Announcements

Homeworks 9 and 10 NOW Available

- HW 9 Due tomorrow 5pm (Tue 11/22)
  - Explore Binary Numbers and Gigapan
- HW 10 Due after Thanksgiving (Wed 11/30)
  - Watch TED talks about TECHNOLOGY and write essay
- Have enough info to complete both

No lecture this Wednesday

Exam 2 graded and returned

UNIVERSITY of WISCONSIN-MADISON
Computer Sciences Department

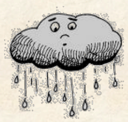
CS 202: Introduction to Computation

Professor Andrea Arpaci-Dusseau

How does a computer… act so logically?

Motivating Example

Ben only rides his bike to class if he overslept, but even then if it is raining he'll walk and show up late (he hates to bike in the rain). But if there's an exam that day he'll bike if he overslept, even in the rain.



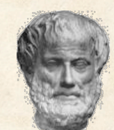
It's raining,

Ben overslept,

and there's an exam

Will Ben bike today????

Propositional Logic: History

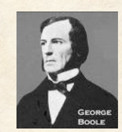


Aristotle

- Law of contradiction
  - ..it will not be possible to be and not to be the same thing
- Law of excluded middle
  - Everything must either be or not be
- Man is mortal, Socrates is a man, therefore, Socrates is mortal

Stoic Philosophers (3rd century BC)

- Basic inference rules
  - If *p* then *q*; not *q*; therefore not p



De Morgan and Boole (19th century)

- Symbolic logic – "automated", "mechanical"

C. Shannon (1930s)

- Proposal to use digital hardware
- 1 = True; 0 = False

Simple Example: Boolean Variables

Ed goes to the party if and only if Stella does

Choose "Boolean variables" for 2 events
Each boolean variable is either TRUE or FALSE

E: Ed goes to party
S: Stella goes to party

Relationship between E and S?

E = S

Simple Example: Logical OR

Ed goes to the party if and only if Dan goes or Stella goes

Choose "Boolean variables" for 3 events:

- **E**: Ed goes to party
- **D**: Dan goes to party
- **S**: Stella goes to party

E = D **OR** S

E is TRUE if one <u>or both</u> of D and S are TRUE

WARNING: In English OR has additional meaning!

- Example: You can eat an orange OR an apple
- Use term "Exclusive OR" or "XOR" for this usage

Simple Example: Logical NOT, AND

Ed goes to the party if and only if Dan does not and Stella does.

Choose "Boolean variables" for 3 events:
E: Ed goes to party
D: Dan goes to party
S: Stella goes to party

E = (NOT D) AND S

Alternately: E = $\overline{D}$ AND S

Boolean Expressions

Composed of Boolean variables (True=1, False=0)

Three Basic Operators: **AND**, **OR**, and **NOT**

Examples

- D AND (P OR (NOT Q))
- C OR D OR E

Boolean Algebra Shorthand

A AND B A • B (AB)	A OR B A + B	NOT A $\overline{A}$

Truth Table: AND

Lists the truth value of the Boolean expression for all combinations of values for the input variables.

Boolean Expression

E = D AND S

E = D • S

E = (DS)

Write E for all possible values of input variables D, S.

D	S	E
0	0	0
0	1	0
1	0	0
1	1	1

Truth Table: OR

Boolean Expression

E = D OR S

E = D + S

D	S	E
0	0	0
0	1	1
1	0	1
1	1	1

Truth Table: NOT

$E = \overline{D}$

Write E for all possible values of D

D	E
0	1
1	0

Truth Table Example: (NOT D) AND S

$E = \overline{D}$ AND S

May be easier if show NOT D in as row of Table

D	S	E
0	0	0
0	1	1
1	0	0
1	1	0

Truth Table Example: D OR (NOT S)

Boolean Expression E = D OR $\overline{S}$

What is E?!?

D	S	E
0	0	1
0	1	0
1	0	1
1	1	1

What does Boolean Logic have to do with Computers?

1) Reasoning in many algorithms uses boolean logic

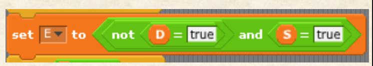
How can one use boolean logic in Scratch programs?

2) Modern computers are themselves built from boolean logic

Basic building block, logic gates, perform boolean logic;

Boolean Logic in Scratch: E = (NOT D) AND S

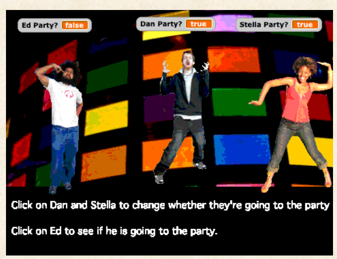
Three Variables: E, D, S



- No real booleans
- Set D and S to true or false
- Value of E depends on D, S

Will Ed go to the party…

- D=False, S=False?
  - No
- D=False, S=True?
  - Yes
- D=True, S=False?
  - No
- D=True, S=True?
  - No



Order of Operations

Are these two expressions the same?

- E = (NOT D) AND S
- E = NOT (D AND S)

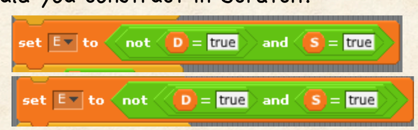
In first expression, when does Ed go to party?

- D does not go AND S goes

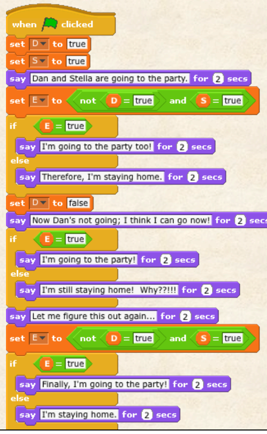
In second expression, when does Ed go to party?

- D does not go AND S goes
- D goes AND S does not go
- D does not go AND S does not go

How would you construct in Scratch?



Variables are NOT formulas!



What is output of script?

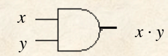
Lesson:

- Variables hold VALUES
- Value of variable determined when "set"
- Variables do not change except when "set"
- Variables do not change when subexpressions change

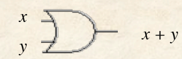
Boolean gates

Shannon (1939)

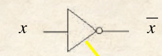
High voltage = 1
Low voltage = 0



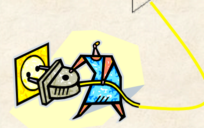
AND: Output voltage is high if **both** of the input voltages are high; otherwise output voltage low.



OR: Output voltage is high if **either** of the input voltages are high; otherwise output voltage low.



NOT: Output voltage is high if the input voltage is low; otherwise output voltage high.

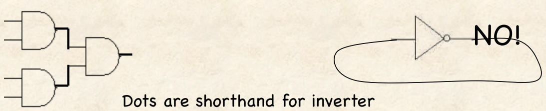


(implicit extra wires for power)

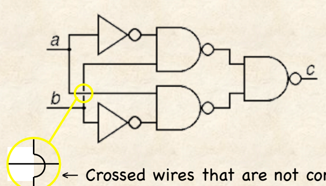
Combinational Circuits

Boolean gates connected by wires with no cycles

- Wires transmit voltage (and hence values)



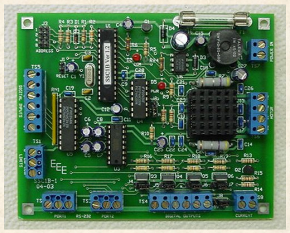
Dots are shorthand for inverter



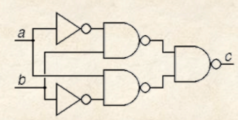
← Crossed wires that are not connected are sometimes drawn like this.

Circuits compute functions

Every combinational circuit computes a Boolean function of its inputs



Inputs



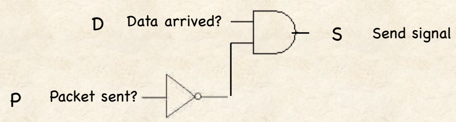
Outputs

Combinational circuits and control

How would you express:

- "If data has arrived and packet has not been sent, send a signal"
- Need 3 boolean variables: D, P, S

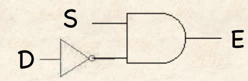
S = D AND (NOT P)



Three Equivalent Representations

Boolean Expression $E = S \text{ AND } \overline{D}$

Boolean Circuit



Truth table:
Value of E for every possible D, S.
TRUE=1; FALSE= 0.

D	S	E
0	0	0
0	1	1
1	0	0
1	1	0

Ben Revisited

Ben only rides to class if he overslept, but even then if it is raining he'll walk and show up late (he hates to bike in the rain). But if there's an exam that day he'll bike if he overslept, even in the rain.

What boolean variables do we need?

B: Ben Bikes (Output)
R: Raining
E: Exam today
O: Overslept

Can you create Boolean expression for B in terms of R, E and O?

Ben Revisited

Ben only rides to class if he overslept, but even then if it is raining he'll walk and show up late (he hates to bike in the rain). But if there's an exam that day he'll bike if he overslept, even in the rain.

What boolean variables do we need?

B: Ben Bikes (Output)
R: Raining
E: Exam today
O: Overslept

$$B = O \cdot \overline{R} + O \cdot E$$

Ben's truth table

Ben only rides to class if he overslept, but even then if it is raining he'll walk and show up. But if there's an exam that day he'll bike if he overslept, even in the rain.

O	R	E	B
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$B = O \cdot \overline{R} + O \cdot E$

Ben's truth table

Ben only rides to class if he overslept, but even then if it is raining he'll walk and show up. But if there's an exam that day he'll bike if he overslept, even in the rain.

O	R	E	B
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

$B = O \cdot \overline{R} + O \cdot E$

Truth table → Boolean expression

Sum of Products:
Use **OR** of all input combinations that lead to TRUE output

$B = O \cdot \overline{R} \cdot \overline{E} + O \cdot \overline{R} \cdot E + O \cdot R \cdot E$

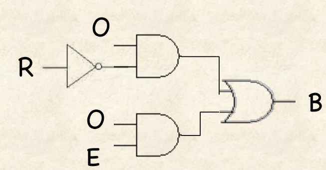
Can simplify expression:

$B = O \cdot \overline{R} + O \cdot E$

O	R	E	B
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Boolean Expression → Gates

B = O· NOT R + O·E



AND, OR, and NOT gates can implement every Boolean function!

Reverse Engineer Boolean Expression

How to determine hidden function?

Approach: Explore all input combinations

D	S	E
0	0	1
0	1	1
1	0	1
1	1	0

E = NOT (D AND S)

$E = \overline{D}\,\overline{S} + \overline{D}S + D\overline{S}$

Today's Summary

Today's topics

- Boolean logic: Operates on True (1) and False (0)
  - Operators: AND, OR, NOT
- Three equivalent representations:
  - Boolean expressions
  - Combinational circuits
  - Truth Tables

Announcements

- No lecture this Wednesday!!
- Homework 9 due tomorrow by 5:00
- Exam 2 returned

Exam 2 Distribution

Exam 2 Class Statistics

Number of submitted grades: 81 / 81
Minimum: 0 %
Maximum: 97 %
Average: 77.62 %
Mode: 95 %
Median: 82 %
Standard Deviation: 17.44 %

Grade Distribution