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Extended Mathematical Programming: Competition and Stochasticity

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Optimization has evolved into a mature discipline over the past 60 or so years and can now be used to model and solve a host of problems arising in a variety of application areas.

While the demonstrated value of optimization in solving standard models of increased size and complexity is of critical importance, we firmly believe that the real value of optimization lies not in solving a single problem, but rather in providing insight and advice on the management of complex systems. Such advice needs to be part of an interactive debate with informed decision makers. An underused aspect of optimization models is as part of a process for finding "holes" in a model—those features that an optimization code can quickly identify and exploit, but that are indicative of problems with the underlying model or analysis.

We believe that optimization processes can be used to develop inputs for other parts of a solution (data processing); that strong optimization theory (of duality, for example) can enable more effective solution schemes that exploit model structure; and that the interplay between collections of models treating stochastic effects, competition between independent agents, and a mixture of continuous and discrete approaches can provide richness for describing, solving, and adapting the underlying system being modeled. Mechanisms that allow decision making in such circumstances can be informed by game theory and techniques from distributed computing.

Exploiting these fundamental properties of an optimization system has the potential to influence many application domains in significant ways. To answer major design questions, small dynamic models need to be developed that are "level of detail" specific; these models need to have interfaces to other "subservient" models that provide appropriate aggregation of data and enhance overall understanding of the underlying complexities.

Example: Next-generation Electric Grid

As an example, we look at the technological and economic trends implying significant growth in our nation's reliance on the power grid in the coming decades; well-accepted estimates cite 35% growth in electricity demand in the U.S. over the next 20 years [3]. Planning and operating the next-generation electric grid involves decisions at time scales ranging from perhaps 15 years, for major grid expansion, through 5-minute markets, and must also account for phenomena occurring in fractions of a second. A representation of the decision process over time scales of interest is shown in Figure 1.

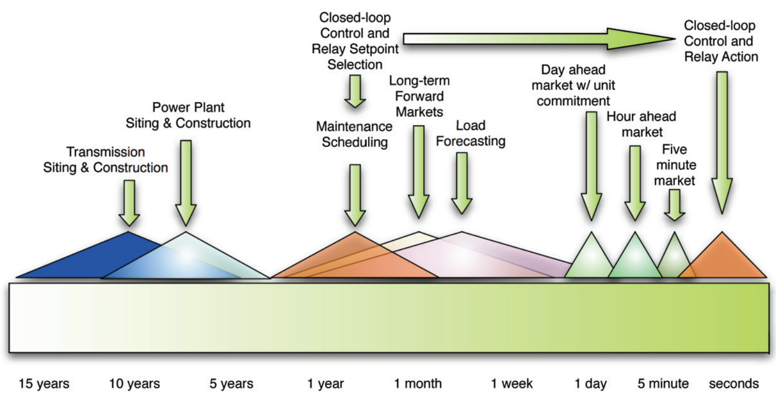

Figure 1. *Representative decision-making time scales for electric power systems.*

What makes this setting particularly interesting is that behaviors at very short time scales (e.g., requirements for grid resilience against cascading failures) potentially impose constraints on decisions at longer time scales, such as maintenance scheduling and grid expansion. We argue here against building a single "monster model" that tries to capture all these scales, and propose rather that a collection of coupled or layered models be used for both planning and operation, interfacing via information/solution sharing over multiple time scales and layers of decision making. Such approaches have been successful in other application domains [1].

We propose a hierarchical approach to this problem, formulated as follows. If we let x represent the decision on transmission line expansion and assume that the regional transmission organization (RTO) can postulate a distribution of future demand scenarios (at the decades scale), then the RTO problem is:

$$\min_{x \in X} \sum_{\omega} \pi_{\omega} \sum_{i \in N} d_i^{\omega} p_i^{\omega}(x),$$

where ω runs over the scenarios, π gives the probabilities, and d_i^{ω} is the resulting demand in such a scenario at a given node in the network. The relationship $x \in X$ re-flects budgetary and other constraints on the RTO's decision, and the function $p_i^{\omega}(x)$ is a response (LMP) price at node i in the given scenario to the expansion by x.

A simple example is depicted in Figure 2, in which blue nodes are demand locations, and yellow and green nodes represent generation capacity under the control of different firms. Changes in the variable x would change the capacities of existing arcs in the network, or introduce new ones. Clearly, the key to solving this problem is to generate a good approximation to the response price, which is the interface to the lower levels of the hierarchy. We believe that a lower-level equilibrium model involving both generator firms and economic dispatch problem solution in every scenario is one way to generate such a response.

Transmission expansion decisions influence the capital investment decisions made by generating companies, again at a time scale of 1–10 years. For each firm f, we denote generator expansion by y_f and propose that this will be determined by an optimization principle:

$$\min_{y_f \in Y_f} \sum_{\omega} \pi_{\omega}$$
$$\sum_{j \in G_f} C_j(y_j, q_j^{\omega}) - r(h_f - \sum_{j \in G_f} y_j).$$

Here G_f denotes the generators in firm f's portfolio, Y_f represents budgetary and other constraints faced by the generator firm, C_j is the cost function for generator j, and r is the interest rate for unused capital. Notice that q_j^{ω} is a parameter in this problem—the actual dispatch is determined by an economic dispatch (ED) model operated by the independent system operator (ISO). These independent models can be solved and potentially validated, including feedback from key decision makers.

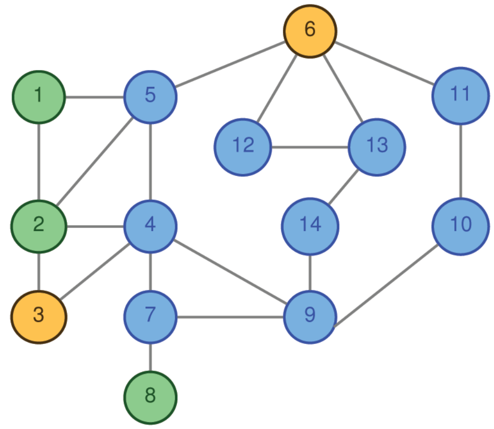

Figure 2. *Transmission line expansion model.*

In addition to the multiple time scales in the decision process, the problem is compounded by uncertainties in estimates and structural makeup of the system. For example, plug-in hybrid electric vehicles are a tangible technology that could dramatically alter the patterns, nature, and quantity of electricity use in the U.S., and yet the ultimate market penetration of the technology is highly uncertain. Similarly, future grid penetration for non-traditional energy sources, such as wind and solar, and for carbon-sequestration-equipped coal plants also remains highly uncertain. We capture the flavor of this uncertainty through d_j^{ω} in the ED model:

$$\begin{aligned}
&\min_{z,\theta,q^{\omega}} \sum_{f} \sum_{j \in G_f} C_j\left(y_j, q_j^{\omega}\right) && \text{s.t.} \\
&q_j^{\omega} - d_j^{\omega} = \sum_{i \in I(j)} z_{ij} && \forall j \in N\left(\perp p_j^{\omega}\right) \\
&z_{ij} = \Omega_{ij}\left(\theta_i - \theta_j\right) && \forall (i,j) \in A \\
&-b_{ij}(x) \le z_{ij} \le b_{ij}(x) && \forall (i,j) \in A \\
&\underline{u}_j\left(y_j\right) \le q_j^{\omega} \le \overline{u}_j\left(y_j\right).
\end{aligned}$$

This formulation uses a DC model of the power flow equations (other, more accurate approximations could easily be utilized). The multiplier on the flow balance constraints is $p_j^{\omega}(x)$. With this formulation, we are able to treat uncertainty by stochastic (or robust) optimization at a submodel level and with evolving resolution.

The collection of all these optimization models (generator expansion and scenario-dependent ED) forms an equilibrium problem (according to the Nash definition), once each optimization model is replaced by its Karush–Kuhn–Tucker conditions. Solution techniques that exploit the model structure facilitate finding global solutions of the underlying nonconvex problems, while still maintaining an overall equilibrium.

Finally, power grid dynamics operate at time scales of milliseconds to minutes. At this level, efficient nonlinear optimization must be carried out to match the varying demand for electricity with the ever increasing and uncertain supply of energy, without interruptions or catastrophic cascading failures of the system.

In short, there is clearly a need for optimization tools that effectively inform and integrate decisions across widely separated time scales, by different agents with differing individual objectives, in the presence of uncertainty. We contend that the additional knowledge gained from understanding the effects of one decision on another in a structured fashion will facilitate better management and operation of the system when it is built and provide understanding and information to the operators as to the consequences of their decisions.

What Is EMP?

While much progress has been made in developing new modeling paradigms (such as stochastic and robust programming, mixed integer nonlinear optimization, second-order cone programming, and optimization of noisy functions), the ability of application experts to utilize these advances from within modeling systems has remained limited. The extended mathematical programming (EMP) framework was created to provide these benefits for problems that fall outside the classical framework [2].

EMP annotates existing equations/variables/models so that a modeler can provide or define additional structure, including equilibrium and multi-agent optimization and variational inequality submodels, bilevel programs, disjunction and other constraint logic primitives, random variables, and extended nonlinear programs. EMP has tools that automatically create the different realizations or model extensions described in a rigorous manner, pass them on to the appropriate solvers, and interpret the results in the context of the original model. This makes it possible to model more easily and directly, to conduct experiments with formulations otherwise too time-consuming to consider, and to avoid errors that can make results meaningless or worse.

Acknowledgments

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