1. Among all rectangles contained in a given circle, show that the one that has maximal area is a square.

2. Divide the number 8 into two nonnegative parts $x$ and $y$ so as to maximize $xy(x - y)$.

3. Consider the problem

$$\max_{x_1^a x_2^a \ldots x_n^a} \quad \text{subject to} \quad \sum_{i=1}^n x_i = 1, \ x_i \geq 0, \ i = 1, \ldots, n,$$

where $a_i$ are given positive scalars. Find a global maximum and show that it is unique.

4. Consider the half space defined by $H = \{x \in \mathbb{R}^n: a^T x + \alpha \geq 0\}$ where $a \in \mathbb{R}^n$ and $\alpha \in \mathbb{R}$ are given. Formulate and solve the optimization problem for finding the point $x$ in $H$ that has the smallest Euclidean norm.

5. Solve the problem

$$\min_{x} x_1 + x_2 \text{ subject to } x_1^2 + x_2^2 = 1$$

by eliminating the variable $x_2$. Show that the choice of sign for a square root operation during the elimination process is critical; the “wrong” choice leads to an incorrect answer.