Motivating Exercise

Play 20 questions in pairs
- Repeat few times
- Person A thinks of a number between 1 and 100
- Person B guesses number with YES/NO questions
  - Try to ask as few questions as possible!
  - Record how many guesses needed
- Switch roles and play few times more

Repeat with numbers between 1 and 1000
- Handout sheet of number grid may be useful
- Cross off guesses or numbers that secret can’t be

How many guesses on average did it take you?

Best algorithm for searching?

**BINARY SEARCH**

Guess number midway between “lo” and “hi”

(lo starts out at 1, hi at 1000, midway = 500)

Ask “Lower than this midway number?”

If Yes then
  - Set hi = midway - 1
  - Guess number ½ btwn lo and hi (< 250?)
ELSE
  - Set lo = midway
  - Guess number ½ btwn lo and hi (< 750?)
Repeat

Play guessing game again with 1000 numbers – should need 10 or fewer guesses!
How would you implement Binary Search for Key?

Exercise Guessing Game:
What is the secret to be guessed?
- Integer between 1 and 1000 partner is thinking of
- Ask if guess is lower (or higher) than secret

Binary Search for Specified Key:
What is the secret to be guessed?
- Secret is index in List holding key we are looking for
- Guess position in list
- Ask if item at guess is lower (or higher) than key

Different Assumptions for Linear vs. Binary Search?

Binary search assumes list is sorted!
- Does not work unless items in list are in order

Trade-off: For a fast search, should application spend time to sort data or not?
- Will look at sorting algorithms later...

Review: Linear Search

Algorithm checks every element in list (in order) to see if it is the one...

Variables
- Valuable Numbers List
- Key: Input - What we are searching for
- Key Index: Output - Index where we found Key
- index: local variable

Binary Search in Scratch

Goal: Same inputs and outputs to script, but faster

Same Variables
- Valuable Numbers List
- Key: Input - What we are searching for
- Key Index: Output - Index where we found Key
- index: local variable

Faster: Use index to skip around List efficiently

Invariant (condition always holds true)
- lo <= index of Secret key <= hi
- True before loop begins
- True every time after
Running Binary Search: Ex 1

10/19/11

Looking for key 85

<table>
<thead>
<tr>
<th>Loop</th>
<th>Index</th>
<th>Item</th>
<th>Item &gt; Key?</th>
<th>Lo</th>
<th>Hi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td></td>
<td>16</td>
<td>Yes</td>
<td>1</td>
<td>16</td>
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<td>1</td>
<td></td>
<td>9</td>
<td>No</td>
<td>10</td>
<td>16</td>
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<td>No</td>
<td>10</td>
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<td>3</td>
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<td>7</td>
<td>Yes</td>
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<tr>
<td>4</td>
<td></td>
<td>6</td>
<td>=</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

Round: rounds up to nearest integer (hint: round(8.5) = 9)

Looking for key 33

<table>
<thead>
<tr>
<th>Loop</th>
<th>Index</th>
<th>Item</th>
<th>Item &gt; Key?</th>
<th>Lo</th>
<th>Hi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
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<td>16</td>
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<td>16</td>
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<tr>
<td>1</td>
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<td></td>
<td>6</td>
<td>=</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

Running Binary Search: Ex 2

How many guesses to find Key with Linear Search?

How many loops?
N = Elements in List

- Best case (minimum)?
  - 1 loop!
- Worst case (maximum)?
  - N loops
- Average case?
  - N/2 loops
  - O(N)
  - Just like Find Max

Looking for key 34

<table>
<thead>
<tr>
<th>Loop</th>
<th>Index</th>
<th>Item</th>
<th>Item &gt; Key?</th>
<th>Lo</th>
<th>Hi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
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<td>6</td>
<td>=</td>
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<td>6</td>
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</table>

What does Key Index equal when script ends?
Key Index still 0
Can use to signify that key not in List

Running Binary Search: Ex 3

Looking for key 34

<table>
<thead>
<tr>
<th>Loop</th>
<th>Index</th>
<th>Item</th>
<th>Item &gt; Key?</th>
<th>Lo</th>
<th>Hi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
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<td>6</td>
<td>=</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

What does Key Index equal when script ends?
Key Index still 0
Can use to signify that key not in List
How many Guesses Needed?

<table>
<thead>
<tr>
<th>N</th>
<th>16</th>
<th>32</th>
<th>64</th>
<th>128</th>
<th>256</th>
<th>512</th>
<th>1024</th>
<th>2048</th>
<th>4096</th>
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</thead>
<tbody>
<tr>
<td>Linear</td>
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<tr>
<td>Binary</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Decision Tree for 1..16

How many questions needed to find answer between 1 and 16 (16 numbers)?

Only 4 questions for 16 numbers

Decision Tree for 1..32

How many questions needed to find answer between 1 and 32 (32 numbers)?

Double range of numbers covered

Just one more question --> 5 questions

Decision Tree for 1..64

How many questions needed to find answer between 1 and 64 (64 numbers)?

Double again range of numbers covered

6 questions
**Decision Tree for N items**

- How many questions needed for N integers?

  - Each level of tree corresponds to one question
  - How deep (or high) is tree of N integers?
    - 1 question → 2 numbers
    - 2 questions → 4 numbers
    - 3 questions → 8 numbers
    - 4 questions → 16 numbers
    - 5 questions → 32
    - 6 questions → 64
    - Q questions → 2^Q numbers

**Game of 20 Questions**

- How many objects can you choose between with 20 questions?

  - 2^20
  - This is approximately 1 million objects (1,048,576)

**Complexity of Binary Search**

- How many iterations of
- What assumptions does binary search make?

**Check-Up**

- Is it possible for index of secret key to be < lo?

- How many iterations of repeat loop are needed for a list containing 1024 (1K) elements?

  - 1048576 (1024 K) elements?
Announcements

How to efficiently search for element in a List
- $O(N)$ guesses to find using Linear Search
- $O(\log_2 N)$ guesses to find using Binary Search
  (depth of tree)
  - Assumes data is sorted!

Homework 5: Due Friday at 5pm
- Gallery will open Friday morning for Extra Credit submissions; submit by Friday midnight
- Vote over weekend for Round 1
- Can you figure out how Mozart Dice Game works?