Homework 1

CS 726, Semester I, 2009–10

September 3, 2009

First of all read Appendix A of the Nocedal-Wright book. This material covers the prerequisites of this course.

1. For each value of the scalar β, find the set of all stationary points $\{x \mid \nabla f(x) = 0\}$ of the following function of variables x_1 and x_2:

$$f(x) = x_1^2 + x_2^2 + \beta x_1 x_2 + x_1 + 2x_2$$

   Which of these stationary points are global minima?

2. In each of the following questions fully justify your answers using the optimality conditions:

   (a) Show that the 2-dimensional function $f(x, y) = (x^2 - 4)^2 + y^2$ has two global minima and one stationary point which is neither a local minimum or a local maximum.

   (b) Find all local minima of the 2-dimensional function $f(x, y) = \frac{1}{2}x^2 + x \cos y$.

   (c) Show that the 2-dimensional function $f(x, y) = (y - x^2)^2 - x^2$ has only one stationary point, which is neither a local minimum or a local maximum.

   (d) Find all local minima and all local maxima of the 2-dimensional function $f(x, y) = \sin x + \sin y + \sin(x + y)$ within the set

$$\{(x, y) \mid 0 < x < 2\pi, 0 < y < 2\pi\}.$$

3. Let $f \colon \mathbf{R}^2 \to \mathbf{R}$ be defined by

$$f(x) = x_2^2 - a x_2 \|x\|^2 + \|x\|^4$$

   where $0 < a < 2$. Show that $f(x) > 0$ for all $x \neq 0$ so that the origin is the unique global minimum. Show also that there exists a $\bar{\gamma} > 0$ such that for all $\gamma \in (0, \bar{\gamma}]$, the level set $L_\gamma = \{x \mid f(x) \leq \gamma\}$ is not convex. Hint: Show that for $\gamma \in (0, \bar{\gamma}]$, there is a $p > 0$ and a $q > 0$ such that the vectors $(-p, q)$ and (p, q) belong to L_γ but $(0, q)$ does not belong to L_γ.