Homework 8

CS 726, Semester I, 2011–12

December 7, 2011

The computational part of this assignment should be submitted electronically using the instructions on the course web page. The assignment name is hwk8 and you should hand in a file with the following name:

hwk8.m

The remaining parts can be handwritten (or submitted in hwk8.txt) and should be submitted at class time on December 14.

1. Solve the equality constrained quadratic problem given at the end of the hwk7.m file using the "quadprog" code within Matlab. Update the options for that code to use the active set algorithm. Compare the results of this run (both solution values and multiplier values) to a code that simply forms the KKT conditions and solves using an LDL' factorization as detailed in class. Modify the file hwk8.m to encode your answers to the above explicitly there.

2. Using the data provided in hwk8.mat (and loaded at the end of hwk8.m), modify the file hwk8.m to use quadprog to solve the general quadratic problem given by that data. Note that ineq and eq give the indices of the less than or equal constraints, and the equality constraints respectively. Experiment with a number of different options for quadprog, and indicate in the update hwk8.m file which options are the best for this problem.

3. Among all rectangles contained in a given circle, show that the one the has maximal area is a square.

4. Consider the unconstrained problem

$$\min_x f_0(Ax + b)$$

   Show that the Lagrange dual function is the constant p^*, so that while we do have strong duality, i.e. $p^* = d^*$, the Lagrangian dual is neither useful or interesting.

5. We define the conjugate function of f by:

$$f^*(y) = \sup_{x \in \operatorname{dom} f} (y^T x - f(x))$$

   Show that if you reformulate the problem from the previous question as:

$$\min f_0(y) \text{ s.t. } Ax + b = y$$

   then write down the Lagrangian, the Lagrange dual function in terms of the conjugate of f_0 and finally the dual problem of the reformulated problem. Note that this dual is more useful and interesting than that of the previous question.

6. Apply the construction of the previous question to

$$\min \log \left(\sum_{i=1}^{m} \exp(a_i^T x + b_i) \right)$$

Note that you should be able to work out an explicit form for the conjugate function. You may assume that $0 \log 0 = 0$.