Lecture 28: How can computation... guess what will usually happen?

Motivating Exercise: Monty Hall Problem

Suppose you’re on a game show
You’re given choice of prize behind 1 of 3 closed doors:
• Behind one door is a car
• Behind the other two doors are goats.

You pick a door, say Number 1. The host, who knows what’s behind the doors, opens another door, say Number 3, which has a goat.

He asks, “Do you want to switch to door Number 2?”

Should you switch your choice???

Official (Non-ambiguous) Phrasing

Suppose you’re on a game show and you’re given the choice of three doors. Behind one door is a car; behind the others, goats. The car and the goats were placed randomly behind the doors before the show.

The rules of the game show are as follows:
After you’ve chosen a door, the door remains closed for the time being.
The game show host, Monty Hall, who knows what is behind the doors, now must open one of the two remaining doors, and the door he opens must have a goat behind it.
If both remaining doors have goats behind them, he chooses one randomly.
After Monty Hall opens a door with a goat, he will ask you to decide whether you want to stay with your first choice or to switch to the last remaining door.
Imagine that you chose Door 1 and the host opens Door 3, which has a goat. He then asks you “Do you want to switch to Door Number 2?”

Is it to your advantage to change your choice?
What is your probability of winning if you don’t switch? If you do switch?

Three Approaches to Solving Monty Hall Problem

1. Analyze with probabilities
2. Play game many times with people
3. Simulate with computation
Option 1: Analyze with probabilities

- Chance of winning if keep choice?
  - 1/3 chance of winning
- Chance of winning if switch choice?
  - 2/3 chance of winning

Option 2: Play Game with People

- Two people participate, alternate roles
  - Contestant
  - Game show host
- Simulate behavior of game multiple times
  - Each contestant tries both strategies 10 times
    - Keep vs. switch
    - Record number of times win vs lose w/ each strategy
- Think about algorithm both contestant and host are using

Record Success Rate

<table>
<thead>
<tr>
<th></th>
<th>Don’t Switch</th>
<th>Switch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Win Tally</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>Lose Tally</td>
<td>7</td>
<td>4</td>
</tr>
</tbody>
</table>

Why not best to use people?

- Slow to do many trials
- People bad at picking random numbers
- People give inadvertent clues
- Cards might look different -> can guess car
Option 3: Computer Simulation
As perform more trials, by law of large numbers, percentage win approximates probability of winning.

Repeat Many Trials (Strategy == Switch or Keep)
- Car placed behind random door
- Contestant picks random door
  - If (Contestant door == Car door)
  - Monty opens 1 of 2 other doors at random
- Else (Contestant did not pick car door)
  - Monty opens goat door
- If (Strategy == Switch)
  - Contestant switches choice to closed door
- If (Contestant door == Car door)
  - Increment Win Tally

Monty Hall: Scratch Program
6 trials: expect to win how many times?
- If Strategy == Keep?
  - 2 times
- If Strategy == Switch?
  - 4 times
- What actually happens?

100 trials: expect to win how many times?
- If Strategy == Keep?
  - 33 times
- If Strategy == Switch?
  - 67 times
- What actually happens?

Monty Hall: Better Intuition?
Easier to understand when scale number of doors
Imagine 70 doors
You pick 1 (purple)
Monte Hall opens 68 doors revealing goats (black)
Do you switch or not?
Do you think car is behind 1 you originally picked or 1 he is not showing you??

Probability Simulations in other Domains
Any game of chance: cards, dice, coin flips, luck-based board games

How to measure probability of getting heads or tails?
Coin Flips: Version 1

**HEADS and TAILS**: Constants

Perform multiple **Trials**

For each trial:
- Generate Data
- Evaluate Success

Success/Trials approaches probability with many trials

Coin Flips: Version 2

What is probability of getting all heads?
- As a function of the number of flips?

**Probability of N=4 heads in a row?**
- \( \text{HHHH} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = 1/16 \)
- Probability = \(1/2^4\)

Coin Flips: Version 3

What is the probability of obtaining a continuous string of \(N\) heads in \(M\) coin flips?
- \(N = 3, M = 4\)
- Success? HHTH
- Success? HHHT
Coin Flips: Version 3

HEADS and TAILS: Constants
Perform multiple Trials
Perform M random coin Flips
Measure often get N (Row) heads in a row
- Row Count shows how many
- Reset to 0 on TAILS

Success percentage approximates probability with many trials

Calculation of Pi:
Monte Carlo Simulation

- Calculate pi given ratio of samples falling in unit circle vs square
- Circle area: \( \pi r^2 \)
- Square area: \( 2r \times 2r = 4r^2 \)
- \( \pi = \frac{\text{Hits}}{\text{Trials}} \times 4 \)

Sport Simulations

- Vary win rate (k%) for games
- Repeat large number of trials
- Pick random game winner with given probability
- See who wins

Today’s Summary

- Today’s topic:
  - Computation can be used to simulate behavior of simple systems with random component
  - Measured successes after many trials approximates probability

- Announcements:
  - Homework 6 late turn in: 5pm today (lab 12-2)
  - Homework 7 will be available today (pencil and paper)