Homework 6

CS 726, Semester I, 2011–12

November 11, 2011

The programming portion of this assignment should be submitted electronically using the instructions on the course web page. The assignment name is hwk6 and you should hand in exactly 2 files with the following names:
GaussN.m, consLS.m

1. Modify your Newton (or other) program from previous Homework to solve nonlinear least squares problems via a Gauss Newton method. You can use the same stepsize procedure, simply modifying the code in Newton that computes the direction. You should use line search parameters $c_1 = 10^{-4}$ and $c_2 = 0.9$ and always initialise your StepSize routine with $\alpha = 1$.

```
function [inform,x] = GaussN(fun,x,nparams)
```

Within your code you need to solve for the Gauss Newton Step, $J'Jd = -J'\bar{r}$. Based on the value in `nparams.lsmethod`, calculate this step using one of the three methods "chol", "qr" and "svd" (you will need to implement all three of these, and use if then else or switch statements to determine which to use on a particular run).

Use `hwk6.m` and `objnls.m` that can be found on the course website to test your code on the following two problems:

$$r_i(x) = y_i - \left(x_1 + \frac{u_i}{v_i x_2 + w_i x_3} \right)$$

where $u_i = i$, $v_i = 16 - i$, $w_i = \min(u_i, v_i)$ and

$$y = (0.14, 0.18, 0.22, 0.25, 0.29, 0.32, 0.35, 0.39, 0.37, 0.58, 0.73, 0.96, 1.34, 2.10, 4.39)$$

and

$$r(x) = a + Hx + \frac{50}{2}((x-1)'B(x-1))d$$

with 1 as a vector of 1's of appropriate dimension and

```
a = [0.13294; -0.244378; 0.325895];
d = [2.5074; -1.36401; 1.02282];
H = [-0.564255 0.392417; -0.404979 0.927589; -0.0735084 0.535493];
B = [5.66598 2.77141; 2.77141 2.12413];
```

These residuals and their Jacobians are calculated in `resida.m`, `residb.m`, also available on the website. The routines are similar to the objnls.m file with a header:

```
function varargout = resida(x,mode)     function varargout = residb(x,mode)
```

2. Test your code on the nonlinear least squares problems that are given in the AMPL files `penalty1.mod` and `penalty2.mod` for which the residuals are encoded in `residc.m` and `residd.m`. Compare your results to any other code of your choice running on these problems (you can use the 'objampl" files to get function, derivative and Hessian evaluations for these). Contrast the solution methods, and explain why they perform so differently.

3. Suppose $A \in \mathbf{R}^{m\times n}$ with $m \leq n$ and suppose A has full row rank. Show that the set of solutions of $Ax = b$ is given by

$$x_0 + \ker(A), \quad \text{where } \ker(A) = \{x \mid Ax = 0\}$$

and x_0 satisfies $Ax_0 = b$. Show how to construct a basis for $\ker(A)$ using a QR factorization of A^T. Use this result to solve:

$$\min_x \|Cx - d\|_2^2 \text{ st } Ax = b$$

for the matrices given at the end of hwk6.m. The input and output parameters should be easy to infer by inspection of hwk6.m.