CS635 – Problem Set #8

Due Date: March 27, 2009

Instructions for Handing In Homework

Formulate the following problems in GAMS and solve them. Submit this assignment electronically using the instructions on the course web page. You should hand in exactly 4 files with the following names: `hw8-1.gms, hw8-2.gms, hw8-3.gms, hw8-3.lst`.

Ensure you use self-explanatory variable names, and that you comment any assumptions made to clarify your model formulation. Also, ensure that answers required for the question are displayed as the last line of the listing file produced, or in a put file that you generate.

1 Lights Out

In Tiger Electronic's handheld solitaire game Lights Out, the player strives to turn out all 25 lights that make up a 5 x 5 grid of cells. On each turn, the player is allowed to click on any one cell. Clicking on a cell activates a switch that causes the states of the cell and its (edge) neighbours to change from on to off, or from off to on. Corner cells are considered to have 2 neighbours, edge cells to have three, and interior cells to have four.

1.1 Problem

Formulate and solve an integer program for finding a way to turn out all the lights in as few turns as possible (starting from the state where all lights are on). Hints: The order in which the cells are clicked doesn't matter. A cell should not be clicked more than once.

1.2 Problem

What if each cell has a three-way bulb? (Repeatedly clicking on a single three way bulb changes its state from off to low, from low to medium, from medium to high, from high to off, and so on.) Which is easiest (a) turning off all the lights when they're all on their high setting, (b) turning them off when they're all on medium, or (c) turning them off when they're all on low?

2 Discounting Goods

There are three suppliers of a good, and they have each quoted various prices for various quantities of a given product. We want to buy at least total cost, yet not buy too much from any one supplier. To alleviate over reliance on a single supplier, the maximum percentage of the total supply from suppliers 1 and 3 is 40%, but only 35% from supplier 2 is allowed.

Each supplier offers decreasing prices for increased lot size, in the form of incremental discounts. Assume the cost function has three break points for each supplier s_i.

```
COST 1   2   3
s1   9.2 9   7
s2   9   8.5 8.3
s3   11  8.5 7.5
```

and that the break points occur at the following levels:

```
BR 0 1   2   3
s1 0 100 200 1000
s2 0 50  250 2000
s3 0 100 300 4000
```

2.1 Problem

Determine how we should buy 600 items in total to minimize total cost.

3 Piecewise Linear Networks

Consider a transportation network with 3 commodities. Demand exists between certain origin-destination (OD) pairs for each commodity.

```
option seed=0; set nodes /1*100/;
parameter offset(nodes); offset(nodes) = round(uniform(2,5));

alias (i,j,nodes); set arcs(nodes,nodes);
arcs(i,j) = no; arcs(i,i+1) = yes; arcs(i,i+offset(i)) = yes;

set k /1*3/;
parameter demand(nodes,k) /
  1.1 -70, 6.1 70, 3.2 -25, 100.2 25, 4.3 -20, 8.3 20, 54.1 -70, 55.1 70
  23.2 -25, 89.2 25, 10.3 -20, 89.3 20, 20.3 -10, 50.3 10 /;
```

The cost of sending commodities over each arc can be modeled using log(x+1) where x represents the sum of the flows of each commodity over that arc. Total flow on each arc must be between 0 and a given capacity.

```
parameter capacity(i,j); capacity(i,j) = uniform(75,85);
```

These data statements can be downloaded from the class web page.

3.1 Problem

By approximating the concave function log(x+1) using a piecewise linear function, formulate and solve this problem for the data given below. Note that the breakpoints for each log function should be 0,5,10,100. Use appropriate cplex options if necessary to ensure the optimal solution is found to within a 10% relative tolerance, and give lower and upper bounds on the optimal solution value. Hand in the lst file for this problem.