 <1.9> \\
\text{F} & 0.2 <2.8> \\
\end{array}
\]

\[
P(D) \begin{array}{cccc}
\text{F} & \text{F} & \text{F} & 0.2 <2.8> \\
\text{F} & \text{F} & \text{T} & 0.1 <1.9> \\
\text{F} & \text{T} & \text{F} & 0.2 <2.8> \\
\text{F} & \text{T} & \text{T} & 0.1 <1.9> \\
\text{T} & \text{F} & \text{F} & 0.5 <2.8> \\
\text{T} & \text{F} & \text{T} & 0.8 <8.2> \\
\text{T} & \text{T} & \text{F} & 0.5 <6.4> \\
\text{T} & \text{T} & \text{T} & 0.5 <6.4> \\
\end{array}
\]

\[
P(A|D) P(C|D) P(D|A,B,C) \propto \begin{pmatrix} (0.2) \times (0.3) \times (0.4) \end{pmatrix} = 0.024 \sim 0.63 \\
\begin{pmatrix} (0.2) \times (0.7) \times (0.1) \end{pmatrix} = 0.014 \sim 0.37 \\
P(A) P(C|A) P(D|A,B,C)
\]

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Problem 3 – Reinforcement Learning (20 points)

Consider the deterministic reinforcement environment drawn below (let $\gamma = 0.1$). The numbers on the arcs indicate the immediate rewards. Assume we learn a Q-table. Also assume all the initial values in your Q table are 0.

\[ Q \]
\[
\begin{array}{c}
A \rightarrow B & 1.5 \\
B \rightarrow C & 3.5 \\
C \rightarrow E & 2.5 \\
E \rightarrow A & 0.5
\end{array}
\]

a) Suppose the learner follows the path $A \rightarrow B \rightarrow C \rightarrow E \rightarrow A$. Using one-step, standard Q learning, show the calculations that produce the new Q table entries and report the final Q table on the graph above.

\[
\begin{array}{c}
A \rightarrow B & 1.5 \\
B \rightarrow C & 3.5 \\
C \rightarrow E & 2.5 \\
E \rightarrow A & 0.5
\end{array}
\]

b) After the learning in part (a), what will be the next two states, in order, visited by the agent if it performs no exploration steps? Show the results of one-step, standard Q learning after the agent takes this path.

\[
\begin{array}{c}
A \rightarrow D \rightarrow C
\end{array}
\]

\[
\begin{array}{c}
A \rightarrow D & 2.5 \\
D \rightarrow C & -2.75
\end{array}
\]
Problem 4 – Computational Learning Theory (20 points)

Consider the problem of PAC-learning 2DNF concepts from $n$ binary variables. 2DNF concepts are disjunctions of conjunctions, where each conjunction consists of at most 2 literals. (Recall that a literal is a binary variable or its negation.) We wish to show that 2DNF is PAC-learnable.

(a) How many 2DNF concepts are there over $n$ binary variables? (Your answer should be a function of $n$, for example $2n+1$ or $3^n$.)

\[
\begin{align*}
\binom{2^n}{2} &= \frac{2^n(2^n-1)}{2} \\
&= \frac{2n\binom{2n-1}{2}}{2}
\end{align*}
\]

plus 2n single-literal conjunctions. Each subset of these results in a 2DNF formula, so \[\binom{2n^2+n}{2}\] OK if you used exactly instead of at most and said $2^{2n^2-n}$

(b) Describe an algorithm for the consistency problem for 2DNF that runs in time polynomial in the number of examples $m$ and number of variables $n$.

Construct a disjunction of all possible conjunctions of at most 2 literals

Delete any conjunction consistent with (satisfied by) a negative example
Problem 5 – Neural Networks (20 points)

Given the neural network below, calculate and show the weight changes that would be made by one step of the online version of backpropagation for the training instance $x=(1,1), y=(1,1)$. Assume that the hidden and output units use sigmoid functions, the network is being trained to minimize squared error, and the learning rate is 0.1 (with no momentum term). Edges from constant 1 on the side indicate bias parameters.

$O_c = \frac{1}{1 + e^{-5}} = 0.5$
$O_b = \frac{1}{1 + e^{-38}} = 0.38$
$O_A = \frac{1}{1 + e^{0.35}} = 0.35$
$O_B = \frac{1}{1 + e^{0.95}} = 0.095$

$\delta_A = (0.35)(1.65)(1.65) = 1.48$
$\delta_B = (0.95)(1.905)(1.905) = 3.778$

$\delta_C = \delta_A \cdot [1 - 2(0.48)(1 - 2(0.778))] = -0.113$
$\delta_B = \delta_A \cdot [1 - 2(0.48)(1 - 2(0.778))] = -0.0018$

$\Delta_{AC} = (1)(1.148)(1.5) = 0.0074$
$\Delta_{AD} = (1)(1.148)(1.38) = 0.0056$
$\Delta_{AB} = (1)(1.148)(1) = 0.148$
$\Delta_{BC} = (1)(1.0778)(1.5) = 0.0039$
$\Delta_{BD} = (1)(1.0778)(1.38) = 0.0029$
$\Delta_{bb} = (1)(1.0778)(1) = 0.0078$

$\Delta_{O_c} = (1)(-0.113)(1) = -0.113$
$\Delta_{O_b} = (1)(-0.0018)(1) = -0.0018$