Homework 9

CS 730, Semester I, 2005–06

November 11, 2005: Assignment due in class Monday, November 21

1. For the problem
$$\min f(x) \text{ subject to } g(x) \leq 0$$
where $f\colon \mathbf{R}^n \to \mathbf{R}$ and $g\colon \mathbf{R}^n \to \mathbf{R}^m$, consider the Absolute Value penalty
$$p_a(g(x)) = \|g(x)_+\|_1$$
and the Courant penalty
$$p_c(g(x)) = \|g(x)_+\|_2^2$$
Consider now the problem
$$P1 : \min x^3 - x \text{ subject to } 0 \leq x \leq 1$$
   (a) Sketch the Absolute value and Courant penalty terms for P1.

   (b) For each positive integer k, compute the minimizer x_k of
   $$P_k(x) = f(x) + kp_c(g(x))$$
   for the f and g defined in P1.

   (c) For each positive integer k, compute the minimizer x_k of
   $$F_k(x) = f(x) + kp_a(g(x))$$
   for the f and g defined in P1.

   (d) Use P_k to solve
   $$P2 : \min x_1 + x_2 \text{ subject to } x_1^2 - x_2 \leq 2$$
   (e) Show that F_k has no stationary points off the parabola
   $$x_1^2 - x_2 = 2$$
   for problem P2 with $k > 1$ and compute the minimizer of $F_k(x)$.

2. Consider the problem: $\min_{x \in S} f(x)$, where $f\colon \mathbf{R}^n \to \mathbf{R}$ and $S \subset \mathbf{R}^n$ is possibly empty. Suppose that some *fixed* $\bar{x} \in \mathbf{R}^n$ solves the penalty problem
$$\min_{x \in \mathbf{R}^n} f(x) + \alpha Q(x), \text{ for all } \alpha > \bar{\alpha},$$
where $Q(x)$ is a penalty function such that:
$$Q(x) = 0 \text{ for } x \in S, \text{ otherwise } Q(x) > 0.$$
What can you say about:

   (a) $Q(\bar{x})$?

   (b) $f(\bar{x})$?