CS 525, Semester II, 2011–12
Midterm Examination
Wednesday 7th March, 2012

Each problem can be solved in 4 tableaus or less including the initial tableau.

1. (a) Find all solutions of \( Ax = b \) where:
\[
A = \begin{bmatrix}
1 & 2 & 1 \\
-2 & -3 & 1 \\
4 & 2 & 2 \\
3 & 5 & 1
\end{bmatrix}, \quad b = \begin{bmatrix}
-1 \\
-3 \\
-22 \\
-1
\end{bmatrix}
\]

(b) Express a linear dependence relationships among the rows of \( A \).

2. Consider the following linear program:
\[
\begin{align*}
\text{min} & \quad 6x_1 - x_2 \\
\text{subject to} & \quad x_1 - 2x_2 \geq -4, \\
& \quad x_1 - x_2 \geq -7, \\
& \quad x_1, x_2 \geq 0.
\end{align*}
\]

(a) Solve this problem.

(b) Write down the dual of the given problem and the KKT conditions.

(c) Find a dual solution \( u^* \) (by inspection of the KKT conditions).
3. Solve the following linear program using the simplex method

\[
\begin{align*}
\text{max} & \quad 3x_1 + x_2 + x_3 + 10 \\
\text{subject to} & \quad x_1 + x_2 + 2x_3 = 5 \\
& \quad 2x_1 - x_3 \leq 2 \\
& \quad -3 \leq x_1 \leq 0 \\
& \quad x_2 \geq -2
\end{align*}
\]

Explicitly justify why you carry out the particular pivots. Is the solution unique?

4. Consider the problem:

\[
\begin{align*}
\text{min} & \quad p'x \\
\text{subject to} & \quad Ax \geq b, \quad \text{where} \quad b = \begin{bmatrix} -1 \\ 0 \\ 2 \\ 1 \end{bmatrix} \\
& \quad x \geq 0
\end{align*}
\]

(a) Write down the Phase I problem (as an optimization problem, not as a tableau).

(b) Write down the dual of the Phase I problem, call it (D0).

(c) Prove that (D0) has a solution.

(d) What can you say if the optimal value of (D0) is 1.5?

Be sure to quote any results you use to justify your comments.