1. Show that for a given set \( C \) and \( x \in C \) that \( N_C(x) \) is closed.

2. Let \( K \) be a nonempty closed convex cone in \( \mathbb{R}^n \) and let \( x \in \mathbb{R}^n \). Show that
   
   (a) \( \pi(x) \) is the projection of \( x \) on \( K \) if and only if
   
   \[ \pi(x) \in K, \quad \langle (x - \pi(x)), \pi(x) \rangle = 0, \quad x - \pi(x) \in K^\circ. \]

   (b) The following two statements are equivalent:
   
   i. \( x_K \) and \( x_{K^\circ} \) are the projections of \( x \) onto \( K \) and \( K^\circ \) respectively,
   
   ii. \( x = x_K + x_{K^\circ} \) with \( x_K \in K \) and \( x_{K^\circ} \in K^\circ \), and \( \langle x_K, x_{K^\circ} \rangle = 0 \).

3. Show that if \( P \) is a polyhedral set in \( \mathbb{R}^n \) containing the origin then cone(\( P \)) is a polyhedral cone. Given an example showing that if \( P \) does not contain the origin then cone(\( P \)) may not be a polyhedral cone.

4. Let \( f: \mathbb{R}^n \to \mathbb{R} \) be a quadratic function of the form
   
   \[ f(x) = \langle x, Qx \rangle + \langle c, x \rangle, \]

   where \( Q \in \mathbb{R}^{n \times n} \) is symmetric and \( c \in \mathbb{R}^n \), and suppose \( C = \{ x: Ax \leq b \} \). Use the Minkowski-Weyl representation of \( C \) to show that the following are equivalent:

   (a) \( f \) attains a minimum over \( C \).

   (b) \( f^* = \inf_{x \in C} f(x) > -\infty \).

   (c) For all \( y \) such that \( Ay \leq 0 \), we have either \( \langle y, Qy \rangle > 0 \), or else \( y \in \ker(Q) \) and \( \langle c, y \rangle \geq 0 \).

5. If \( D, E \) and \( F \) are matrices each having \( n \) columns, then exactly one of the following systems is solvable:

   (a) \( Dx > 0, \quad Ex \geq 0, \quad Fx = 0 \).

   (b) \( DTu + ETv + FTw = 0, \quad u \) and \( v \) nonnegative and \( u \) not zero.
6. Show that if $T$ is a nonempty polyhedral cone in $\mathbb{R}^n$ and $A \in \mathbb{R}^{n \times m}$ then

$$A^{-1}(T) = A^T(T^\circ).$$

7. Let $A$ and $D$ be linear transformations from $\mathbb{R}^n$ to $\mathbb{R}^p$ and $\mathbb{R}^q$ respectively. Let $K$ be a nonempty polyhedral cone in $\mathbb{R}^p$ and $L$ be a nonempty closed convex cone in $\mathbb{R}^q$. Show that the following are equivalent:

(a) $Ax \in K$ implies $Dx \in L$.
(b) $A^T(K^\circ) \supseteq D^T(L^\circ)$. 