

CS635 – Problem Set #3

Due Date: February 13, 2009

Instructions for Handing In Homework

Formulate the following problems in GAMS and solve them. Submit this assignment electronically using the instructions on the course web page. You should hand in exactly 6 files with the following names: `hw3-1.gms`, `hw3-1.txt`, `hw3-2.gms`, `hw3-2.txt`, `hw3-3.gms`, `hw3-3.txt`.

Use an editor to extract the required lines of output to the “txt” file, namely the model and solution status, the optimal values for the variables and the optimal value of the objective function. Ensure you use self-explanatory variable names.

1 GlassCo

Glassco manufactures wine glasses, beer glasses, champagne glasses and whiskey glasses. Each type of glass uses time in the molding shop, time in the packaging shop, and a certain amount of glass. The resources required to make each type of glass are given in Table 1.

Table 1: GlassCo Resource Requirements

	WINE GLASS	BEER GLASS	CHMPGNE GLASS	WHISKEY GLASS
Molding time	4 minutes	9 minutes	7 minutes	10 minutes
Packaging time	1 minute	1 minute	3 minutes	40 minutes
Glass	3 oz	4 oz	2 oz	1 oz
Selling price	\$6	\$10	\$9	\$20

At present, 600 minutes of molding time, 400 minutes of packaging time and 500 oz of glass are available.

1.1 Problem

Write down and solve the LP (in GAMS) that Glassco should solve, assuming the company wishes to maximize revenue. You should use appropriate sets to index both the variables you choose and the constraints that you formulate.

1.2 Problem

Write down (and solve) the dual of this LP problem, in the same GAMS file. You should set up two separate models and include just those equations needed in each model in the model statement.

Please also make note how the marginals of the constraints in Problem 1.1 are related to the level of the variables in Problem 1.2. What about the marginals of the constraints in Problem 1.2 and the variables in Problem 1.1?

2 Efficient farming

A farmer can lease land up to 1000 acres. She has to pay \$6 per acre (per year) if she leases up to 600 acres. For any land beyond 600 acres, she can lease at \$8 per acre. She grows corn on the land. She can grow corn at normal level or at an intense level (more fertilizer, frequent irrigation, etc.) Normal level yields 70 bushels/acre. Intense level yields 100 bushels/acre. The normal and intense levels require, respectively, 7 and 9 hours of labor per acre, and \$20 and \$30 in materials (such as seed, fertilizer, water, etc.) per acre. (On each acre, some amount can be at the normal level and some at the intense level.) Harvesting requires 0.1 hours of labor per bushel harvested. Harvested corn can be sold up to any amount at the rate of \$2.50 per bushel. The farmer can raise poultry at the same time. Poultry is measured in poultry units. To raise one poultry unit requires 24 bushels of corn, 20 hours of labor, and 25 cubic feet of shed space. She can either use the corn she herself has grown or buy corn from the retail market. She gets corn at the rate of \$3.20/bushel from the retail market. She can sell at the price of \$180 per poultry unit in the wholesale market up to 200 units. Any amount of poultry over 200 units sells for just \$165 per unit. She has only one shed for raising poultry, and it has 15,000 cubic feet of space. She and her commune can contribute 4000 hours of labor per year at no cost. If she needs more labor, she can hire it at \$4/hour up to 3000 hours. For any amount of labor over 3000 hours, she has to pay \$6 per hour.

2.1 Problem

Formulate and solve a GAMS model to maximize net profit. Briefly explain the computational advantages of your particular formulation.

3 Milk production

Happy Milk Distributors (HMD) purchases raw milk from farmers in two regions: A and B. Prices, butterfat content and separation properties of the raw milk differ between the two regions. HMD processes the raw milk to produce cream and milk to desired specifications for distribution to the consumers.

Region A Raw Milk Milk from region A is 54 cents per gallon up to 500 gallons and 58 cents per gallon in excess of 500 gallons. There is no upper bound on the amount that can be purchased. Raw milk from Region A has 25% butterfat and when separated (at 5 cents per gallon) it yields two “milks”, one with 41% butterfat and another with 12% butter fat.

The volume of milk is conserved in *all* separation processing.

Region B Raw Milk The purchase price for milk from region B is 38 cents per gallon up to 700 gallons and 42 cents per gallon thereafter. Raw milk from Region B has 15% butterfat and when separated (at 7 cents per gallon) yields two “milks”, one with 43% butterfat and another with 5% butterfat.

Production Process After the milk is purchased and collected at the plant, it is either mixed directly or separated and then mixed. The mixing is done at no cost, and its purpose is to produce cream and milk to specifications. For example, some of the raw milk from Region A may be separated and then mixed, and some of it may be mixed directly (i.e., without

having been separated).

Demand and Selling Price The demand and selling price are described in the following table:

	Minimum req'd % of butterfat	Max vol dem (gallons)	Selling price (cents/gallon)
Cream	40	250	150
Milk	20	2000	70

For example, all the cream produced must have at least 40% butterfat, it sells at \$1.50 per gallon, and as much as 250 gallons of the cream produced can be sold.

3.1 Problem

Assuming free disposal, *formulate* a linear program in GAMS which when solved enables HMD to maximize its profit.

3.2 Problem

Suppose that the purchase price for milk from region B were 50 cents per gallon up to 700 gallons and 30 cents per gallon thereafter. Can you still formulate the profit maximization problem as a linear program? If not, why not?