You need to learn the concepts and formulae highlighted in red. The rest of the text is for your intellectual enjoyment, but is not a requirement for homework or exams.

Chapter 1
REFLECTION and REFRACTION

SPECULAR REFLECTION OF LIGHT

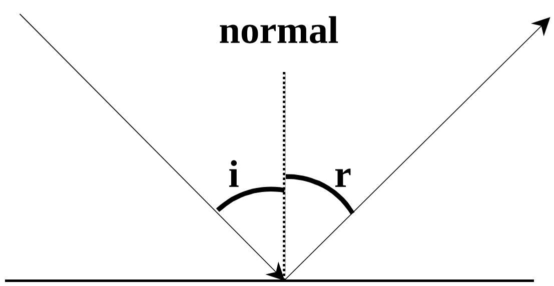
1. Imagine a mirror surface, which is perfectly flat, polished and reflecting. Now imagine a line, perpendicular to the mirror surface, called the *normal*. The normal forms angles of 90 degrees from the mirror surface. When light illuminates a mirror such as this, it is reflected.

In the diagram below, a ray of light coming from the left is incident (or "incoming") on the mirror surface. The angle marked **i** is called the *angle of incidence*, and it is measured from the normal (in this particular diagram, i = 45°). The incident ray is reflected by the mirror into another ray: the reflected ray of light with an *angle of reflection* **r**, again, measured from the normal. ***The angle of incidence and the angle of reflection are equal***, or

i = r.

This is the **law of specular reflection**, from the Latin word "speculum", meaning mirror. The law of specular reflection is valid for any value of the angle of incidence **i**, not only for the 45° angle reported in the diagram. This law is all you really need to know about specular reflection.

Specular Reflection: ***i = r***



i = angle of incidence
r = angle of reflection

All polished and smooth surfaces reflect light as a mirror. Usually mirrors are a combination of a glass pane, which can be made very flat and smooth, coated on its back by a reflective metal layer.

All materials, even the most insulating and transparent ones, such as uncoated glass, reflect light, at least to some extent. To see this you can do a simple experiment.

Take a transparent pane of glass and standing in front of it in a dark room. Even though the pane is transparent, there is no source of light behind the pane, therefore there is no light for the pane to transmit towards your eye. If you now place a source of light, for instance a lit candle between you and the pane, you can easily see the small glass reflectivity. Light from the candle travels in all directions. The rays that from the candle go toward the pane are reflected back, and your eye can see the reflection of the candle. Of course, since the glass is transparent, most of the light from the candle goes through the glass and you don't see it, but a small proportion of that light is reflected back towards you. Since the room is dark, you can see the small amount of light forming the reflected ***virtual image***.

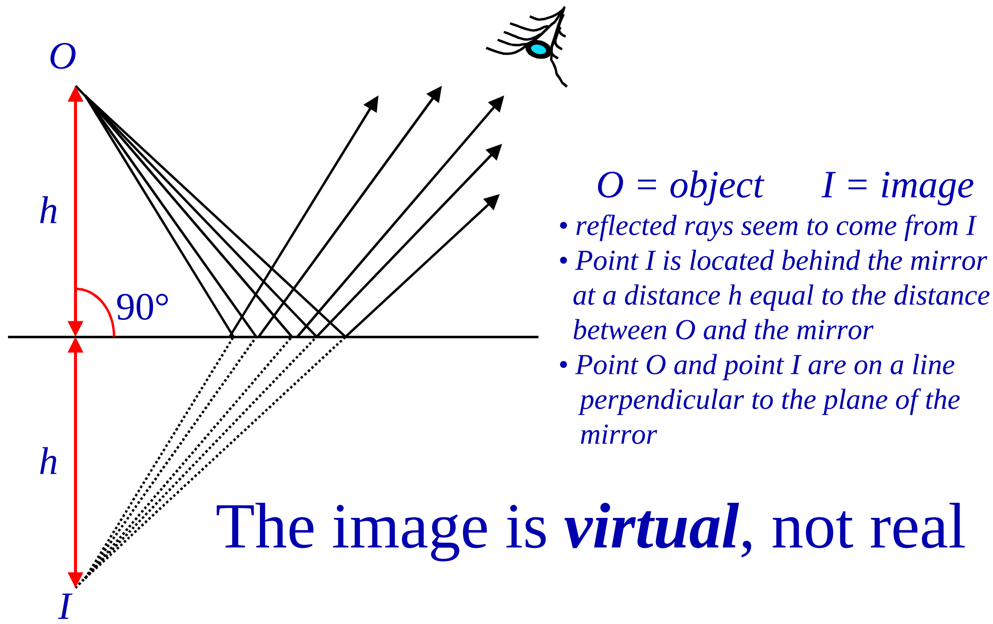
The same phenomenon is constantly visible in everyday life. When you are standing outside in daylight and look at a building all windows reflect the outside world, including the sky, other buildings and trees?
This happens because the window panes are very flat and slightly reflecting, and there is more light outside than inside the building, so they reflect the outside world exactly as the candle was reflected in the previous example. The same happens at night, although in the opposite direction. When there is artificial illumination inside, and it is dark outside, people from outside can see in, through the window panes very well, while from inside at night, you can't see outside, and you only see the reflection of yourself and the rest of the room. If you want to see outside, you must turn the light off inside. The general rule is, therefore, that you can see from darker to brighter, through a window pane, but not vice versa, because of reflection. Conversely, you only see reflection if you stand on the bright side.

Another observation is that the reflectivity of polished surfaces is greater at grazing incidence, that is, when the angle of incidence (and the one of reflection) are close to 90°.

Most objects around us are not perfectly polished, and are not good specular reflectors. When they are illuminated they do not reflect light according to the law of reflection. Non-polished objects ***diffusely reflect*** (or ***scatter***) light in all directions around them. This means that you do not have to be in a specific position, at a specific angle to see an illuminated object. You can be at any angle from it, and still see it. This is quite convenient!

Every non-polished, illuminated object can be considered as a source of light rays. Obviously, not many objects emit light, as do the sun, incandescent light bulbs, neon lights, fluorescent lights, or computer monitors. Nevertheless, when illuminated by a specific source of light, non-polished objects *diffusely reflect* light in all directions. From the geometric point of view, they can be schematically represented as the center of diverging rays, i.e. sources of light rays.

2. Imagine an object in front of a mirror. The diverging rays of light that originate from the object, marked O in the diagram below, are reflected by the mirror. The reflected rays of light *behave as if* they originated from a point I. The point I is called the ***virtual image*** of the object O.

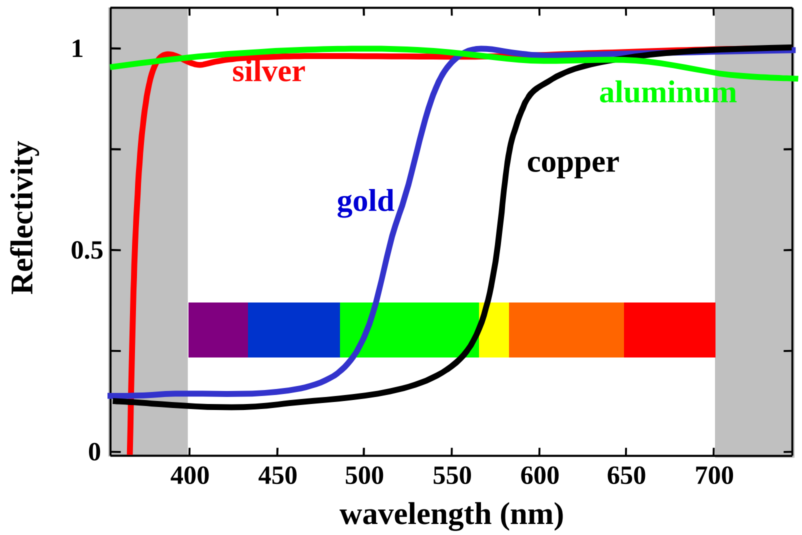


To find the image position of any object, draw the normal to the mirror and measure the object distance from the mirror. The image I is located at the same distance as the object from the mirror, but on the opposite side. This is why it is called a *virtual image*. In other words, ***there are no real rays of light that diverge from the point I. In the above drawing, it is only the artificial extensions of the rays (dashed lines) that originate from I.***

Any ray from O that strikes the mirror is reflected, so that it seems to come from I. If you now remove the mirror, and place the point object O at the point I, it will look identical to the reflected point O. The eye cannot tell the difference between rays coming from a virtual image I and a real object at the same position.

When you look at yourself in the mirror, what you see is a virtual image of yourself, and that virtual image is behind the mirror. If a friend of yours looks behind the mirror, there will be no image of you behind the mirror.
Another important observation is that vertically mounted mirrors invert left and right, but not top and bottom. Horizontally mounted mirrors only invert top and bottom, but not left and right.

3. Why is gold yellow? Why are silver and aluminum gray? Why is copper red?



Above are the ***reflectivity curves*** for these metals. As we will see later on, the wavelengths between 400 nm and 700 nm are the ***visible range***, below 400 nm is ultraviolet, above 700 nm is infrared and we cannot see them. From the reflectivity curves, you can see that gold does not reflect blue, it absorbs blue, and therefore appears yellow. Copper absorbs both blue and green and therefore appears red, while aluminum and silver have fairly flat reflectivity curves all over the visible range, and they appear gray. In other words aluminum and silver reflect all the light colors illuminating them, without absorbing any. That's why silver and aluminum are the best materials for mirrors. Silver was the only material to make mirrors in the old days, and that's why mirrors were so expensive. Today the technology to deposit thin smooth layers of aluminum on glass has been developed, and we can all afford mirrors.

REFRACTION OF LIGHT

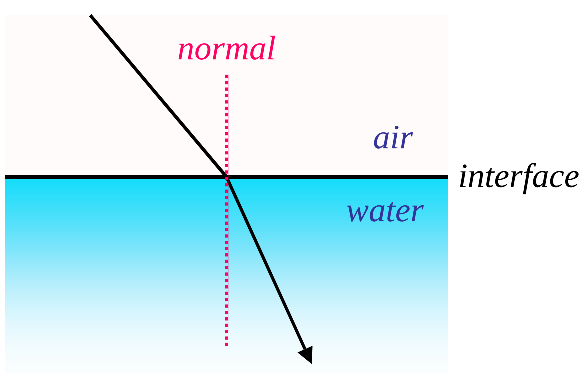
4. Light travels along straight lines in a uniform medium. If a ray of light passes from one medium to another (from air into water or glass, for instance), along a direction that is normal to the interface, the ray travels straight. If, however, a ray passes from one medium to another, *in a direction different from normal*, the direction of the light ray changes in the second medium: *the ray is kinked*. The abrupt deflection, or kinking, of light rays at the interface between two media is called ***refraction***.

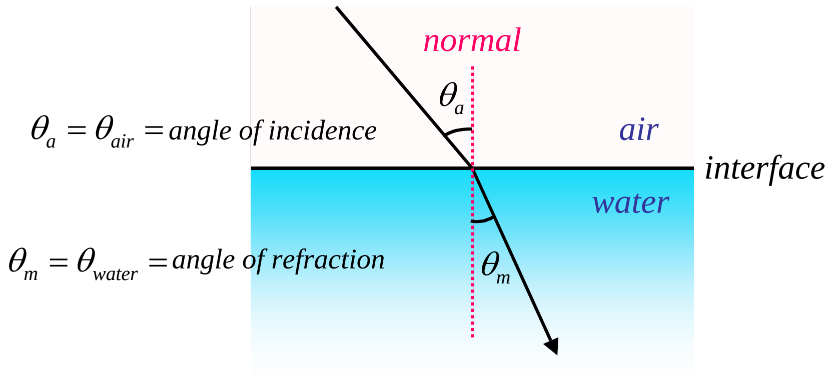
The behavior of light in a medium, such as the speed of light and the refraction angle, depends on the ***index of refraction*** of the medium, **n**. This is an intrinsic property of each material, and depends on its density. In general, the denser the medium the higher the index of refraction n. Vacuum, and, to a good approximation, air have n = 1. For water n = 1.33, and for glass n = 1.52. The index of refraction n is a dimensionless number, that is, it has no units. In most media n is a number between 1.0 and 2.5. A table of the indices of refraction of common materials is shown in the next page.

material	index of refraction n
vacuum	1.00000
air at 50°C	1.00024
air at 0°C	1.00029
water	1.33
plexiglass	1.50
glass (crown)	1.517
glass (flint)	1.89
zircon	1.97
diamond	2.42

In reality there is an additional complication: the index of refraction is not constant for each material, but depends a bit on the wavelength of the light. This phenomenon is called ***dispersion***. For example: the index of refraction of crown glass for short wavelengths (400 nm) is n_{400}=1.59, at longer wavelengths (700 nm), the index becomes n_{700}=1.58, a very small difference. However, as we will describe later on, this makes it possible to separate the spectral colors using a crown glass prism.

Let us consider a light ray crossing the interface between a medium with index of refraction, n = 1.0 and entering a second medium with index of refraction n > 1. In the diagram below, a ray of light is traveling in air, and reaches the interface between air and water. The ray is kinked towards the normal. The index of refraction is related to the density of a material, and a dense material has a larger index of refraction than a less dense material (e.g. air). Whenever light passes from a less dense to a denser medium, it kinks *towards the normal*, when it passes from a denser to a less dense medium it kinks *away from the normal*.





Let us consider the angles that the ray forms with the normal to the interface at the point of incidence: θ_a the angle in air, and θ_m the corresponding angle in the medium (water). The behavior of the ray as it is refracted at the interface can be described as follows:

a. The angle in air (θ_a), and the angle in the medium are on opposite sides of the normal (see diagram above).

b. The angle in air is always larger than the angle in the medium. Air is less dense than the medium (e.g. glass, water, plexiglass, acrylic, etc..)

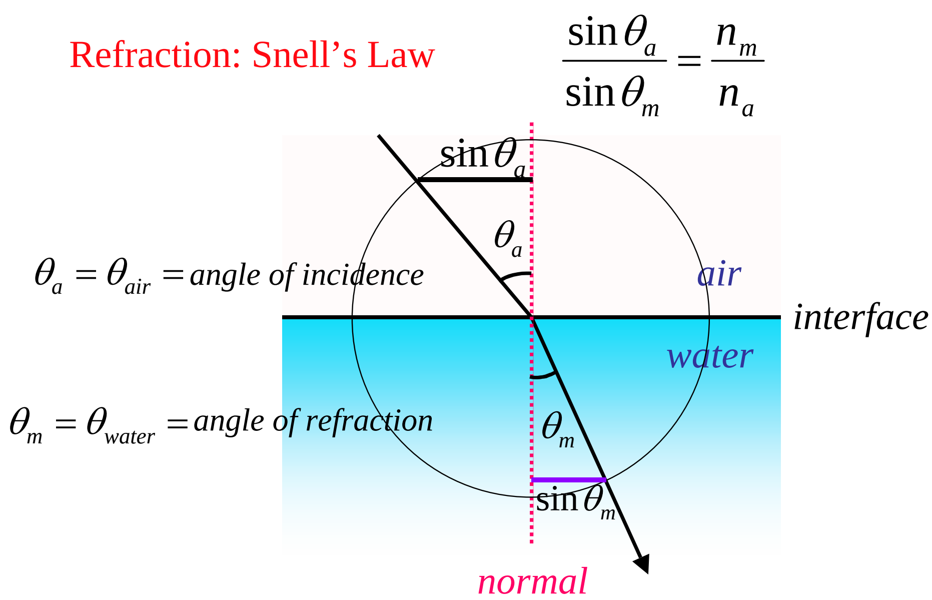
c. The angles θ_a and θ_m do not change if the light ray goes from air to the medium, or from the medium to air. The ray of light is reversible. ***Light rays are always reversible***, in this and in all cases we will encounter later.

d. The angles θ_a and θ_m can be calculated using Snell's law.

SNELL'S LAW

$$\frac{\sin\theta_a}{\sin\theta_m} = \frac{n_m}{n_a}$$

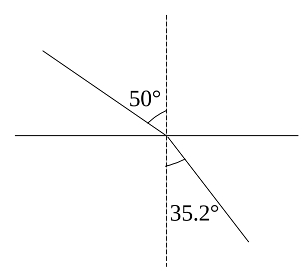
In the case of air and water, Snell's law is:



TOTAL INTERNAL REFLECTION

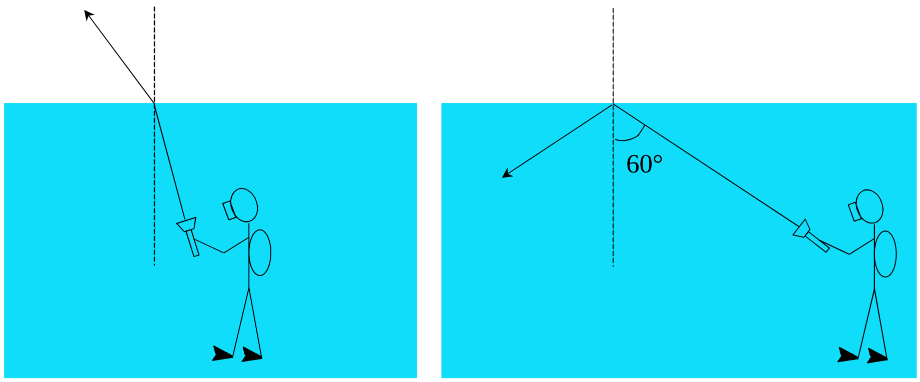
5. To any angle in air θ_a, corresponds a particular angle θ_m in the medium. For example, for rays traveling from air into water (index of refraction n = 1.33) Snell's law gives the following results:

angle in air θ_a	angle in water $\theta_m = \sin^{-1}(\theta_a/n)$
0°	0°
10°	7.5°
20°	14.9°
30°	22.1°
40°	28.9°
50°	35.2°
60°	40.6°
70°	44.9°
80°	47.8°
90°	48.7°



The table goes up to 90° only. This is because the angle in air, θ_a, can never be greater than 90°; if it were greater, it would not be in air!

The table can be also used for a ray of light traveling in the opposite direction, from water into air. For instance, a diver with a waterproof flashlight could experiment with rays of light, and verify that the angles reported in the table are still accurate, when going from water to air.



Remember, ***rays of light can always be inverted***. All optics formulas, including Snell's law, the table of values in the previous page, and all drawings do apply to inverted rays. The difference for Snell's law, is that you now start from the angle in water, and calculate the angle in air. So, looking at the table in the previous page, start from right and read the corresponding angles on the left. There is one peculiarity, however. Because $n_{water} > n_{air}$, the angles in water θ_m do not go all the way to 90°. The table does not tell us what happens with a ray with a θ_m of 60° for example. What happens is simply the following:

$\theta_m = 49°$ corresponds to $\theta_a = 90°$

$\theta_m = 60°$ corresponds to $\theta_a > 90°$ but this is ***impossible*** because the ray would not be in air, so the ray is not refracted, but ***reflected*** at the interface water-air.

Above a certain angle, called the ***critical angle*** θ_c or ***angle of total internal reflection***, all light is reflected from the interface. No light is refracted. This occurs only when the ray of light passes from a denser to a less dense medium, that is, from higher to lower index of refraction.

The angle of total internal reflection for the water/air interface is 49°. For glass/air it is 41°, for diamond is 24.4°.

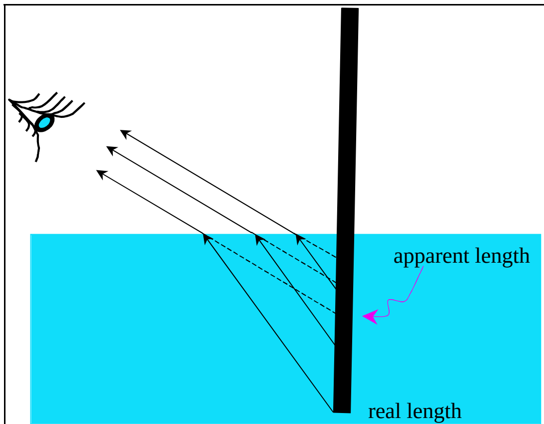
This means that, if the angle of incidence is greater than the angle of total internal reflection, $\theta_m > \theta_c$, ***no light is refracted***. In the figure above, since the angle of incidence is 60°, which is greater than 49°, the light is totally reflected.

It is easy to calculate the angle of total internal reflection. At this angle, $\theta_a = 90°$, and $\sin 90° = 1$. Snell's law in this case appears as: $n_{water} = \dfrac{1}{\sin\theta_{c(water)}}$

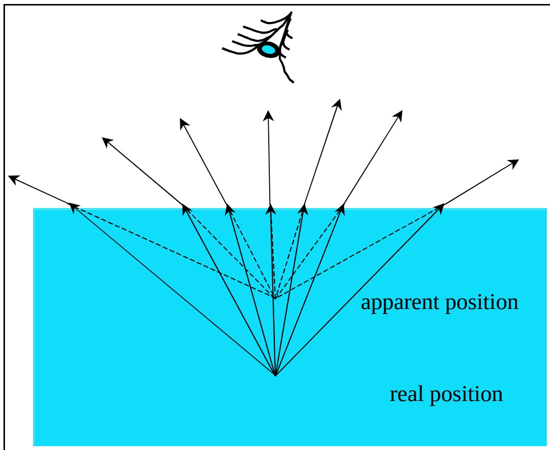
Hence, $\sin\theta_{c(water)} = \dfrac{1}{n_{water}}$

from which we can calculate $\theta_{c(water)} = \sin^{-1}(\dfrac{1}{n_{water}}) = \sin^{-1}(0.75) = 49°$.

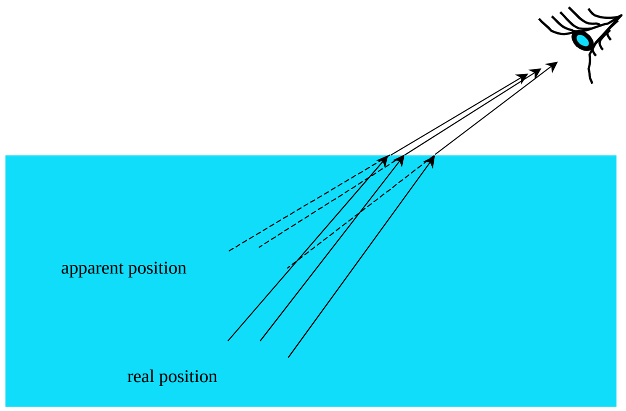
With the aid of the above information, we can anticipate and draw the behavior of a light ray as it crosses a transparent medium. Below and in the next page, we report a few examples of refraction in some common situations. In all these examples, for simplicity, we only show the rays coming from one point of the object. Imagine that there are similar rays coming from each point on the object.



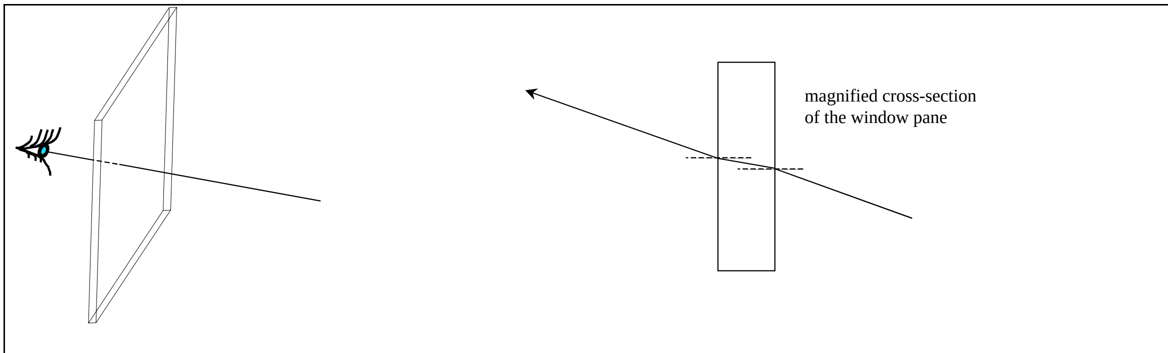
Because of refraction, a vertical stick under water appears to be shorter. If the stick is not vertical, it also appears to be bent at the air-water interface, so that the bottom of the stick always appears closer to the surface than it is in reality.



A fish underwater, seen from the top, appears to be closer to the surface than it really is. The apparent size does not change, though.



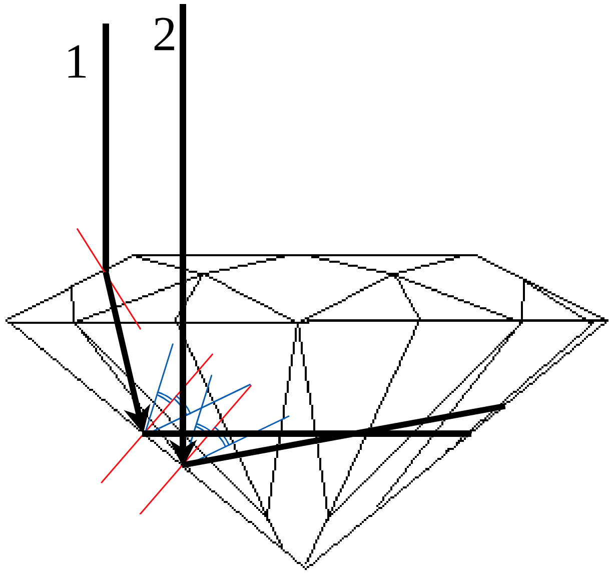
A fish seen not from the top, but at an angle, appears to be in a different position than it really is: closer to the surface and compressed vertically. Experienced spear-fishermen keep refraction into account when they aim and launch their spear.



Here a ray crosses a glass pane with parallel sides. The ray coming from the right is kinked towards the normal (dashed line) when entering the glass, and away from the normal when exiting on the left. The incoming and outgoing rays are parallel to each other. Since the window pane is thin, there is almost no shift in the image position. A thicker block of glass or plexiglass shifts the rays much more than a window pane.

REFLECTION AND REFRACTION IN DIAMONDS

A brilliant cut diamond is so sparkly due to the skillful use of refraction and total internal reflection. Let us look at the detailed path of some rays of light (adapted from Jillian F. Banfield's web site: http://socrates.berkeley.edu/~eps2/).



Consider two rays of light (1 and 2 in the diagram on the left) passing the air-diamond interface. Ray 1 is kinked towards the normal (red), because diamond is denser than air. Ray 2 enters perpendicular to the surface, therefore it is not kinked, and goes straight through the surface. They both reach the inner surface by the arrow heads, and undergo total internal reflection. The angle of total internal reflection is shown by the blue cones in the diagram. Both rays 1 and 2 are ***outside*** this angle, and are therefore totally internally reflected.

The angle of total internal reflection is calculated, as before, using Snell's Law. The index of refraction for diamond is $n_{diamond} = 2.42$.

Snell's law for diamond is:

$$\frac{\sin\theta_{air}}{\sin\theta_{diamond}} = \frac{n_{diamond}}{n_{air}}$$

Total internal reflection in diamond takes place when $\theta_{air} = 90°$, which corresponds to

$$\sin\theta_{diamond} = \frac{n_{air}}{n_{diamond}} = \frac{1}{2.42} = 0.41 \text{ and } \sin^{-1} 0.41 = 24.4°$$

Whenever $\theta_{diamond} \geq 24.4°$ light is completely reflected within the diamond, and no light is refracted outside.

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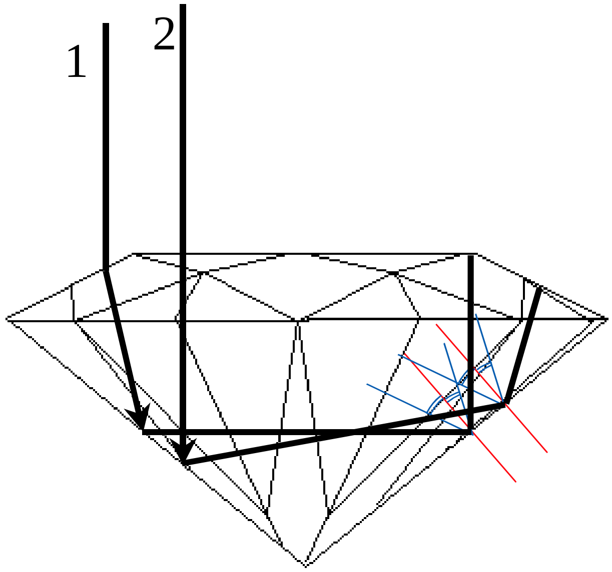
<td>
</td>
<td>After the first total internal reflection, the two rays 1 and 2 reach the other side of the diamond bottom surface, and are, again, outside the angle of total internal reflection. They are therefore completely reflected towards the top of the diamond.

From this diagram, you can now see why the bottom sides of the diamond must be at an angle!</td>
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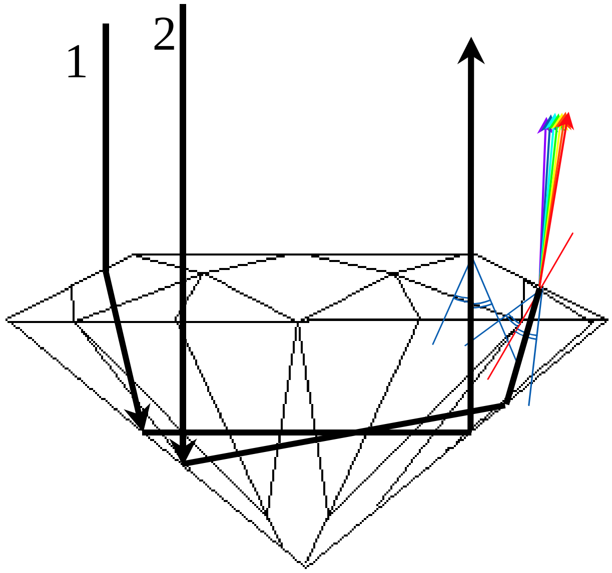
<td>
</td>
<td>Ray 1 is reflected vertically and reaches the top surface perpendicularly. It therefore goes straight through the surface. Ray 2 reaches the surface at an angle, within the blue cone of total internal reflection, and is therefore refracted outside the diamond. The index of refraction n is slightly different for each light wavelength, therefore each color of the spectrum is refracted with a slightly different angle. Violet rays are kinked more than red rays. This phenomenon, called dispersion of light, will be described in detail in Chapter 6. In jewelry dispersion is called the fire of a gemstone.</td>
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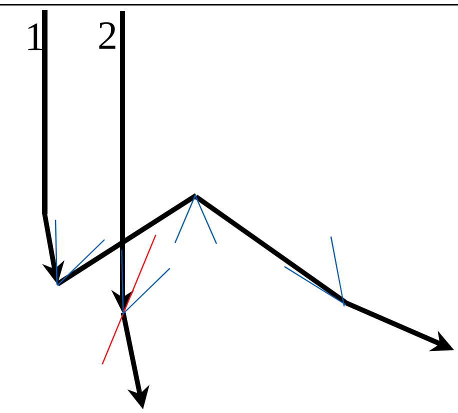
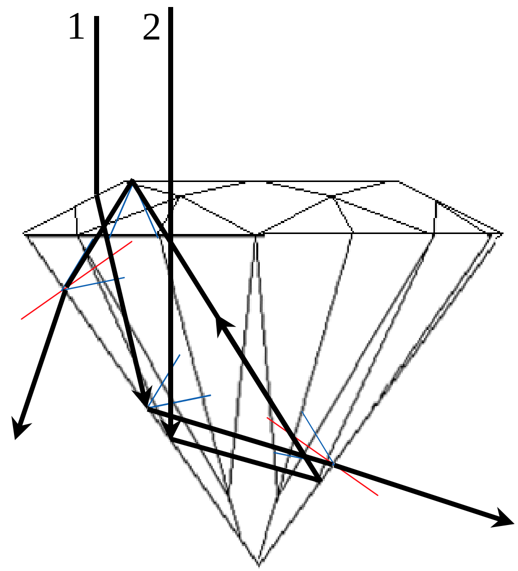
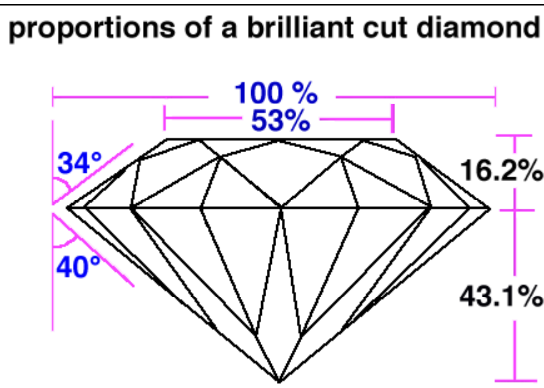
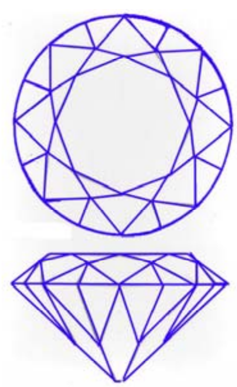


If the diamond is cut too shallow or too deep, light leaks from the bottom facets, and the diamond appears dark and dull. The cones defining the angle of total internal reflection are shown in blue.



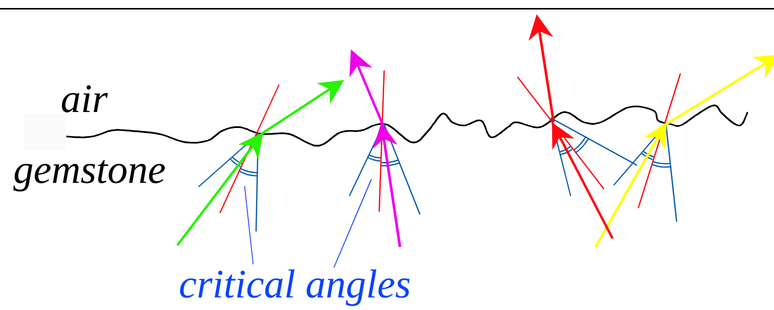


For the diamond to appear sparkly the proportions of the brilliant cut must be very accurate.



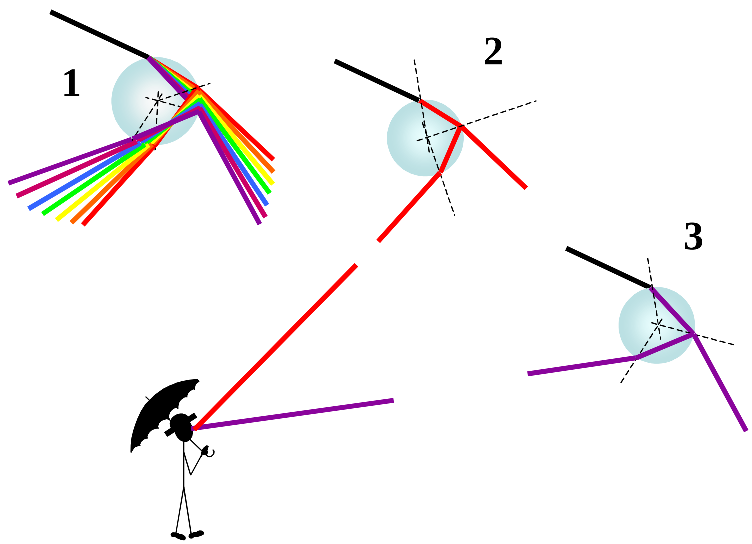
The brilliant cut: top view of the circular crown on top, and side view at the bottom.

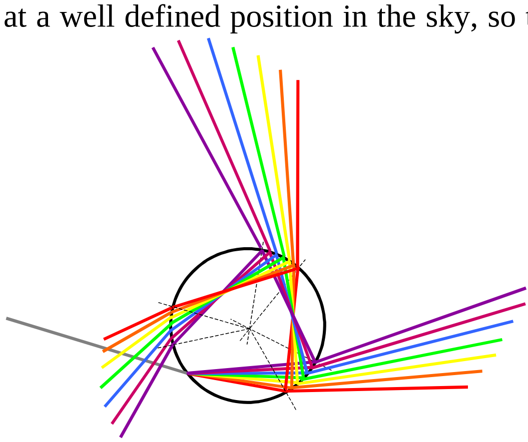
Furthermore, the surface of the diamond facets, after cutting the gem, must be polished. If they are not polished the angles of total internal reflection are not all oriented equally with respect to the surface, and most of the light is lost to refraction.



THE RAINBOW

Now that we have familiarized ourselves with reflection, refraction and dispersion, we can understand how rainbows form. In general, to be able to see the rainbow in front of you the sun must be behind you. Myriad water droplets, either rain or moisture, must be present in the air for the rainbow to appear, as shown in the diagram below. The black lines represent light rays coming from the sun. Three of the droplets are magnified here. Droplet 1 shows how the white light from the sun (black line, here) is ***refracted*** inside the droplet (toward the normal), and ***dispersed***, so that violet rays are deflected more than red rays. All light rays are then partly ***reflected*** on the inner surface of the droplet, following the law of reflection. Notice that this is ***not*** total internal reflection, in fact light rays are partly reflected and partly refracted outside the droplet (away from the normal). The reflected rays then reach the droplet surface again, and will be partly reflected again (not shown in droplet 1) and partly refracted outside the droplet. Droplets 2 and 3 show the red and the violet light rays only, to clarify angles and directions of refraction and internal reflection. In reality, all colors come out of all droplets at all times, so it is incorrect to assume that 2 is a red droplet and 3 is a violet one. They only appear to have those colors to the man with the umbrella, because they are at a well defined position in the sky, so that the refracted-reflected-refracted rays reach his eyes at the correct angle to appear red and violet, respectively. Notice how the violet rays come out of the droplets at a shallower angle, while the red rays come out at a steeper angle. This means that red is always at the outside arches of the rainbow, while violet is always inside.





On rare occasions ***double rainbows*** can be observed in nature. The second rainbow always lies outside the first one, and has inverted color order. The second rainbow is formed by the second internal reflection, and a fourth refraction outside of the droplet. The above droplet shows a schematic of all reflections and refractions. Notice that at each refraction some of the light is lost in a direction that cannot be observed from ground, therefore the second rainbow is always dimmer than the first. At this point it may be a bit difficult to imagine how these two rainbows can form at once. The diagram below shows a droplet which could "potentially" generate both rainbows. In reality, no droplet alone can generate either a single or a double rainbow. You have to imagine myriad droplets, and then the ones that are at the correct position for the observer to see them red will be appear red, the next ones will appear orange, yellow, green, and so forth. Notice that the doubly-reflected rays that form the second rainbow have an inverted color order with respect to the first rainbow. Inside the droplet, again for simplicity, only violet and red rays are shown. The final refracted rays, however, are the seven commonly known colors of the spectrum. Notice that this is another simplification. There are not seven colors in the rainbow, but an infinity of them, associated with an infinity of continuously increasing wavelengths between 400 and 700 nm. Water droplets disperse all these wavelengths with continuity as well. No abrupt separations between colors can be observed in the rainbow or any other dispersed spectrum.