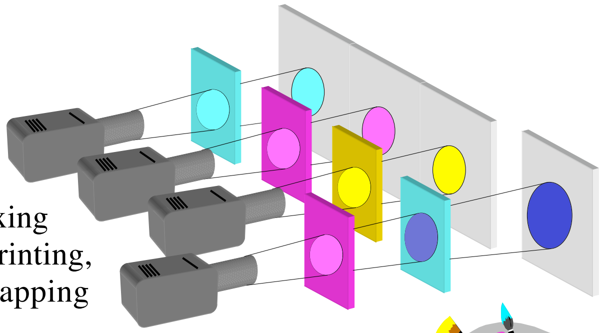
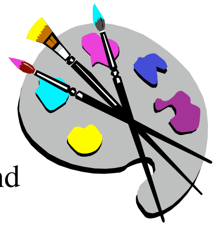
You need to learn the concepts and formulae highlighted in red. The rest of the text is for your intellectual enjoyment, but is not a requirement for homework or exams.

Chapter 8
SUBTRACTIVE COLOR MIXING

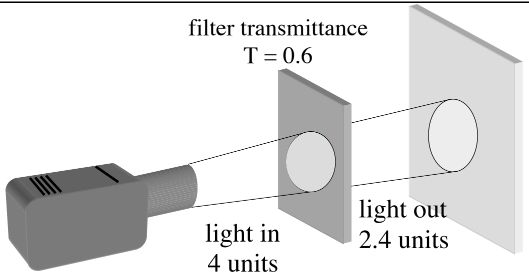


Subtractive color mixing works for mixing pigments in paints or inks, for color printing, for color photography, and for overlapping multiple filters in front of projectors.



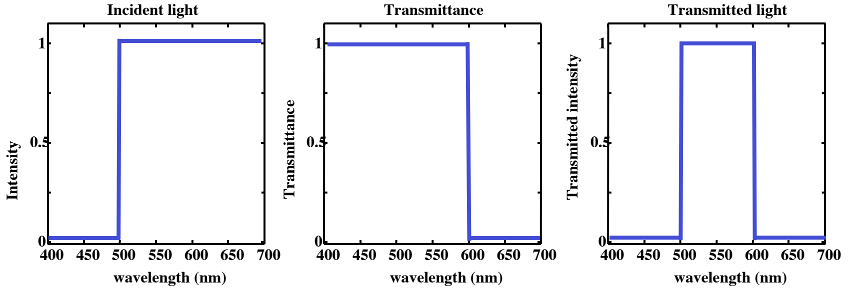
FILTERS AND PIGMENTS

If light of a given spectral distribution passes through a ***filter***, the spectrum of the light is modified. The filter absorbs part of the light, and transmits the remainder.



The ***transmittance*** **T** of a filter is the fraction of the incident light transmitted through the filter. The filter in the figure on the left has T = 0.6. If incoming light has an intensity of 4 units, then the transmitted light has an intensity of 4 × 0.6 = 2.4 units. The filter absorbs 40%, and transmits 60% of the light.

The transmittance of a filter is usually different for different wavelengths (colors) of light. If it is the same for all wavelengths, the filter is a ***gray filter***. Let us look at a colored filter.



The diagram above shows the spectra of colored light incident on a colored filter, the transmittance curve of the filter and the corresponding spectrum of the transmitted light. You should by now be able to decide what color the incident and transmitted lights are in the plots above.

Once again, we are using a simplified version of the spectrum, in which the range of wavelengths between 400 and 500 nm is referred to as blue, while green and red are 500-600 nm and 600-700 nm, respectively (B, G and R). Furthermore, the spectra which are in reality curved lines, can be simplified as step functions, with vertical and horizontal lines, and sharp angles, as in the above figure. This approximation is not always perfect to describe the behavior of light and color, but in most cases it works, and is much simpler to use than the real spectral lineshapes. We will later point out cases in which this approximation does not apply.

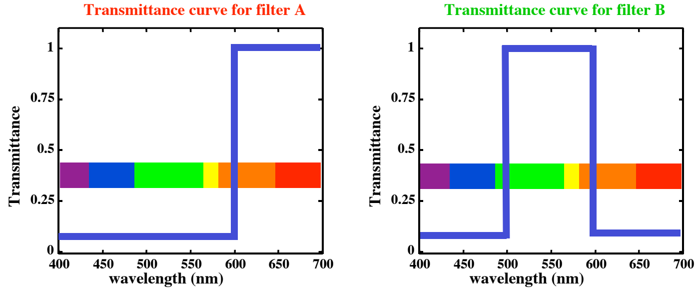
If you were not able to identify the colors in the above spectra, here is a complete explanation for them. The first spectrum was the incident light. This light had no intensity (I = 0) in the B region, and intensity I = 1 in the G and R regions of the spectrum. This is therefore a mixture of G+R lights. As described in the Chapter 7, G and R lights add to form Y light. The incident light is yellow. The filter, instead, transmits all light in the B and G regions (T = 1), and no light in the R region (T = 0). Since there was no B in the incident light, the only light that the filter can transmit is G. The spectrum of the transmitted light, therefore, has I = 0 in the B and R regions, and I = 1 in the G region. The filter ***subtracted*** red from the incident light. It ***absorbed*** the R component from the incident light. We are now beginning to see why this is called ***subtractive color mixing***.

The table below gives the same information contained in the three spectra of the previous page, in a more schematic way, broken down to the relevant values in each spectral range:

Simplified spectral ranges	Intensity of incident light	Transmittance of the filter	Intensity of transmitted light
B (400-500 nm)	0	1	0
G (500-600 nm)	1	1	1
R (600-700 nm)	1	0	0

Notice in the table that ***if the value is 0 either in the incident light intensity or in the transmittance, the resulting transmitted light intensity is 0***.

When two filters are overlapped, and light passes through both of them, the resulting transmittance depends on the transmittance of each filter separately. Consider two colored filters, A and B, with transmittance curves:



Transmittance curve for filter A

Transmittance curve for filter B

Transmittance curves for two filters, A and B. The spectral colors corresponding to each wavelength are reported to guide the eye in understanding what color the two filters are.

If we place these two filters in front of a white light projector, the resulting transmittance will be ***the product of the transmittances*** for the two filters, in each spectral range.

The resulting transmittance, $T_{A\&B} = T_A \times T_B$, indicates which colors of light are transmitted by both filters. It is equivalent to giving the spectrum of the transmitted light intensity. For the two above filters, we obtain a resulting transmittance of 0 in all three regions, B, G and R: no light is transmitted by those two filters. Again, schematically we obtained:

Simplified spectral ranges	Transmittance of the filter A T_A	Transmittance of the filter B T_B	Resulting transmittance $T_{A\&B} = T_A \times T_B$
B (400-500 nm)	0	0	0
G (500-600 nm)	0	1	0
R (600-700 nm)	1	0	0

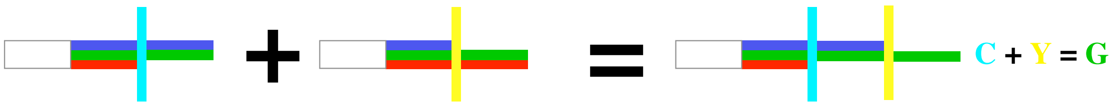
As you may have observed from the transmittance curves for filters A and B, filter A is red while filter B is green. We saw in Chapters 6 and 7 that when adding lights, R + G = Y. We now observe that when adding the same two colors as subsequent filters, the resulting transmittance is 0 over the whole spectrum, that is, the resulting filter is black: no light can be transmitted by the combination of R and G filters.
In other words,

Subtractively, R + G = Black
Additively, R + G = Yellow.

Subtractive color mixing is very different from additive color mixing!
Mixing filters (or pigments, or paints) is very different from mixing lights.

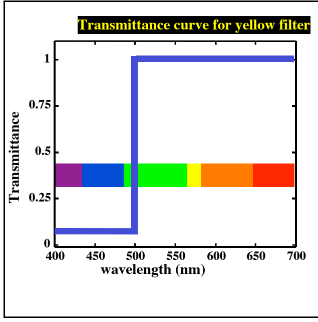
As mentioned, overlapping filters, mixing pigments, or paints, or inks, is called ***subtractive color mixing***. The term is a little misleading, because a filter does not subtract (absorb) a fixed amount of light: it absorbs a fixed ***fraction*** of the light incident upon it. The mathematical operation is not a subtraction. It is rather the **multiplication** of the incident intensity by the transmittance of the filter, or the multiplication of the transmittances of two filters.

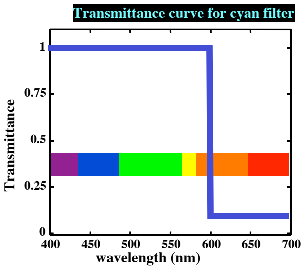
Let us look at more examples. A yellow and a cyan filter overlapped produce green.



White light, coming from the left, can be for simplicity considered to have RGB components only. ***Each filter subtracts its complementary color***. A cyan filter absorbs R (its complementary color) while transmitting B and G (remember that additively B + G = C, therefore the B and G lights transmitted by the filter, added form C). A yellow filter absorbs B, and transmits G and R. Again, additively G+R=Y, so everything works: a C filter transmits C light, a Y filter Y light. The combination of the two filters in sequence, will only transmit G light. If we start from white light, we have R, G and B. The cyan filter subtracts R, the yellow filter subtracts B, and only G is left to be transmitted. (R + G + B) – R – B = G.

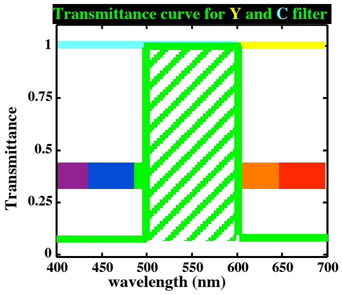
For this example the transmittance curves and table are:





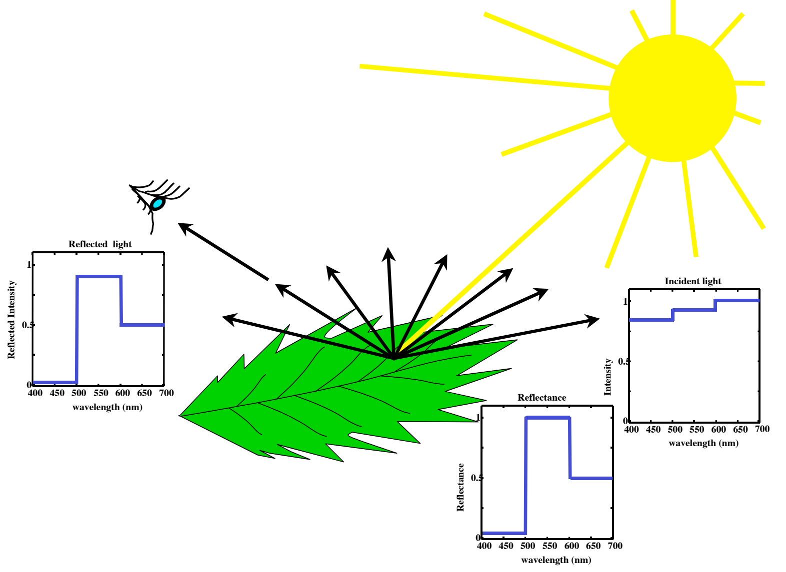
Simplified spectral ranges	T_{yellow}	T_{cyan}	$T_{Y\&C} = T_Y \times T_C$
B (400-500 nm)	0	1	0
G (500-600 nm)	1	1	1
R (600-700 nm)	1	0	0

The resulting transmittance of the two filters can be graphically obtained by combining the transmittance curves of the two filters. The regions in which the two curves overlap is the resulting transmittance (dashed region). In this region neither of the transmittance curves was 0.



The color of familiar objects depends on the ***pigments*** that they contain, and their ***reflectance R*** of the objects. Consider for example the leaf of a tree, illuminated by sunshine. Some of the light from the sun is specularly reflected, unchanged, by the glossy surface of the leaf. The remainder of the light is partly absorbed and partly diffusely reflected, in all directions.

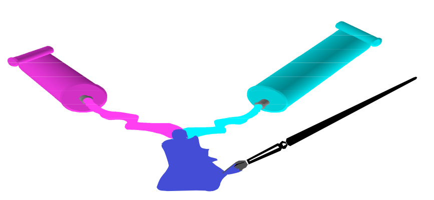
The reflectance is the fraction of the incident light that is diffusely reflected. As in the case of transmittance, the reflectance usually depends on the wavelength of light.

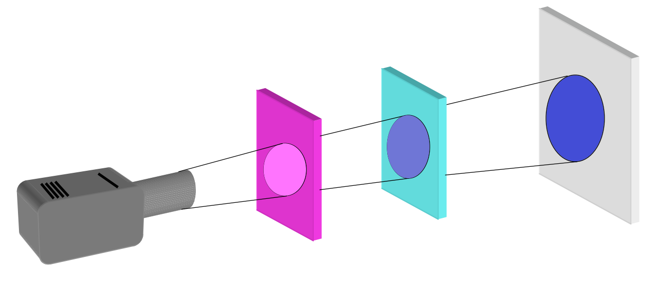


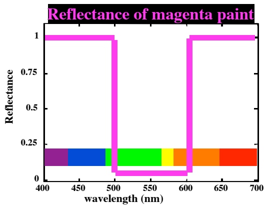
The reflectance indicates the amount of light diffusely reflected by the leaf in the RGB ranges, separately, as shown in the table on the right. The reflectance determines the color of the leaf, as we see it.

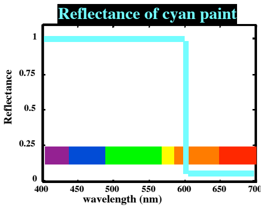
Simplified spectral ranges	Incident intensity	R_{leaf}	Reflected light
B (400-500 nm)	0.8	0	0
G (500-600 nm)	0.9	1	0.9
R (600-700 nm)	1	0.5	0.5

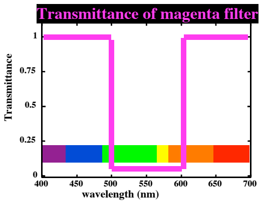
When mixing pigments of two different colors, the resulting color depends on the reflectance curves for the two colors. The reflectance values can be combined in the exact same manner as described above for transmittances of filters. For the case of magenta and cyan described below, it is completely equivalent to mix paints or overlapping filters. In one case we would use the reflectance curve and table, in the other the transmittance, but the mathematical operation (the product in each spectral range), and the resulting colors are identical.

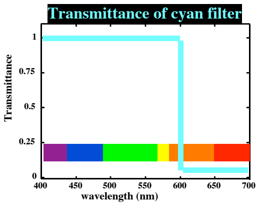












Simplified spectral ranges	R_M	R_C	$R_{M\&C} = R_M \times R_C$
B (400-500 nm)	1	1	1
G (500-600 nm)	0	1	0
R (600-700 nm)	1	0	0

Simplified spectral ranges	T_M	T_C	$T_{M\&C} = T_M \times T_C$
B (400-500 nm)	1	1	1
G (500-600 nm)	0	1	0
R (600-700 nm)	1	0	0

It is worth noticing that the reflectance values in each spectral range correspond exactly to the RGB intensities we discussed in additive color mixing, and all colors, defined *only* by their reflectance or transmittance curves, can be located on the color triangle, after having calculated the fraction of red r and the fraction of green g.

On a computer, magenta and cyan would have the RGB intensities:

$M = 255B + 0G + 255R$
and
$C = 255B + 255G + 0R$

Whereas from the reflectance or transmittance curves of the previous page we would obtain:

$M = 1B + 0G + 1R$
and
$C = 1B + 1G + 0R$

We already discussed the fact that the units and the absolute values of each color intensity (a, b and c) are not relevant, since the only relevant quantities are the fractions of red and green.

The fraction of red and green for M and C are:

$r_M = 0.5$ and $g_M = 0$;
$r_C = 0$ and $g_C = 0.5$.

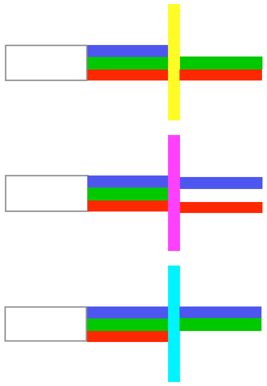
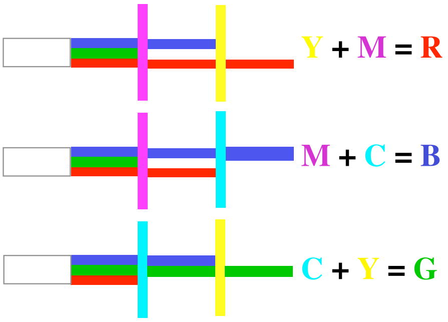
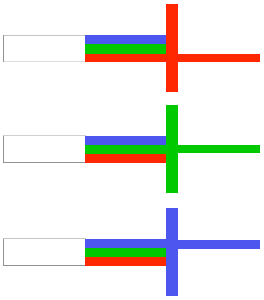
Based on these coordinates, it is obvious where M and C are positioned on the color triangle. When subtractively combining colors of known reflectances, if the resulting reflectance is hard to identify, the fractions of red and green can easily be calculated, and the color can be identified on the color triangle.

SUBTRACTIVE PRIMARY COLORS

The ***subtractive primary colors are cyan, magenta and yellow***. Each one of these colors absorbs one additive primary, its complementary color.

Cyan absorbs red, magenta absorbs green and yellow absorbs blue. All three combined, ***C, M and Y produce black***. Again, these are not the only colors that can be chosen as subtractive primaries. Any other three colors that combined produce black are potential primaries, although the mixing potential, that is, the number of colors that can be obtained mixing three primaries is maximum with CMY.

The diagram in the next page shows the absorption of the three subtractive primaries (on the left) when CMY filters are used. The central part of the diagram shows how to obtain the additive primaries using combinations of CMY filters. Paints, inks and pigments in general behave just like filter, but in diffusely reflected light, instead of transmitted light.

Subtractive primaries		
 Each filter (or pigment) absorbs its complementary color and transmits (diffusely reflects) the others.	 Combinations of subtractive primaries to obtain R, G, and B.	 R, G and B filters transmit the R, G and B light. They absorb their complementaries as well, B and G (C), B and R (M), G and R (Y), respectively.

In subtractive color mixing one often obtains colors that seem unexpected or surprising. The ***hue and the purity*** of the light subsequently transmitted by two filters is not intermediary between the hues of the two filters. The same applies for pigments. You can look on the color triangle where colors are, and compare with the filter or paint mixtures obtained above and below.

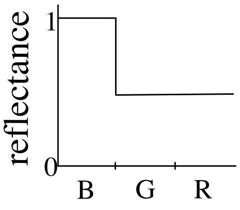
Let us now look at a few examples of subtractive color mixing that show changes in purity or hue.

CHANGE IN PURITY

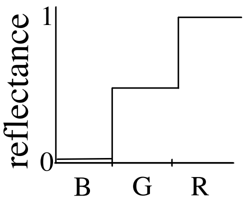
Sky B+	**Orange =**	**Orange** (pure, low intensity)
B=128	B=0	B=0
G=128	G=128	G=64
R=255	R=255	R=128

From the RGB intensities given for each color, we can draw the reflectance curves, and write the reflectance table.

sky blue
(blue + white)



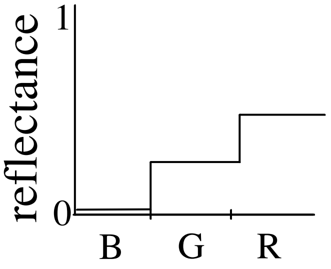
orange
(yellow + red)



Simplified spectral ranges	R_{uB}	R_O	$R_{uB\&O} = R_{uB}$ x R_O
B (400-500 nm)	1	0	0
G (500-600 nm)	0.5	0.5	0.25
R (600-700 nm)	0.5	1	0.5

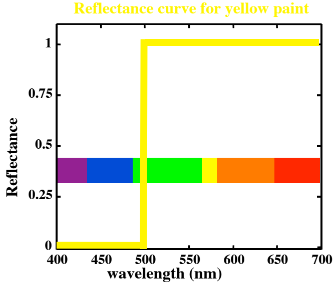
Sky blue is unsaturated B (uB in the table), or low purity blue, i.e. B + white. The B reflectance curve therefore will have white in it, that is, a certain amount (0.5 in this example) of all 3 spectral components, R, G and B. Adding white is equivalent to adding a horizontal line in the reflectance curve, or shifting the whole reflectance curve up. Orange, as should be evident from the RGB components, and from the reflectance curve, is saturated. ***As long as at least one of the RGB components is 0, the color is saturated***. As long as the reflectance curve is at 0 intensity in at least one spectral range, there is certainly no horizontal line added to the reflectance, that is, there is no white, and the color is pure.

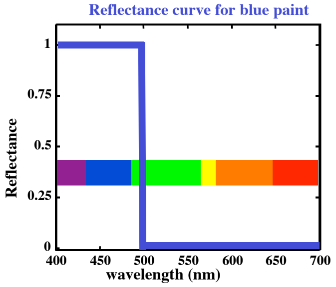
The reflectance curve for orange is a direct consequence of how orange is obtained, additively: 1 part of Y light, and 1 part of R light. But Y = R + G lights, therefore orange must contain 2 parts of R and 1 part of G, as shown in the reflectance curve of the previous page.

The color resulting from the mixing of sky blue and orange is pure (saturated), since its reflectivity in the blue range is 0. The lineshape of the reflectance obtained for this color is reported on the right here. This curve is identical to the one for orange, but with lower intensity: this color is brown.	Brown (low intensity orange) 

BLUE AND YELLOW DON'T MAKE GREEN?

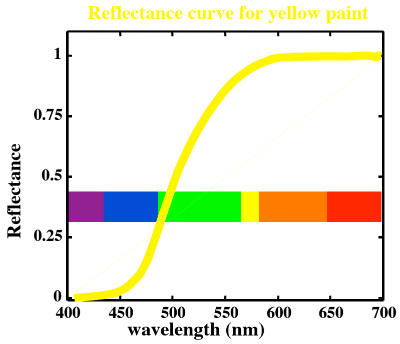
Have you ever tried to mix blue and yellow paints? If you did, you certainly obtained green. Now, let us look at the reflectance curves for blue and yellow paints:

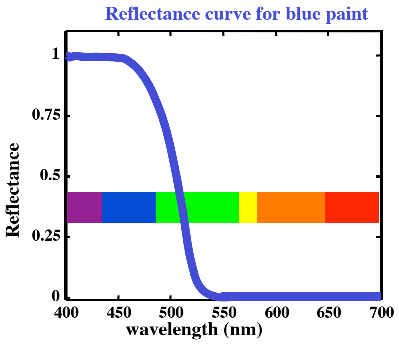


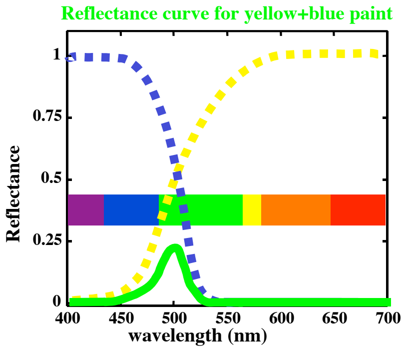


As you can see, there is no spectral region in which the two curves overlap. In other words, at no wavelength the product of reflectivities is non-zero. The result, therefore, should be zero everywhere, that is, black. But if we mix the paints we obtain green: something is wrong here! Have we been always wrong in all the previous examples of color mixing? How can we explain the blue+yellow mystery?

The solution is a bit tricky. Remember that when we introduced the “oversimplified” spectrum, composed by 3 colors only, RGB, we said that that in most cases it works? Well, this is one of the rare cases in which it does not work. We cannot consider the reflectivities of yellow and blue as having 3 values only (0, 1, 1 for yellow, and 1, 0, 0 for blue, in the R, G and B regions respectively). We must consider that the real reflectivity curves have many more values, not only three. In reality, these curves look as shown below.


Reflectance curve for yellow paint


Reflectance curve for blue paint


Reflectance curve for yellow+blue paint

Therefore, when calculating the reflectance of the resulting paint color, we find that it is non-zero in the green region, as shown in the curve on the right. That is why the mixture of blue and yellow appears green.

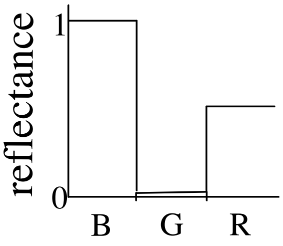
In all other cases of color mixing we discussed before and will discuss later, the oversimplification of considering only 3 colors (RGB) in each reflectance curve does not constitute a problem, and the results can be trusted.

CHANGE IN HUE

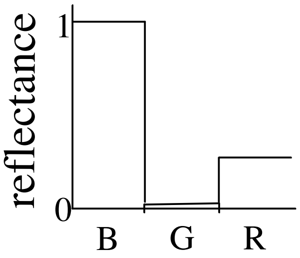
We mentioned before that in most cases mixing pigments or overlapping filters is completely equivalent. There are a few exceptions to that rule. The two examples below only apply to filters. Mixing two parts of the same color, when mixing pigments (in paints or inks) will generate twice the amount of the same color, and there is of course no change in hue.

M-purple +	**M-purple** =	**Bluish-M**
B=255	B=255	B=255
G=0	G=0	G=0
R=128	R=128	R=64

The light transmitted by two subsequent identical magenta-purple filters, is not magenta-purple, it is bluish-magenta. Again, looking at the transmittance table will explain this unexpected result.

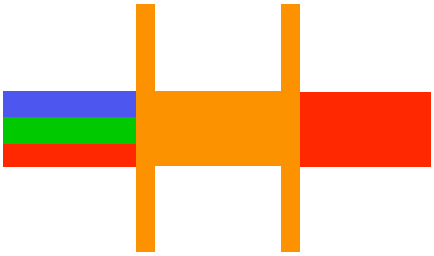

M-purple

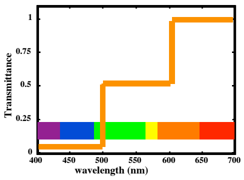
Simpl. ranges	T_{MP}	T_{MP}	$T_{BM} = T_{MP} \times T_{MP}$
B	1	1	1
G	0	0	0
R	0.5	0.5	0.25

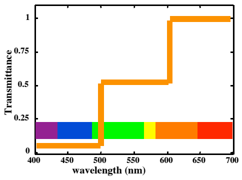

Bluish-M

The same phenomenon occurs when using two orange filters.

Two orange filters when used in combination transmit reddish-orange




Transmittance curve for filter A


Transmittance curve for filter B

Simpl. ranges	T_O	T_O	$T_{RO} = T_O \times T_O$
B	0	0	0
G	0.5	0.5	0.25
R	1	1	1

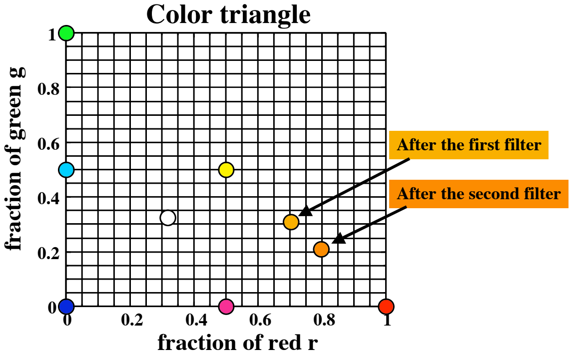
After the first filter

$$r = \frac{1}{1.5} = 0.7 \quad \text{and} \quad g = \frac{0.5}{1.5} = 0.3$$

After the second filter

$$r = \frac{1}{1.25} = 0.8 \text{ and } g = \frac{0.25}{1.25} = 0.2$$

We can then, with r and g, localize the two hues on the color triangle.


Color triangle

After the first filter

After the second filter

Although these surprising changes in hue do not apply to mixing pigments, painters are well aware of them. They will not affect oil or acrylic painting much, but certainly do affect watercolor painting. As painters know, one layer of watercolor on white paper is partly transparent, and therefore acts as the filters in front of the projector. If they paint a second layer over the first, with the exact same color, the hue changes.

COLOR PHOTOGRAPHY

All color films consist of three layers of silver halide (iodide or bromide) emulsion, coated one on top of each other to form a permanent, multi-layered structure supported by the film base. The top emulsion layer absorbs B light and transmits G and R light to the lower layers. The middle and lower emulsion layers respond only to G and R, respectively. Each layer of a color negative film emulsion absorbs a specific color, prescribed by the properties of the dyes embedded in that layer. There is a chemical reaction in that emulsion, which permanently modifies the emulsion after exposure and absorption of light.

Objects of other colors react with two or more of the emulsion layers, according to the spectral distribution of the light they reflected, which illuminated the film during exposure. The original color of the object is reproduced when the negative is printed. ***Color negative film*** produces an image that is complementary with respect to colors and density of the original objects. Subsequently, the negative is printed on color photographic paper, to yield ***a positive color print***.

After exposure, the negative film is processed to yield the color negative. The processing steps are identical for all C-41 type processes and include a color developer, bleach, fixer, bleach/fix, final wash, stabilizer, and drying. Let us look at all steps.

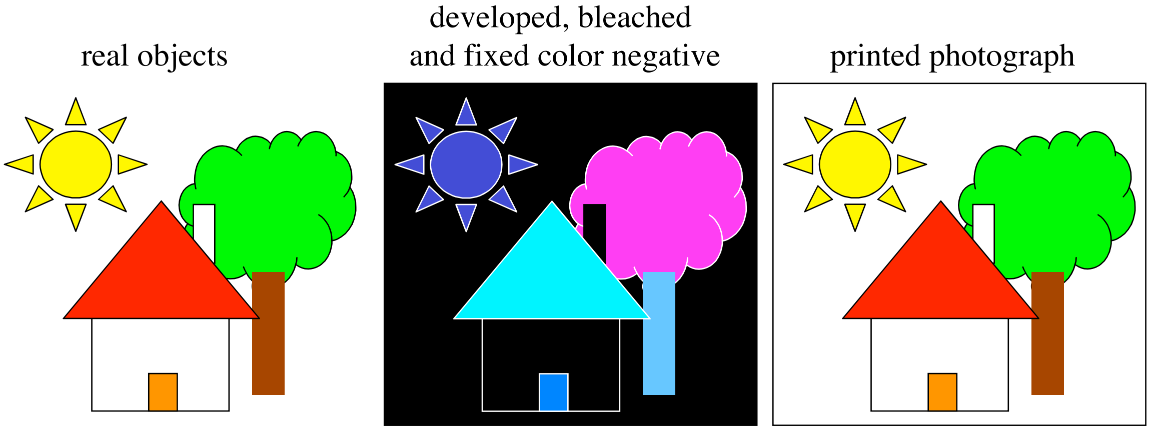
The ***chromogenic developer*** converts light-sensitive crystallites of silver halide compounds in the emulsion layers into metallic silver. While doing this, the developer also oxidizes and combines with dye couplers that are either built into the emulsion layers or added during development. The result is the formation of three dye layers, one from each of the subtractive primary colors: cyan, magenta, and yellow. The blue-sensitive layer of the original film forms a yellow image, while the green and red-sensitive layers form magenta and cyan images, respectively. ***Bleach*** is then utilized to remove all of the silver metal, and each layer is left with only a color image. After bleaching, the film is ***fixed***, and ***washed*** exhaustively to remove all solubilized silver salts, then rinsed in ***stabilizer*** to improve dye stability and harden the emulsion, and dried.

unexposed
color film (side view)

exposed and developed
color film

bleached and fixed
color film

The unexposed film (left) appears gray. Once it has been exposed and developed the B, G and R absorbing layers are colored. After bleaching and fixing the silver halide crystals are removed, and only the color layers remain, where appropriate, to produce a negative of the image.


real objects

developed, bleached and fixed color negative

printed photograph

The processed color film is a negative image in which ***all colors are inverted***, that is, they are complementary to colors in the original objects. These colors, e.g. the cyan roof and the blue sun, can be seen in transmission on the negative, and act exactly like the combination of subsequent CMY filters. The negative has the C layer present only where the roof is located in the image, and the M and C layers where the sun is. The Y layer is absent from both areas of the processed negative. The black areas on the negative have all three layers. White light is then transmitted through the processed film, and projected onto color sensitive paper for printing. The paper is coated with a similar emulsion, which inverts the colors again.