CS 635: Tools and Environments for Optimization Homework 11: Nonlinear Programming Formulation Exercises

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Submit this assignment electronically using the instructions on the course web page. The assignment name is hw11 and you should hand in exactly 3 files with the following names:
hw11-1.gms, hw11-2.gms, hw11-3.gms

1. The variable y represents the yield in a chemical process. There are n process variables $x_1, x_2, \ldots, x_n$ (such as temperature, flow rate, etc) which influence the yield. Data was collected to observe the yield y for various values of the process vector $x^t = (x_1^t, x_2^t, \ldots, x_n^t)$. It is believed that y can be reasonably approximated by a convex quadratic objective function. Formulate the problem of finding the best convex quadratic approximation $Q(x)$ for y using the available data as a nonlinear program, and discuss the important features of your formulation.

   To test your model, ensure that you write GAMS statements to generate random inputs that are consistent with the above hypothesis. Also, write statements to print out the results of the model to show how the solution relates to your random inputs. [Hint: you may want to use the fact that if $Q = R^T R$ then Q is guaranteed to be symmetric positive semidefinite.]

2. Consider the problem

$$\begin{array}{ll} \min_{x_1,x_2,x_3} & 10/(x_1x_2x_3)^2 \\ \text{subject to} & 12(x_1)^2x_2 + 4x_3 \leq 1 \\ & 0.1x_2\sqrt{x_1} + x_2x_3 \leq 1 \\ & (x_1x_2)^{1/3} \leq 1 \\ & x_1, x_2, x_3 > 0 \end{array}$$

   (a) Formulate this problem as a standard nonlinear program and solve using GAMS. Note that you will have to impose small lower bounds on the variables since it is impossible to impose strict positivity of variables directly. Experiment with various starting points to ensure you find a good solution.

(b) Reformulate the problem using an (invertible) nonlinear change of variables. For an appropriate choice you can remove the positivity constraint entirely and the resulting problem will be a convex program. Solve this problem in GAMS and invert the solution given to determine the values of the original variables. Is this globally optimal? You may add comments to the gams file if you wish.

3. What's the biggest circle you can make from a 4 foot by 8 foot piece of plywood if you are willing to make one straight line cut and one join? In other words, you can lop off one end and glue it on to the other edge. But just once. Assume that the cut has to be 90 degrees.

   This is a real life problem. It came up the other day when I was talking to my friends about building a card table. I don't want to make too many cuts and do too much gluing. But 4 feet is too small for a card table.