Motivating Exercise

Play 20 questions in pairs
- Repeat few times
  - Person A thinks of a number between 1 and 100
  - Person B guesses number with YES/NO questions
  - Record how many guesses needed
- Switch roles and play few times more

Repeat with numbers between 1 and 1000
- Handout sheet of number grid may be useful
- Two scraps of papers for markers

How many guesses on average did it take you?
- What algorithm works well?

Best algorithm for searching?

BINARY SEARCH

Guess number midway between “lo” and “hi”
(Lo starts out at 1, hi at 1000, midway = 500)
Ask “Lower than this midway number?”
If Yes then
  - Guess number between old lo and old midway − 1 (250?)
  - Set hi = midway − 1 and repeat
ELSE
  - Guess number between old midway and old hi (750?)
  - Set lo = midway and repeat
How would you implement Binary Search in Scratch?

Exercise Guessing Game:
What was the secret to be guessed?
Integer between 1 and 1000 partner was thinking of

Binary Search for Specified Key:
What is the secret to be guessed?
Secret is index in List holding key we are looking for

Different Assumptions for Linear vs. Binary Search?

Binary search assumes list is sorted!

Data organization very important
• Can you think of any other organization techniques for helping people look up data in large lists?

Trade-off: Should application pay cost to sort data or not?
• Will look at sorting later...

Review: Linear Search

Algorithm checks every element in list (in order) to see if it is the one...

Variables
• Key: Input - What we are searching for
• Key Index: Output – Index where we found Key
• index: local variable

How many guesses to find Key? (with Linear Search)

How many loops?
N = Elements in List
• Best case (minimum)? – 1 loop!
• Worst case (maximum)? – N loops
• Average case?
  – N/2 loops
• Answer for average: O(N)
  – Just like Find Max
Binary Search in Scratch

Use index to skip around
List efficiently

Invariant (condition always holds true)
lo <= Index of Secret key <= hi
True before loop begins
True every time after

Running Binary Search

Looking for key 85
Not found! Key Index still 0

Running Binary Search

Looking for key 33

Running Binary Search

Looking for key 34
How many Guesses Needed?

<table>
<thead>
<tr>
<th>N</th>
<th>16</th>
<th>32</th>
<th>64</th>
<th>100</th>
<th>250</th>
<th>500</th>
<th>750</th>
<th>1000</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Binary</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Decision Tree for 1..16

How many questions needed to find answer between 1 and 16 (16 numbers)?

Only 4 questions for 16 numbers

Decision Tree for 1..32

How many questions needed to find answer between 1 and 32 (32 numbers)?

Double range of numbers covered

Just one more question --> 5 questions

Decision Tree for 1..64

How many questions needed to find answer between 1 and 64 (64 numbers)?

Double again range of numbers covered

6 questions
How to be a good player of 20 questions?

This is approximately 1 million objects! \(2^{20}\)

Each level of tree corresponds to one question

<table>
<thead>
<tr>
<th>Questions</th>
<th>Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>32</td>
</tr>
<tr>
<td>6</td>
<td>64</td>
</tr>
<tr>
<td>Q</td>
<td>(2^Q) numbers</td>
</tr>
</tbody>
</table>

How many questions needed for \(N\) integers?

Decision Tree for \(N\) items

20 Questions

How many objects can you choose between with 20 questions?

\(2^{20}\) This is approximately 1 million objects! \((1,048,576)\)

How to be a good player of 20 questions?

Binary Search in Scratch

Use index to skip around
List efficiently

<table>
<thead>
<tr>
<th>Operations</th>
<th>How many iterations?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(O(\log_2 N)) guesses to find using Binary Search</td>
<td>(2) if's per loop Stop if item at index = key</td>
</tr>
<tr>
<td>(O(N)) guesses to find using Linear Search</td>
<td>(2) if's per loop Stop if item at index = key</td>
</tr>
</tbody>
</table>

Today’s Summary

Today’s Topics
- How to efficiently search for element in a List
- \(O(N)\) guesses to find using Linear Search
- \(O(\log_2 N)\) guesses to find using Binary Search (depth of tree)
  - Assumes data is sorted!

Reading:
- Invitation pp 55-66 and 80-88 (Searching and complexity)
- "The Pattern on the Stone" Ch 5: Algorithms and Heuristics, pp 77-90

Announcements
- Homework 3 Due Wednesday