

Homework 7

CS 726, Semester I, 2009–10

November 9, 2009

1. Apply Newton's method with a constant stepsize to minimize the function $f(x) = \|x\|^3$. Identify the range of stepsizes for which convergence is obtained, and show that it includes the unit stepsize. Show that for any stepsize within this range, the method converges linearly to $x^* = 0$. Explain this fact in light of the results proven in class for local convergence of Newton's method.
2. Consider the steepest descent method

$$x_{k+1} = x_k - s(\nabla f(x_k) + e_k),$$

where s is a constant stepsize, e_k is an error satisfying $\|e_k\| \leq \delta$ for all k , and f is the positive definite quadratic function

$$f(x) = \frac{1}{2}(x - x^*)^T Q(x - x^*).$$

Let

$$q = \max\{|1 - sm|, |1 - sM|\},$$

where m and M are the smallest and largest eigenvalues of Q respectively and assume that $q < 1$. Show that for all k , we have

$$\|x_k - x^*\| \leq \frac{s\delta}{1 - q} + q^k \|x_0 - x^*\|.$$

3. Consider the Nelder-Mead simplex method applied to a strictly convex function f . Show that at each iteration, either $f(x_{n+1})$ decreases strictly, or else the number of vertices x_i of the simplex such that $f(x_i) = f(x_{n+1})$ decreases by at least one.