Exercise 3-0-2. Consider the problem

\[
\begin{align*}
\text{min} & \quad z = p'x \\
\text{subject to} & \quad Ax \geq b, \quad x \geq 0
\end{align*}
\]

where

\[
A = \begin{bmatrix}
0 & -1 \\
-1 & -1 \\
-1 & 2 \\
1 & -1
\end{bmatrix}, \quad b = \begin{bmatrix}
-5 \\
-9 \\
0 \\
-3
\end{bmatrix}, \quad p = \begin{bmatrix}
-1 \\
-2
\end{bmatrix}
\]

(i) Draw the feasible region in \( \mathbb{R}^2 \).

(ii) Draw the contours of \( z = -12, z = -14, z = -16 \) and determine the solution graphically.

(iii) Solve the problem in MATLAB using Example 3-0-1 as a template. In addition, trace the path in contrasting color that the simplex method takes on your figure.

3.1 The Phase II Procedure

The simplex method is typically split into two phases. Perversely, we treat the second phase first of all. It can be summarized by the following steps. Note that \( B(r) \) is the label on \( x \) corresponding to the \( r \)th row of the tableau, and \( N(s) \) is the label on \( x \) corresponding to the \( s \)th column of the tableau.

1. Put the problem into standard form (3.1).

2. Construct an initial feasible tableau. (For now we shall assume the origin is feasible. Later we will show how to use a Phase I procedure to obtain a feasible tableau if one exists, when the origin is infeasible.)

3. Use the pricing rule to determine the pivot column \( s \). If none exists, stop (a), tableau is optimal.

4. Use the ratio test to determine the pivot row \( r \). If none exists, stop (b), tableau is unbounded.

5. Exchange \( x_{B(r)} \) and \( x_{N(s)} \) using a Jordan exchange on \( H_{rs} \).