Lecture 7:
How does a computer... represent information and data?

Big Idea #1:
Universal Computing Device
All computers (given enough time and storage) are capable of computing exactly the same things

Theme 2: How do computers...
... Represent data and information?
... Represent numbers, words, pictures, and movies?
... Act so logically?
... Manipulate and remember data?
... Execute instructions?
... Access data quickly?
... Run multiple programs simultaneously?
... Store data permanently?
... Send messages?
... Find web pages?

From Theory to Practice
Theory: Computer can compute anything that’s possible to compute (given enough time and storage)
Practice: Solving problems involves computing under constraints
• Time
  - weather forecast, next frame of animation, ...
• Cost
  - cell phone, automotive engine controller, ...
• Power
  - cell phone, handheld video game, ...
Big Idea #2: Abstractions Hide Complexity

How do we solve a problem using a computer?
Sequence of transformations between layers of abstraction

Abstractions Hide Complexity

More and more layers...

Example:
Many Choices at Each Level

Tradeoffs: cost, performance, power (etc.)
How do computers... Represent data?

Lowest level: modern computer = electronic machine
- Works by controlling the flow of electrons

Easy to recognize two conditions:
- presence of voltage – state "1"
- absence of voltage – state "0"

More difficult to detect and control analog values

<table>
<thead>
<tr>
<th>Digital Values</th>
<th>&quot;0&quot;</th>
<th>Illegal</th>
<th>&quot;1&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analog Values</td>
<td>0</td>
<td>0.5</td>
<td>2.4</td>
</tr>
</tbody>
</table>

Representing Data

Recognize this photo? Was a vote cast or not?

Modern Computers = Binary Digital Systems

Binary (base two) system:
- has two states: 0 and 1

Basic unit of information is *binary digit, or bit*
- Can be represented in any technology with two states

Bits in Computers

Transistors and wires: electrons flowing or not?

Capacitors and memory: holding a charge or not?

Optical CD-ROMs and DVDs: Reflecting or not?

Hard disk drive: Magnetized north or south?
How Can Bits represent Data?

Everything in computer is represented with 1 and 0
- All text you see or type, movies you watch, music you listen to
- Everything stored on disk, CD, or flash drive
- Everything you send between computers
  - Email, web pages
- Even instructions computer uses to run programs

What kinds of data must bits represent?

Logical: True, False
- Straight-forward: Two states
  - True: 1, False: 0

Numbers
- Signed, unsigned, integers, floating point, complex, rational, irrational, ...

Text
- Characters, words, strings, ...

Images
- Pixels, colors, shapes, movies ...

Sound

Instructions

Unsigned Integers

Approach 1: Non-positional notation
- Represent a number (“5”) w/ string of ones (“11111”)
- Problems?

Unsigned Integers: Weighted Positional Notation

- Position and base determines value of symbol
- Example: Decimal numbers (base-ten)
  - Base-ten implies digit can be one of 10 different symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9
  - Position weight

329 = 3x10^2 + 2x10^1 + 9x10^0

2563 = 2x10^3 + 5x10^2 + 6x10^1 + 3x10^0
Unsigned Integers: Weighted Positional Notation

- Same properties hold for binary numbers (base-two)
- Base-two: Each digit holds two different symbols: 0 or 1

<table>
<thead>
<tr>
<th>Base-10</th>
<th>Base-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>329</td>
<td>101</td>
</tr>
</tbody>
</table>

Most significant

Least significant

$3 \times 100 + 2 \times 10 + 9 \times 1 = 329$

$1 \times 4 + 0 \times 2 + 1 \times 1 = 5$ (decimal)

<table>
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<th>Base-10</th>
<th>Base-2</th>
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<tbody>
<tr>
<td>2563</td>
<td>1011</td>
</tr>
</tbody>
</table>

Most significant

Least significant

$2 \times 1000 + 5 \times 100 + 6 \times 10 + 3 \times 1 = 2563$

$1 \times 8 + 0 \times 4 + 1 \times 2 + 1 \times 1 = 11$ (decimal)

Converting Binary to Decimal

Alternate way to view binary numbers

- What would 01001 be in decimal?
  
  $2^3 = 8$
  $2^2 = 4$
  $2^1 = 2$
  $2^0 = 1$

  $0 + 1 \times 8 + 0 \times 4 + 1 \times 2 + 1 \times 1 = 11$ (decimal)

Converting Decimal to Binary

- How would you make decimal 5 in binary?
  
  $4 + 1 \rightarrow 00101$
  $16 \rightarrow 10000$
  $21 \rightarrow 10101$

- How many different numbers can you make with 5 cards?
  
  $2 \times 2 \times 2 \times 2 \times 2 = 32$

- What is the largest number you can make with 5 cards?
  
  $2^5 - 1 = 31$

- What is the largest number you can make with N cards?
  
  $2^N - 1$

Counting in Binary

<table>
<thead>
<tr>
<th>Binary</th>
<th>Decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>0</td>
</tr>
<tr>
<td>001</td>
<td>1</td>
</tr>
<tr>
<td>010</td>
<td>2</td>
</tr>
<tr>
<td>011</td>
<td>3</td>
</tr>
<tr>
<td>100</td>
<td>4</td>
</tr>
<tr>
<td>101</td>
<td>5</td>
</tr>
<tr>
<td>110</td>
<td>6</td>
</tr>
<tr>
<td>111</td>
<td>7</td>
</tr>
</tbody>
</table>

What do you notice about bits in binary number?
Practice Available with Scratch Game

Decision Tree for 0..31

How many questions needed to find answer between 0 and 31 (32 numbers)?
If 0 represents "no" or "false" and 1 represents "yes" or "true" what do you notice about the answers?

What is the height of this tree? (i.e. how many questions?)
5 questions for 32 numbers; $\log_2(32) = 5$
How many bits are needed to represent number between 0..31?
5 bits

Other Useful Units

Bytes
- Collection of 8 bits: 1101 0011
- How many different values represented in a byte?
  - $2^8 = 256$ values
- Abbreviation: Use B for Bytes vs. b for bits

Hexadecimal numbers
- Base-16
- Why might hexadecimal numbers be useful?
  - Two hex digits per byte
- What characters should we use?
  - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, a, b, c, d, e, f
- 1101 0011 in hex is 0xd3

Today’s Summary

Today’s topics
- All computing devices equivalent given sufficient time and storage
- Abstractions of lower layers hide complexity
- Bits: Two states (on vs. off, true vs. false)
- Represent unsigned numbers with binary numbers
  - N bits can represent $2^N$ different values

Reading
- Pages 130-138 of "Invitation to Computer Science"

Announcements
- Play binary number game
- Homework 3 due Friday