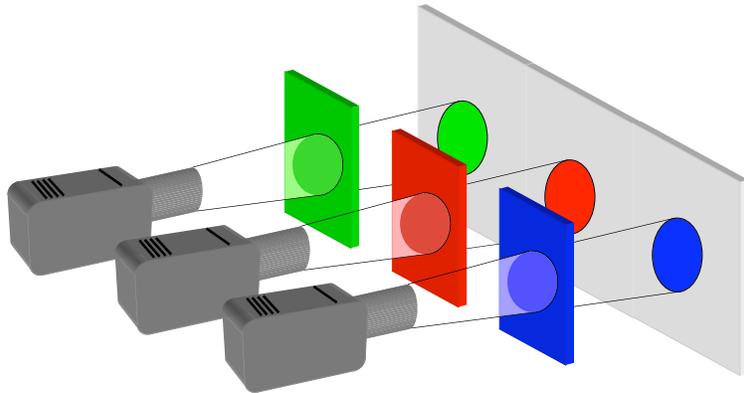


You need to learn the concepts and formulae highlighted in red. The rest of the text is for your intellectual enjoyment, but is not a requirement for homework or exams.

Chapter 7 ADDITIVE COLOR MIXING

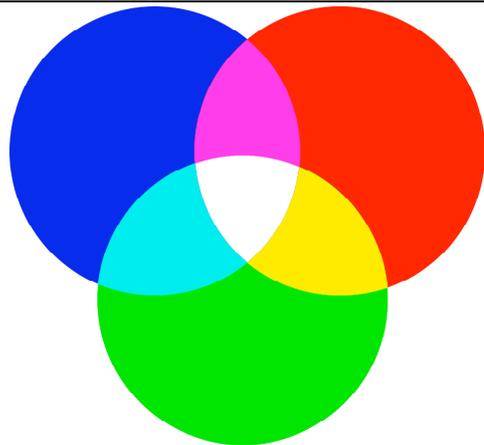
Additive color mixing works for *mixing lights*. The rules of additive color mixing apply to color TV, computer monitors, projectors, stage and disco lights. These rules do not work at all when mixing pigments and paints. The three projectors above are illuminating three filters with white light. The red, green and blue filters transmit light of the same colors. We can now mix these three colored lights, that is, project them onto the same location on the white screen and overlap them, to obtain almost any colors. Let us start with the additive primary colors.



PRIMARY COLORS

Three filters in front of three projectors of *identical intensity* are red (R), green (G) and blue (B). If all three colored lights are projected on a white screen, partly overlapping, they produce the additively mixed colors displayed here on the right. As shown, adding blue to red light produces magenta (M) ($B + R = M$), while adding $B + G =$ cyan (C) and $R + G =$ yellow (Y) are produced. Where R, G and B overlap, white is obtained. R, G and B are therefore three *additive primary colors*. The definition of primary colors is:

- *one primary cannot be matched by a mixture of the other two.*
- *all three, when combined in equal amounts, produce white.*



The usual additive primary colors are R, G and B.

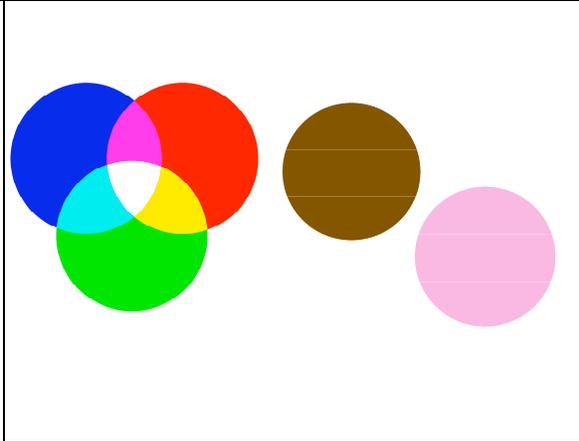
There are many other combinations of three colors that correspond to this definition. An example is already given in the diagram: C, M and Y could also be primaries, but these, as we will see in more detail later, when mixed will only generate a limited number of colors, so they are not the most useful additive primaries.

Two *complementary colors* are *two colors that additively mixed produce white*. Additively mixed implies that we are adding lights of these two colors, as with the above projectors and colored filters. Note in the diagram above, that the complementary colors

appear opposite to each other, and that the complementary to each primary color is the combination of the other two primaries.

ADDING PRIMARY COLORS

It does not seem possible, at a first sight, to mix R, G and B to obtain the two colors reported on the far right, chocolate brown and cyclamen pink. We will describe in detail how to do this quantitatively. For now, suffice it to say that there are accurate combination of R, G and B that produces these two colors. These are: 2 parts of R, 1 part of G, no B, and decreasing the intensity to obtain brown; while pink is a mixture of approximately 2 parts of R, 1 part of G and 1 part of B.



In fact, *almost any color can be obtained with a combination of R, G and B* lights. Let us illuminate the left part of a white screen, in the figure above, with R, G and B lights.

Let us now illuminate the right part of the screen using a projector with a light of an arbitrary color (stimulus S), brown or cyclamen pink in the example above. Alternatively we can place on the screen a colored sheet of paper, and illuminate it with white light. We can now vary the intensities of the three lights on the left, until we obtain a match for brown or pink. A match of the color can be represented by the equation:

$$S = aR + bG + cB$$

Where a, b, c are the intensities of R, G and B lights, respectively. Since the total intensity is of no interest, we can calculate what fraction r of the light is red:

$$r = \frac{a}{a + b + c}$$

And similarly what fraction g is green:

$$g = \frac{b}{a + b + c}$$

These two numbers, r and g, define the mixture of colors projected on the left spot.

In the cyclamen pink example we have:

$$S = 250 R + 170 G + 200 B$$

The total Intensity I is the sum of the intensities for R, G and B (a, b and c):

$$I = a + b + c = 250 + 170 + 200 = 620 \text{ units.}$$

The units in the intensity are irrelevant, because the only numbers that matter to define the color are ratios, that is, the fractions of each color, which are unitless. On computers, in 8 bit color images, a , b and c vary between 0 and 255. There are therefore $256 (= 2^8)$ intensity levels for R, for G and for B. You may try on your computer the numbers above, and verify that you really obtain cyclamen pink. The ratios we need to calculate are:

$$\text{The fraction of red } r = \frac{250}{620} \approx 0.40$$

$$\text{The fraction of green } g = \frac{170}{620} \approx 0.28$$

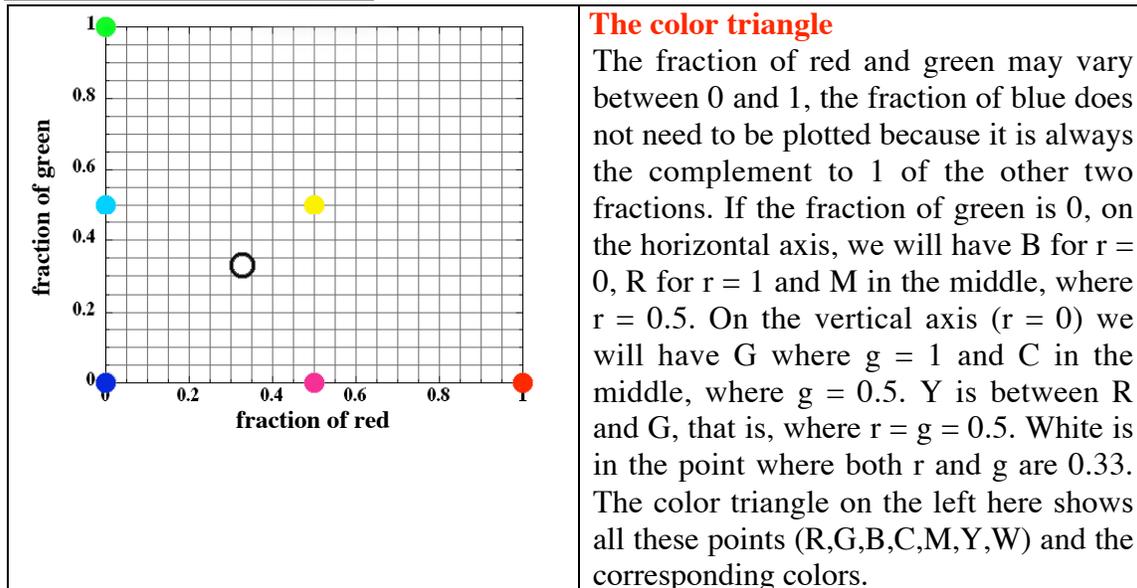
$$\text{The fraction of blue } b = \frac{200}{620} \approx 0.32$$

$$\text{Notice that } r + g + b = 0.4 + 0.28 + 0.32 = 1$$

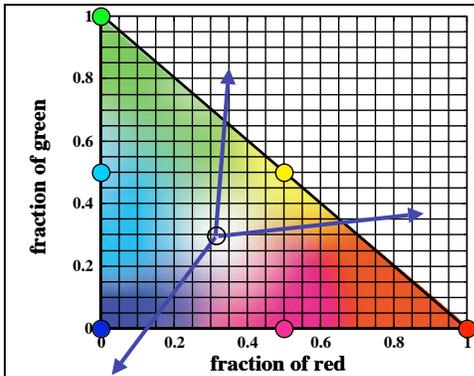
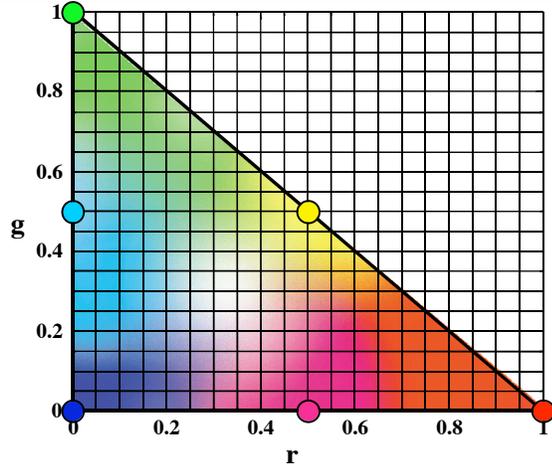
This is always true: *the sum of the fractions r , g , and b always adds up to 1*. We therefore do not need to calculate all three fractions: calculating the fraction of red and the fraction of green will always be sufficient, because the fraction of blue is the complement to 1 of the other two ($b = 1 - r - g$).

We can now plot the fraction of red on the horizontal axis and the fraction of green on the vertical axis of a diagram. This is the chromaticity diagram or *color triangle*.

THE COLOR TRIANGLE



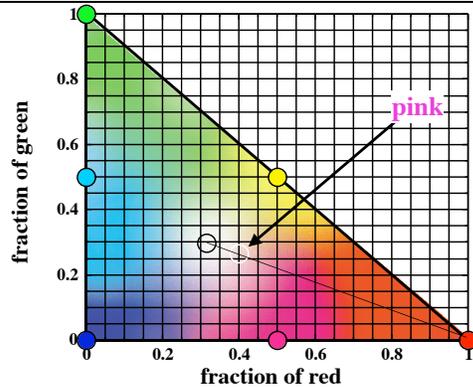
We can find the coordinates of any mixed color on the color triangle. The color triangle rule is: *mixing two colors, the resulting color is always on the line joining the two colors*. That is why C is in the middle point between B and G, M between B and R, and Y between R and G. The color triangle provides a simple way of mixing additive primary colors to obtain the desired color, or anticipating exactly what the resulting color will be, when two colors are mixed in known proportions.



Any mixed color is the sum of the corresponding spectral color plus white. A high purity color has no white, a low purity (pastel) color has lots of white. Along any line starting from white and going towards the edges of the triangle, the color does not change, only the purity changes. Pure colors are at the edges of the triangle, while low purity ones at the center.

Once the fractions of red and green of a color are identified, therefore, to work out what color that is, all we need to do is report the point with coordinates r and g on the triangle, draw a line from white to that point, extend the line until it reaches the edge of the triangle and identify what the color at the edge is. We then estimate how far the point is from the edge and from white, and identify its purity.

For the example of cyclamen pink we found that $r = 0.4$ and $g = 0.28$. We draw the point with these coordinates on the triangle (white circle in the diagram), extend a line from white to it and continue until we reach the edge: the color at the edge (the corner in this case) is red. The point corresponding to our color is much closer to white than to red, we therefore conclude that the color is a low purity red, or low saturation red: pink.



When mixing two colors, we must first identify them on the color triangle, then draw a line between them and, if they were mixed in equal amounts, find the resulting color in the mid-point between the two original colors. If there was more of one color than the other the resulting color will be closer to the most abundant.

<p>According to the color triangle rule, when mixing two colors the resulting color lies along the line connecting the two. If for example we mix 1 part of high purity orange and 2 parts of high purity green, the resulting color lies 1/3 of the way from green and 2/3 of the way from orange, and the resulting color is high purity lime green (thicker white circle). If we mix 2 parts of yellow and 1 part of C, we obtain low purity lime green.</p>	
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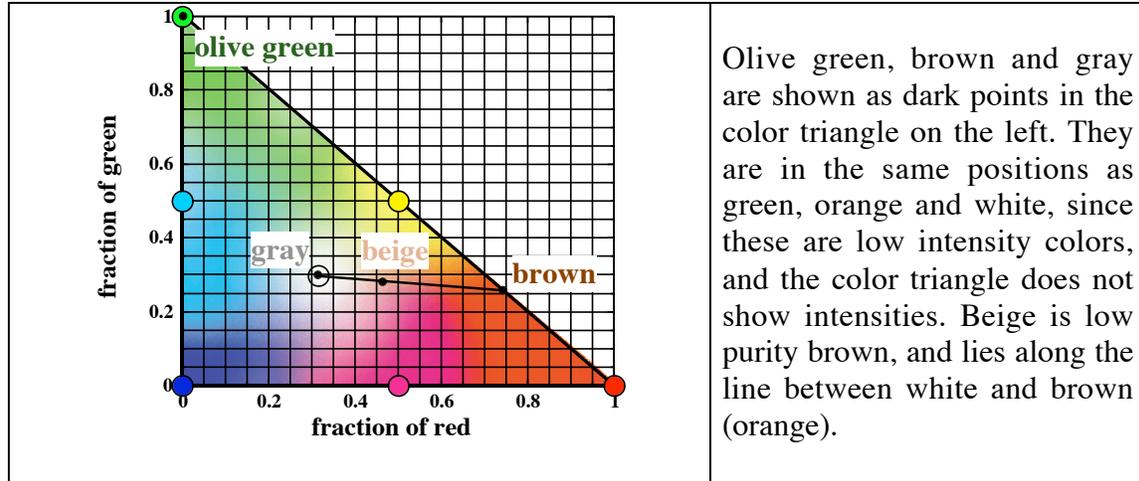
It should now be clear why R, G and B are usually chosen as the additive primary colors: RGB give the largest mixing potential, that is, the largest number of colors can be obtained when mixing them. All the colors in the color triangle can be obtained. *If we chose red, blue and yellow*, for example, we could only obtain the colors within the triangle delimited by the three points where these three colors are on the color triangle, that is, R, O (for orange), Y, W, B, M and the colors intermediate between these. These colors are shown below, on the left. *We could never obtain G or C, mixing only RBY.* Even worse, *if we chose C, M and Y, we could never obtain red, or blue, or green.* This case is shown below, on the right.

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LOW INTENSITY COLORS

Are *all* colors represented on the color triangle? Where are the olive greens, the grays, browns, beige, black? These colors are indeed shown on the triangle, but since *the color triangle does not represent the color intensity*, these low intensity colors are not

immediately evident. Olive green is nothing but low intensity green, brown is low intensity orange, gray is low intensity white, and black is lowest intensity white. Black can actually be lowest (or zero) intensity any color. Black could therefore be anywhere on the color triangle.



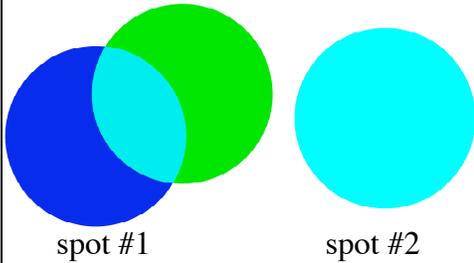
For the above colors the RGB intensities on a 0-255 level basis are:

Green = 0R + 255G + 0B
 Olive green = 0R + 100G + 0B
 White = 255R + 255G + 255B
 Black = 0R + 0G + 0B
 Gray = 123R + 123G + 123B
 Orange = 255R + 123G + 0B
 Brown = 123R + 62G + 0B
 Beige = 223R + 162G + 100B

The colors chosen for typing the color names on the diagram above were obtained using these accurate values. Please try them on your own computer: any graphics program (Adobe Photoshop[®], Adobe Illustrator[®], PowerPoint[®], Canvas[®], etc.) will allow you to enter the RGB numeric values you desire, when you use the “color picker” to select a color.

SPECTRAL COLORS

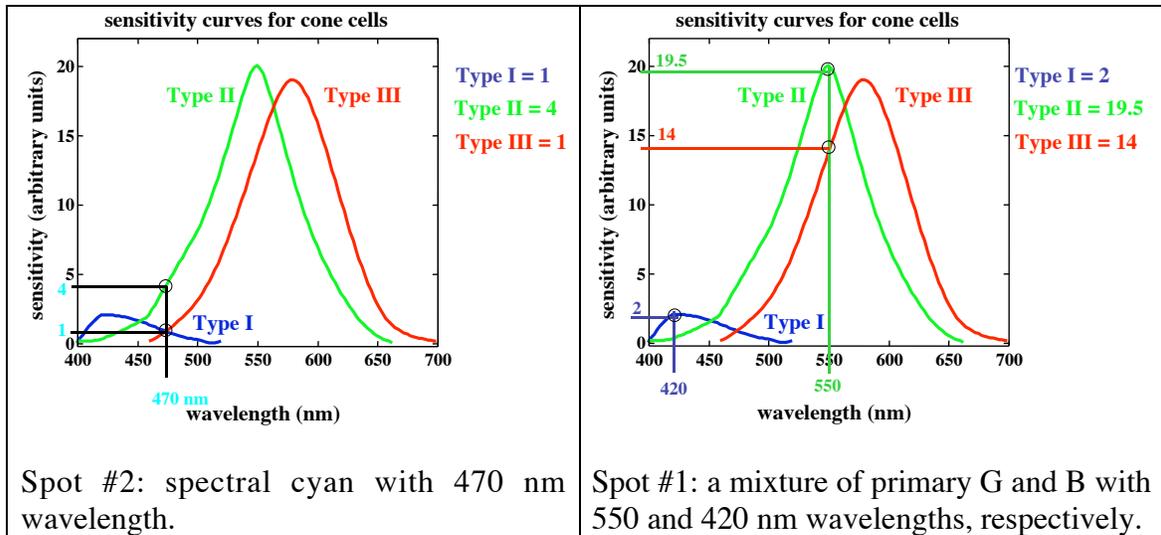
Not all the spectral colors can be matched by mixing RGB. *The hues obtained mixing RGB primaries are less pure than their spectral corresponding colors* (dominant wavelengths). This is particularly evident in the in the *blue-green* region, also called *cyan* or *turquoise*.

<p>Let us project on a screen in spot #1 a combination of RGB and on spot #2 spectral cyan (470 nm), and try to match the color of spot #2 in spot #1. On spot #1 we project blue and green, with wavelengths 420 nm and 550 nm, respectively. We need to use our imagination here, because the color in spot #2 was produced on a computer that works additively, and is therefore not as pure as the spectral cyan with 470 nm. Please imagine the cyan in spot #2 much higher saturation than the mixed cyan of spot #1.</p>	 <p>spot #1 spot #2</p>
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To obtain a perfect match we need to produce the same response in the cone cells of our eye. We will find that the only way to obtain a match is to shine some red light on spot #2 and then match the new color with blue and green light on spot #1. The reason for this is clearer if we look again at the sensitivity curves for the cones.

spot #2	Intensity	wavelength (nm)	response of cones		
			type I	type II	type III
C	1	470	1.0	4	1

We want to match, that is, obtain the same responses for the three types of cones for a mixture of RGB as we obtained for the spectral cyan in spot #2.



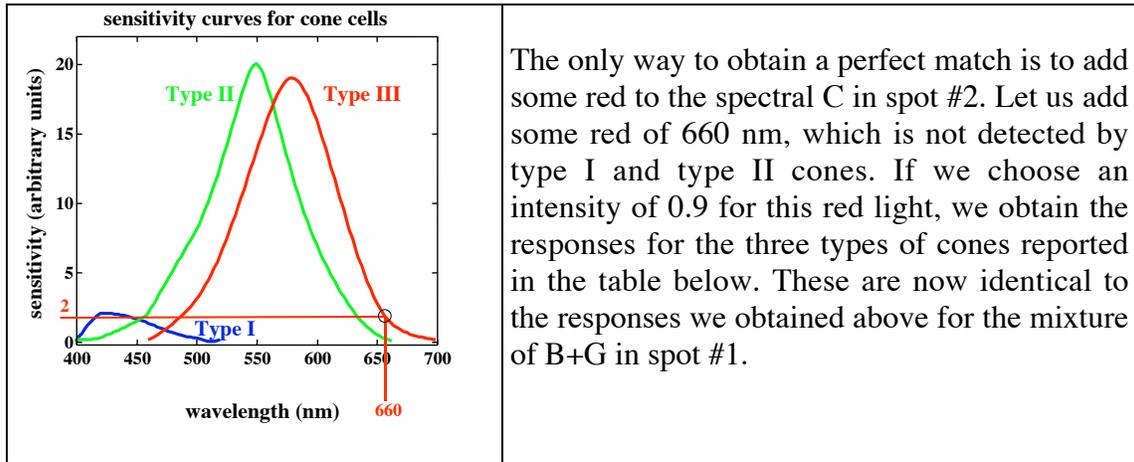
The responses for the three types of cones are:

spot #1	Intensity	wavelength (nm)	response of cones		
			type I	type II	type III
B	1	420	2	0	0
G	1	550	0	19.5	14
B+G		420&550	2	19.5	14

A comparison of the above tables of responses obtained for spots #1 and #2 shows a large discrepancy. Clearly, the intensities of B and G in spot #1 cannot be 1, to obtain a good match with the responses to C in spot #2. Let us try with intensities of 0.5 for B and 0.2 for green:

spot #1	Intensity	wavelength (nm)	response of cones		
			type I	type II	type III
B	0.5	420	$2 \times 0.5 = 1$	0	0
G	0.2	550	0	$19.5 \times 0.2 \approx 4$	$14 \times 0.2 = 2.8$
B+G		420&550	1	4	2.8

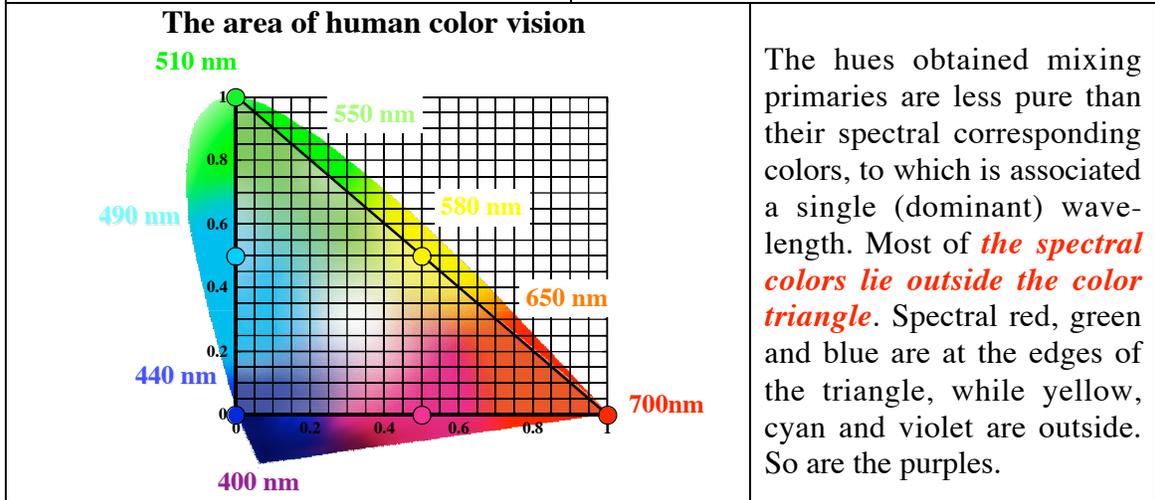
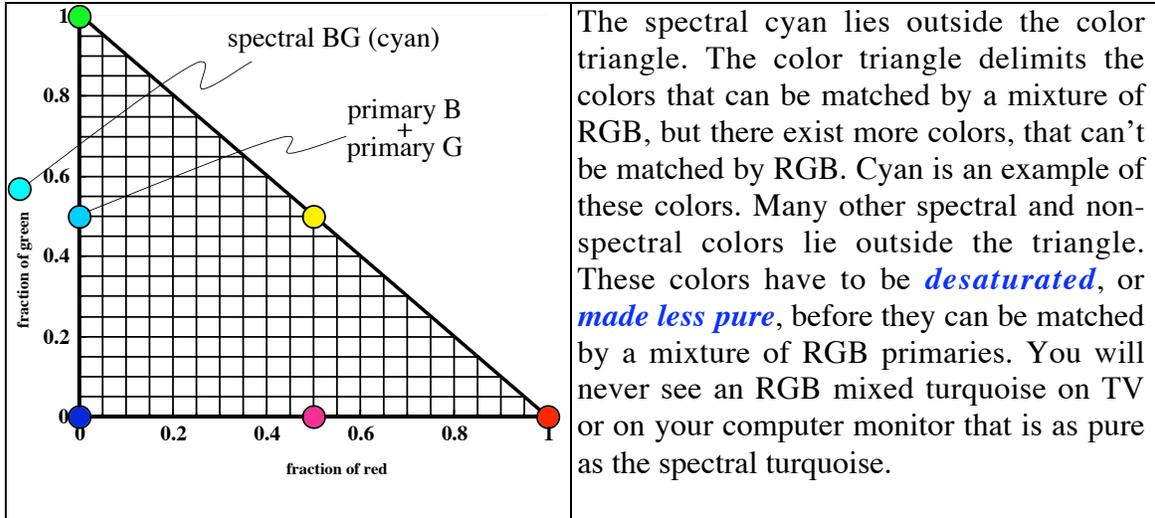
We are getting closer, but our mixture of G+B, still does not generate the exact same response as C: type I and II cones are now identical, but for type III cones we have 2.8 in B+G and 1 in C.



The only way to obtain a perfect match is to add some red to the spectral C in spot #2. Let us add some red of 660 nm, which is not detected by type I and type II cones. If we choose an intensity of 0.9 for this red light, we obtain the responses for the three types of cones reported in the table below. These are now identical to the responses we obtained above for the mixture of B+G in spot #1.

new spot #2	Intensity	wavelength (nm)	response of cones		
			type I	type II	type III
C	1	470	1	4	1
R	0.9	660	0	0	1.8
C+R		470&660	1	4	2.8

Adding red to cyan meant to *desaturate* cyan. Since red is the complementary color to cyan, the addition of red is equivalent to adding some white, and produce a less pure, a less saturated color. Adding red to the spectral cyan we wished to match is equivalent to adding *negative red* to the mixed color. Now, this is physically impossible, because there was no red to begin with in the B+G mixture, so it is not conceivable to subtract red from it. On the other hand, a negative fraction of red can be plotted on the color triangle, or, rather, *outside the color triangle*.



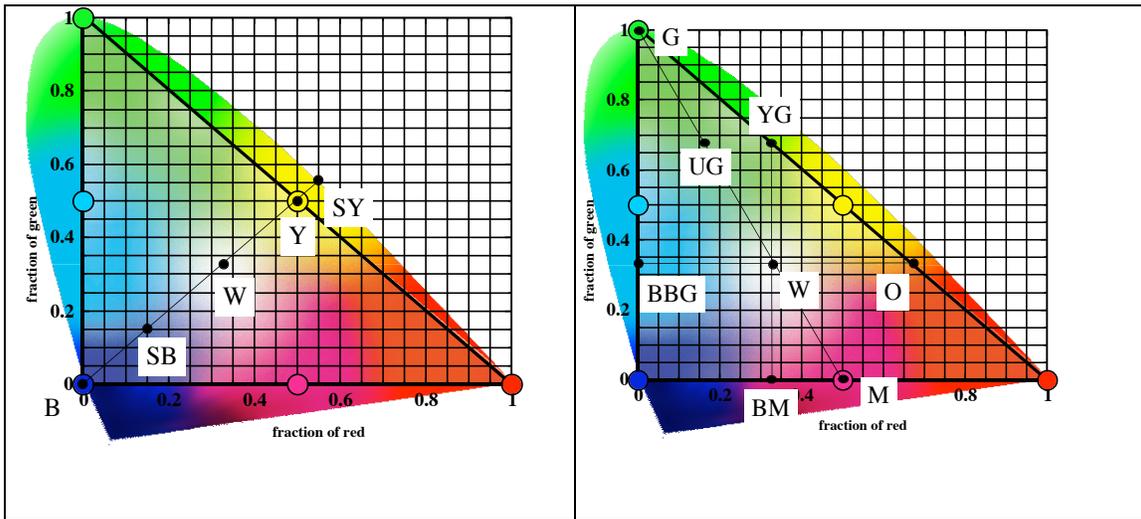
The horse-shoe shape in the figure above, describes *the area of human color vision*. This area is delimited by the curved line along which all spectral colors lie (see corresponding wavelengths in the diagram), and the line between spectral violet and red. In summary, *the color triangle includes only the colors that can be obtained mixing RGB, but the eye can see more colors, including spectral colors and purples*.

NON-SPECTRAL COLORS

Purple and magenta, as well as all the magenta-purple hues are *non-spectral colors*. There is no single (dominant) wavelength associated with them. They are exclusively mixed colors. Magenta is the mixture of spectral red (700 nm) and spectral blue (440 nm), while purple is the mixture of spectral red (700 nm) with spectral violet (400 nm). The mixture of R+B in different proportions generates all the magentas, the reddish-magentas and the bluish-magentas. These hues can be obtained mixing RGB primaries (R+B), therefore you can see them on TV or on the computer monitor. The purple hue cannot be mixed with RGB primaries.

Now that we described the color triangle and the spectral colors, let us look at the complementary colors again. Two complementary colors must produce white when

added. White must be on the line between them. For example, B and Y are complements because when additively mixed they produce white. To be perfectly accurate, **B and spectral Y** are **complementary colors**, not B and Y, since the points B and Y are not equally distant from white, while SY and B are equidistant from W. B and Y are **complementary hues**. We would actually need to add a lower purity blue (sky blue, SB in the figure below) to yellow, to obtain white. B and Y pigments, however do not produce white when mixed, as will be explained in detail in Chapter 8. The line connecting blue and yellow passes through white. Other pairs of complements are shown below, such as bluish-cyan and orange (BBG and O), lime green and bluish magenta (YG and BM), green and magenta (G and M), but many more exist. Again, to be accurate, it is unsaturated green (UG) that complements magenta perfectly.



SUMMARY

Every color can be located on the color triangle by a three-step process. First match the color with three primaries R, G, and B. Second, find r and g.

$$r = \text{fraction of red in the total intensity of light in the match};$$

$$g = \text{fraction of green in the total intensity of light in the match}.$$

Third, plot r on the horizontal axis and g on the vertical axis.

White falls at the point $r = 0.33, g = 0.33$, near the center of the triangle. The more saturated a color (i.e. the greater its purity) the further it falls from white. Spectral colors are the most saturated and fall on a curved line outside the triangle.

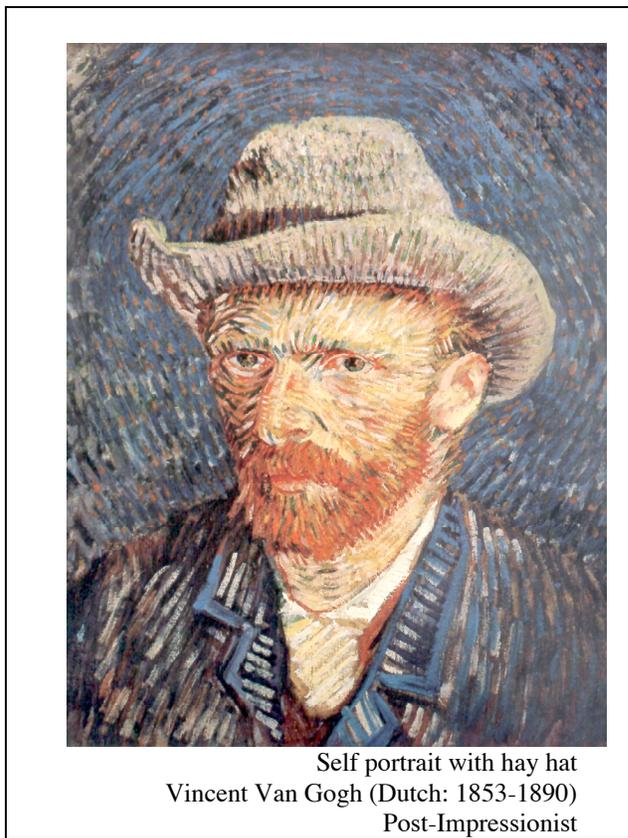
When two colors are added the resulting color always falls on a line connecting the two. This idea is useful in the explanation of complementary colors--colors which produce white when added. Complements always fall on opposite ends of a line through the white point. The hue of a color can be specified by finding the dominant wavelength, or in the case of purples, the complementary wavelength of the color. Every color can be matched

by a spectral color or a purple, plus white. The smaller the amount of white, the higher the purity.

ADDITIVE COLOR MIXING IN PAINTING

When painting and mixing oil paints the rules of additive color mixing don't usually apply. We will describe in Chapter 8 how to mix pigments and paints, with subtractive color mixing. Nevertheless, there have been extraordinary cases in art, in which painters attempted painting additively.

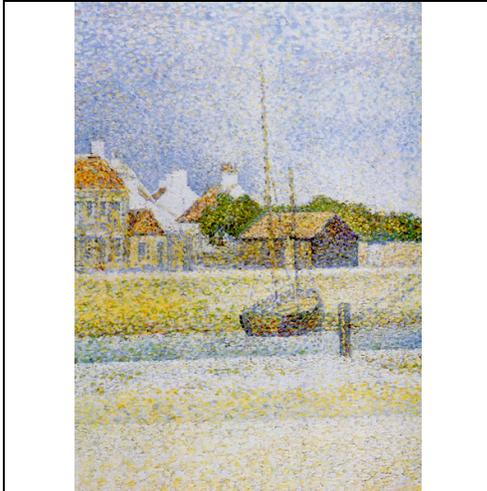
Post-impressionists used a revolutionary technique to enhance color brightness and contrast, which was based on additive color mixing. Van Gogh wanted to achieve the effects of art glass in the windows of gothic cathedrals. These windows have colored pieces of glass adjacent to each other, and they transmit colored lights exactly as our projectors through colored filters.



Self portrait with hay hat
Vincent Van Gogh (Dutch: 1853-1890)
Post-Impressionist

Van Gogh did not know about additive color mixing and mixing lights, but he was striving to achieve the brightness and color contrasts that only mixing lights can achieve. He invented the “*mélange solide*”, French for “solid mixture”, in which instead of mixing oil colors subtractively, he painted adjacent brush strokes with contrasting colors. He described it saying “je cherche à exprimer avec le rouge et le vert les terribles passions humaines”. The terrible human passions are expressed by red and green, not mixed but adjacent. Since the brush strokes in his *mélange solide* are large, much larger than the eye's resolution, the eye does not actually mix lights from adjacent colors, it perceives the striking contrast of very different and very bright colors, as in a gothic cathedral window!

If the regions of two different colors are much smaller, lights from the two different colors side by side mix additively, and create a third color. This resulting color is optically mixed, that is, it is mixed in the eye of the observer. When two colors are mixed in pigments they lose some of their intensity, while when those same two colors are mixed optically they retain their intensity and they may appear brighter. Two other Neo-Impressionists, *the pointillists* Seurat and Signac, explored this possibility, with small color regions: points or dots.



Chenal de Gravelines
Georges Seurat (French: 1859-1891)
Neo-Impressionist

Seurat studied the interactions of colors with each other, and by reducing the number of colors on his palette and applying pigments in very small spots he tried to control their interactions. The Impressionists have often been interpreted as using “broken colors”, but a patch of yellow in an impressionist painting is almost always yellow paint, not strokes of red and green laid side by side to achieve optical mixing. Seurat, however, did construct magenta from small spots of blue and red, and with his system he could also vary the intensity of red, for example, near an orange dot, and determine how much the red will induce in the adjacent color a perception of its complementary, cyan.



St. Tropez, le port
Paul Signac (French: 1863-1935)
Neo-Impressionist

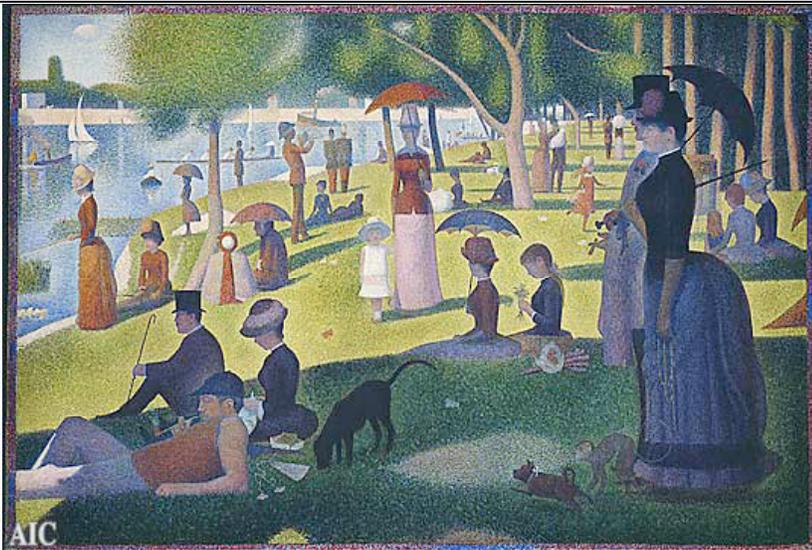
Seurat called his technique *divisionism* because he had divided the colors into their physical constituents, but his theory and practice, together with those of his followers, were soon described as Neo-impressionism and Pointillism. The subject was still Impressionist – ordinary people seen in every day activities, atmospheric conditions, immediacy of the scene – but presentation and technique were new.

The Neo-impressionists had color theory available to them — including the works of Herman von Helmholtz (whose *Manual of Psychological*

Optics had by then appeared in French) and Wilhelm von Bezold's texts, which were published in French in 1876. Seurat and his colleagues had begun to feel that, if they did not become more involved in the science that could explain the optical effects forming the basis of visual arts, art itself would remain intellectually unsatisfying. His task was made easier by the work of the American physicist Nicholas Odgen Rood (N. O. Rood, *Modern Chromatics with Application to Art and Industry*, Chicago 1879) who was also an accomplished painter. Seurat was inspired by N. O. Rood's writings and color theory, and he imitated the behavior of light itself, that is, performed additive color mixing when actually optically mixing pigments. He, however, misunderstood when he thought that “adjacent dots of pure spectral colors could be mixed [in the eye of the observer] to obtain other spectral colors observed in nature, with a brilliance and luminosity [read purity!] unattainable by conventional pigment mixtures”. Scientifically speaking, Seurat's idea was flawed, but he has given us some of the most haunting images in European art. Another problem was with the contrast enhancement of adjacent complementary colors.

We have now learned that lights of complementary colors, when mixed, produce white, *not* pure colors.

The optical mixture of complementary colors kills color, as Signac himself later admitted: “Pointillage makes the surface more lively, but it does not guarantee luminosity, intensity of color, or harmony. The complementary colors, which are allies and enhance each other when juxtaposed, are enemies and destroy each other if mixed, even optically. A yellow and a blue, if juxtaposed, enliven each other, but yellow dots and blue dots make an aggregate which is grey and colorless.” You may verify this effect very simply, next time you go to the Art Institute of Chicago. If you stand in front of Seurat’s “Dimanche après midi sur l’Isle de la Grande Jatte”, and are close enough to see the individual dots, all colors appear very bright and luminous (pure). If you stand far away they all blend together, and the overall effect is a hazy gray patina covering the whole painting.



Dimanche après midi sur l’Isle de la Grande Jatte
Georges Seurat (French: 1859-1891)
Neo-Impressionist