GAMS, Condor and the Grid: Solving Hard Optimization Models in Parallel

Michael C. Ferris
University of Wisconsin

Parallel Optimization

- Aid search for global solutions (typically in non-convex or discrete)
  - Pattern search, evolutionary algorithms
  - Branch and bound/cut
- Enhance speed of computation
  - Decomposition approaches
    - Benders, Dantzig-Wolfe, Lagrangian
  - Linear algebra

What has it achieved?

- Heuristic approaches
- CPLEX (multi-threaded)
  - Parallel MIP, Barrier and Concurrent
- Large Computations
  - TSP cases, NUG30, Seymour
- Software Paradigms
  - MPI, PVM, Tao, MW
- Limited impact to date
  - Gulf between theory and implementation

Reasons for failure

- Optimization has inherent synchronization
- Specialized software
  - Rewrite applications at low level
  - Impractical, impossible
  - E.g. FATCOP does things sub-optimally
- Specialized hardware
  - Expensive, ages quickly, Moore's Law

What is Grid Computing?

A pool of connected computers managed and available as a common computing resource

- Effective sharing of CPU power
- Massive parallel task execution
- Scheduler handles management tasks
- E.g. Condor, Sun N6 Grid Engine, Globus
- Can be rented or distributively owned
- Licensing, communication and security issues

Condor


Condor Project Homepage - Microsoft Internet Explorer
File Edit View Favorites Tools Help
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Address http://www.cs.wisc.edu/condor/
Condor
High Throughput Computing
The goal of the Condor® Project is to develop, implement, deploy, and evaluate mechanisms and policies that support High Throughput Computing (HTC) on large collections of distributively owned computing resources. Guided by both the technological and sociological challenges of such a computing environment, the Condor Team has been building software tools that enable scientists and engineers to increase their computing throughput.
If you find Condor as interesting as we do, consider joining our team of talented and enthusiastic developers.
Condor Week Meetings
European Condor Week 2006 is scheduled for June 26-29, 2006, in Milan, Italy. Please consider joining us for this informative meeting!
Condor Week 2007 will be April 30-May 3, 2007. More details available in 2007.
Information on past Condor Week meetings
Current Releases
Stable series: Condor Version 6.6.11 released March 28nd, 2006
Development series: Condor Version 6.7.20 released June 22th, 2006
Recent News

Condor Features

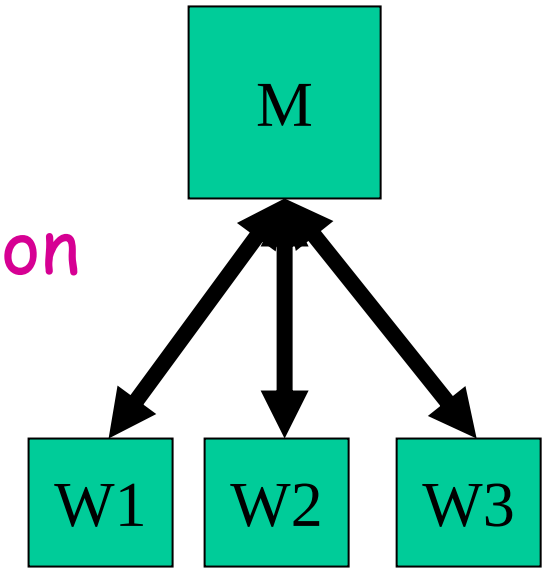
- Uses dedicated clusters and cycles from desktop workstations (> 1000 machines available for "ferris")
- Heterogeneous machines, with or without shared file system
- Machines updated regularly
- Fault tolerance
  - Jobs submitted are eventually executed
- Available for download, configurable

Can we use it effectively?

- High throughput not high performance computing (modify perspective)
- New modeling features of GAMS facilitate use of grid computation and sophisticated solvers
- Optimization expertise shared with computational engines

Master-Worker Paradigm

- Master
  - Generates tasks
  - Responsible for synchronization
- Worker
  - Processes individual tasks
  - Reports back results
- Simple! Dynamic! Fault tolerant!
- No worker inter-communication



Modeling Language as Producer

- Natural representation for Mathematical Programs
- Easily handles large data volume
- Rapidly generate many models
- Examples: AMPL and GAMS

FOR MORE INFO...

See http://www.ampl/com and http://www.gams.com

Typical Application for GAMS

```
demand = 42; cost = 14;
solve mymodel min obj using minlp;
report = var.l;
```

Typical Application for GAMS

```
loop(scenario,
  demand=sdemand(scenario); cost=scost(scenario);
  solve mymodel min obj using minlp;
  report(scenario) = var.l);
);
```

Typical Application for GAMS & Grid

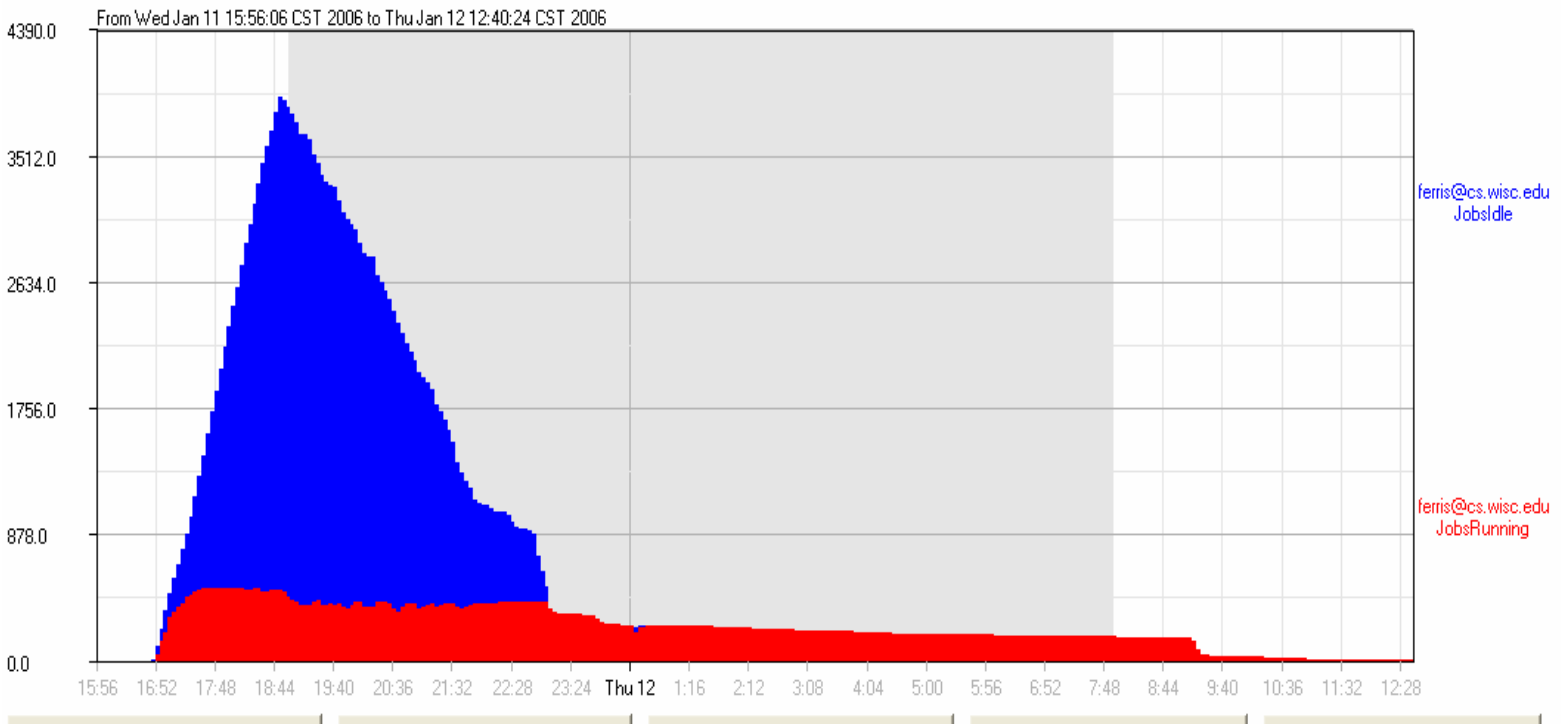
```
mymodel.solvelink=3;
loop(scenario,
  demand=sdemand(scenario); cost=scost(scenario);
  solve mymodel min obj using minlp;
  h(scenario)=mymodel.handle);

repeat
  loop(scenario$h(scenario),
    if(handlestatus(h(scenario))=2,
      mymodel.handle=h(scenario); h(scenario)=0;
      execute_loadhandle mymodel;
      report(scenario)=var.l);
  if(card(h), execute 'sleep 1');
until card(h)=0 or timeelapsed > 100;
```

Massively Parallel MIP

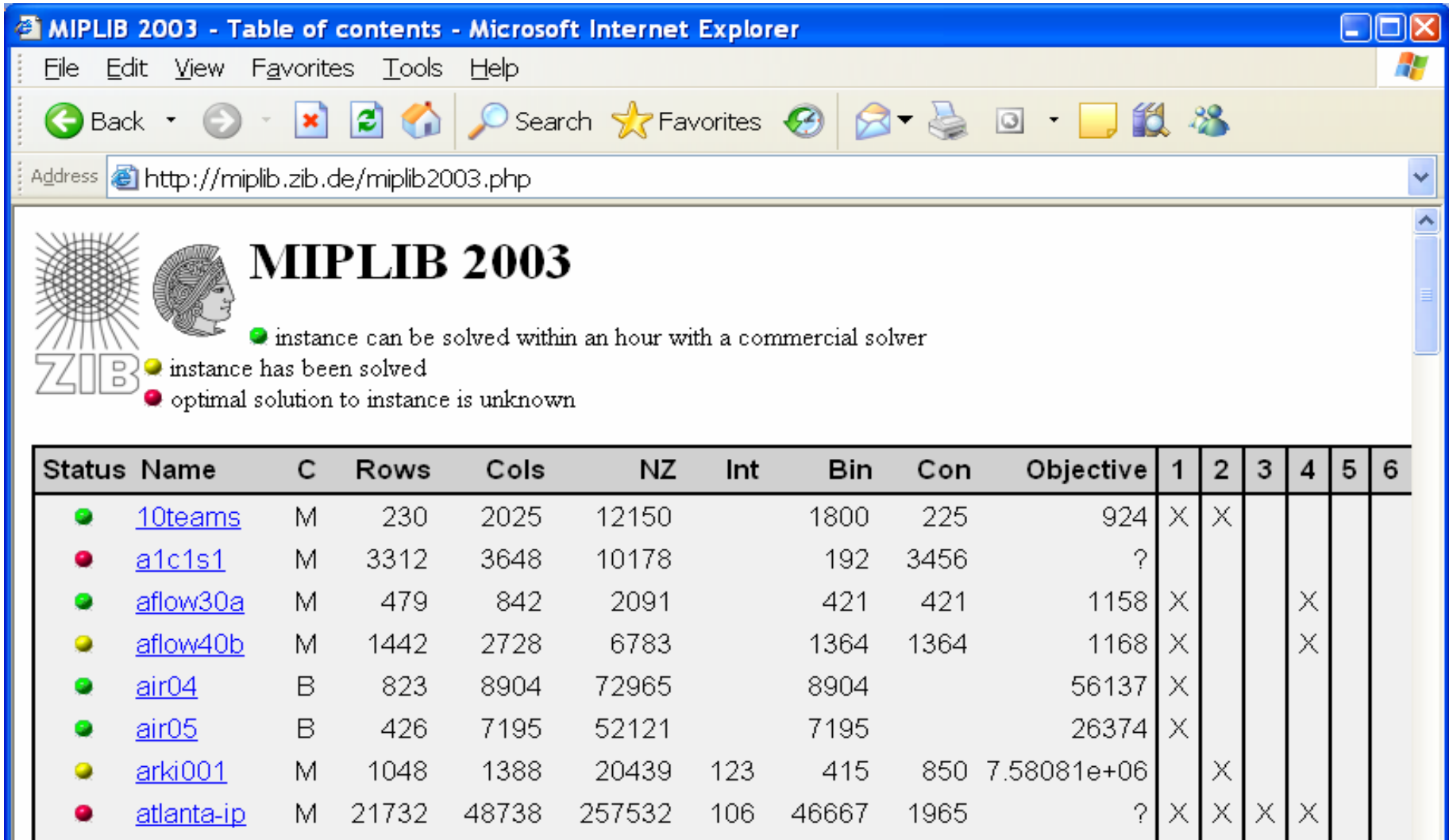
- MIP/B&C Algorithm ideal to parallelize
  - Master/Worker Paradigm (process nodes in parallel)
    - Software: FATCOP/Condor, BCP/PVM, PICO/MPI
  - A-priori subdivision into *n* independent problems
    - Seymour problem solved that way
  - Open Pit Mining (openpit in GAMS Model library)
    - Partition integer variables to subdivide model into 4096 sub-problems

4096 MIPS on Condor Grid



- Submission started Jan 11, 16:40
- All jobs submitted by Jan 11, 23:00
- All jobs returned by Jan 12, 12:40
  - 20 hours wall time, 5000 CPU hours, Peak # CPU's: 500

MIPLIB 2003 has 13 unsolved instances



Status	Name	C	Rows	Cols	NZ	Int	Bin	Con	Objective	1	2	3	4	5	6
●	10teams	M	230	2025	12150		1800	225	924	X	X				
●	a1c1s1	M	3312	3648	10178		192	3456	?						
●	aflow30a	M	479	842	2091		421	421	1158	X			X		
●	aflow40b	M	1442	2728	6783		1364	1364	1168	X			X		
●	air04	B	823	8904	72965		8904		56137	X					
●	air05	B	426	7195	52121		7195		26374	X					
●	arki001	M	1048	1388	20439	123	415	850	7.58081e+06		X				
●	atlanta-ip	M	21732	48738	257532	106	46667	1965	?	X	X	X	X		

Tool and expertise combined

- Initial schemes take over 1 year of computation and go nowhere – even with fastest commercial solver - CPLEX
- Extensions of approach that incorporate both computational strategies and optimization expertise
  - Adaptive refinement strategy
  - Sophisticated problem domain branching and cuts
  - Use of resources beyond local file system
  - Dedicated resources

Problem with a-priori partitioning

- 99% of sub-problems very easy to solve
- 1% (almost) as difficult as the original problem

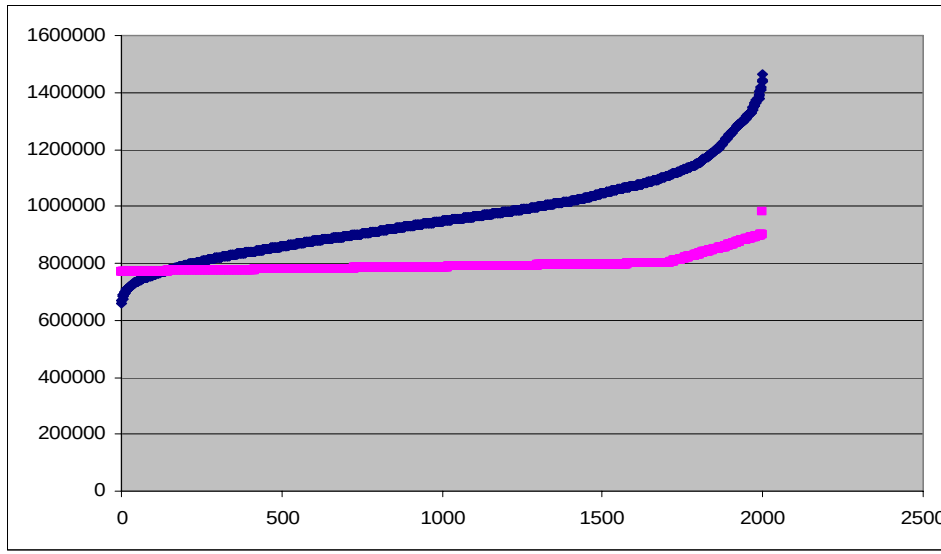
- How can we find *n* sub-problems with similar (but reduced) level of difficulty?
  - B&C Code keeps a list of *open/unexplored* nodes
  - Problem-bounds of these open nodes represent partitioning of the original problem

Node	Nodes Left	Objective	IInf	Best Integer	Cuts/ Best Node	ItCnt	Gap
0	0	29.6862	64		29.6862	165	
100	37	17.0000	14		25.0000	2230	
200	70	21.8429	22		24.0000	4022	

- GAMS/CPLEX Option **`dumptree n`** creates *n* bound files

How difficult is a subproblem?

- What is a good estimate for how difficult a subproblem is?
  - Look at the LP value of a subproblem
  - The smaller the LP value (assuming minimization) the more difficult the subproblem



- **Cplex Default**
- **Cplex Strong Branching**
- **Spend more time in sub-problem generation**

Putting it all together

```
Generate n sub-problems using GAMS/CPLEX with dumpopt n;

loop(n,
  load nth bound file;
  generate and submit nth sub-problem
);

Repeat
  loop(n$(not collected),
    if (n finished,
      load nth-solution and mark n as collected));
  sleep some time;
Until all collected;
```

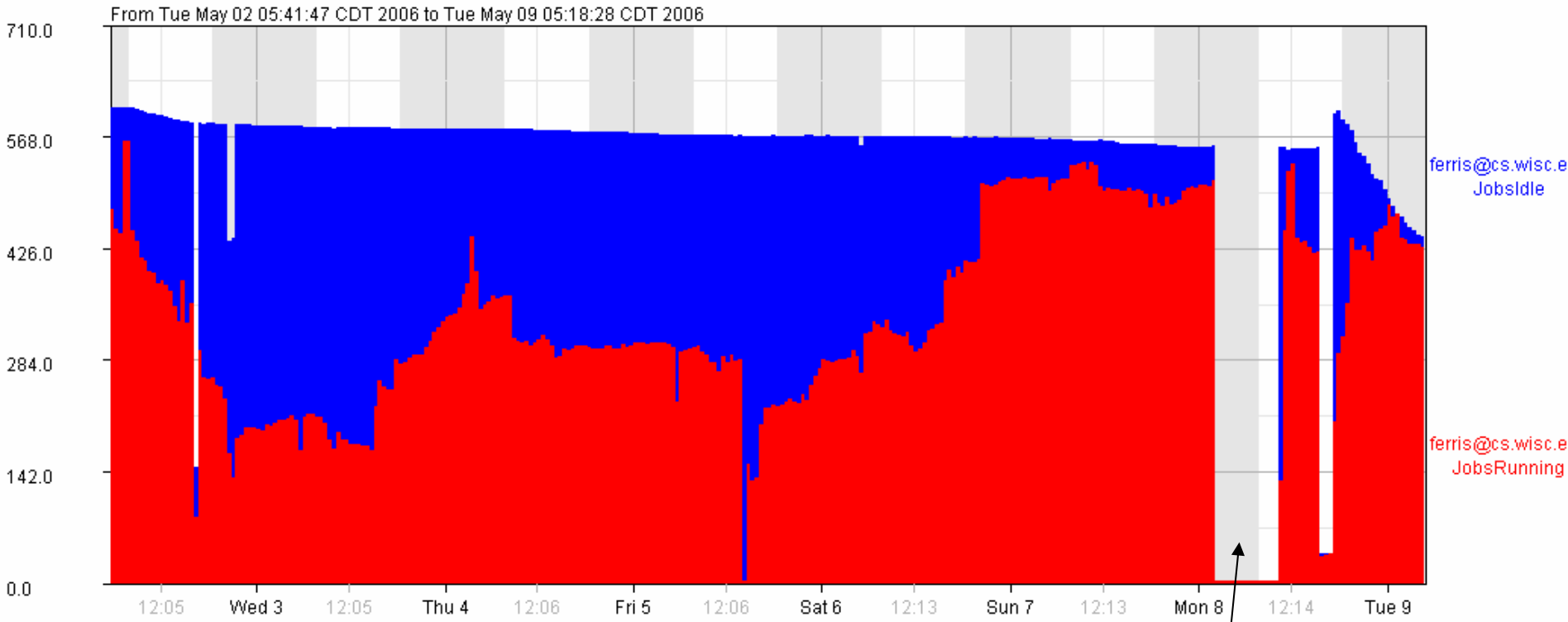
Communication

- Incumbent solution allows pruning of nodes with larger LP solution value
  - How greedy are you?
  - Shared file system (unreliable, unavailable)
  - condor_chirp for inter-worker communication (background process on worker)
- Hence communicate newly found incumbent to all subproblems
  - Subproblems not started: Start with **`cutoff`**
  - Running subproblems: Update **`cutoff`** with a GAMS/CPLEX option file that is read while running (solver option facilitates on-the-fly strategy changes)

Strategy

- Strategy:
  - Have one machine working on good solutions for original problem
    - CPLEX **`mipemphasis 1`** or **`4`**
  - Subproblem emphasis on best-bound
    - CPLEX **`mipemphasis 3`**
  - Repartition longest running jobs
  - Restart from incumbent (cf NLP)

Grid resources used



Partitioned into 1000 subproblems, over 300 machines running for multiple days

main submitting machine died, jobs not lost

Some results

	ROLL3000	A1C1S1	TIMTAB2 (added problem cuts)
#sub-problems	986	1089	3320
objective	12890	11503.4	1096557.
#Cplex B&B nodes	400,034	1,921,736	17,092,215
CPU time used	50h	3452h	2384h
CPU time wasted	0.5h	248h	361h
Wall time	Over night	Over night	Over night

Other Results

- Problem SWATH (TSP type problem)
  + sub-tour elimination cuts:

- Subproblems: 1539 (23 not finished)
- Objective: 467.407
- CPU time used: 36159 hr (4.1 years)
- CPU time wasted: 71557 hr (8.2 years!)
- Nodes explored: 721,718,141

- Second Level Partitioning (subdivide of several of the 23 outstanding problems):

Subproblems: 2000
CPU time used: 2,232 hr
CPU time wasted: 24,000 hr
Nodes explored: 464,006,423

A word of caution

- Go back to original SWATH paper!
- Understand underlying (20 var) TSP with "supernodes"
- 5 rounds of subtour elimination cuts, 32 extra constraints in all
- Problem solved in less than 20 minutes on a single machine using CoinCbc!

Summary

- GAMS/CPLEX dumpopt n
  - a-priori problem partition of MIP
- Use GAMS Grid facilities, Condor, and GAMS/CPLEX to generate, submit, and solve n subproblems
- Communication of updated incumbent is essential
- Solved two previously unsolved problems (ROLL3000, A1C1S1) from MIPLIB2003 over night (with few hundred machines available)
- Brute force has its limits, but with some additional problem specific knowledge (turned into problem specific cuts) one more problem (TIMTAB2) could be solved over night
- Problem knowledge still very useful, solved (SWATH)
- Some problems in MIPLIB2003 will remain unsolved for a while

GAMS/Grid

- Commercial modeling system – abundance of real life models to solve
- Any model types allowed
  - Scheduling problems
  - Radiotherapy treatment planning
  - World trade (economic) models
  - Sensitivity analysis
  - Cross validation, feature selection
- Little programming required
- Separation of model and solution maintained

Scheduling Multistage Batch Plants

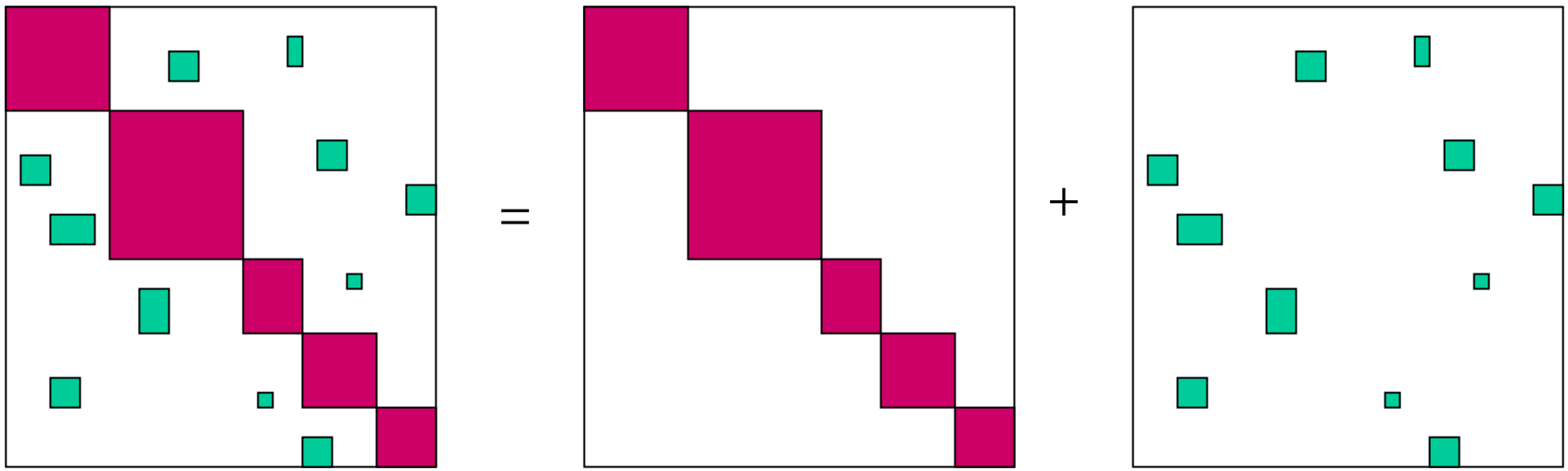
- Solution within 1 day
- Three level decision process (GAMS)
  - Split order into batches
  - Assign batches to processing units
  - Sequence batches over stages
- Instance 1: solved sequentially CPLEX
- Instance 2: solved GAMS/CPLEX/Condor
- Instance 3: gap (1176-1185) after 24h

Adaptive SB Method

- Split model at top level
- Apply (very) strong branching to generate a collection of subproblems
- Solve each subproblem
  - If 2 hour time limit reached, reapply strong branching to subdivide and resolve
- Instance 3 solved - 4 branching levels

Trade/Policy Model (NCP)

- Split model (18,000 vars) via region



- Jacobi, Gauss-Seidel, Asynchronous
- 87 regional subprobs, 592 solves

Other Applications

- Sampling approach for radiotherapy planning
  - Multiple replications to reduce uncertainty and improve model integrity
- Stochastic linear programming
  - Bender's Decomposition (S.K. Lee)
- Cross-validation / feature selection
  - Each fold in parallel, subsets of features tested in parallel

Multiple Solvers/Platforms

- Can use all supported solvers including:
  - CPLEX, XPRESS, PATH, SNOPT, MOSEK
- Runs on multiple platforms using heterogeneous machines for solvers
- Can interleave solutions on host and worker, maintains data confidentiality
- Available right now!

Two Condor Shortcomings

- Condor doesn't run short jobs well
  - lots of time required to schedule jobs in the pool
  - time needed to transmit the executable/data/results
- Condor doesn't deal directly with parallel algorithms
  - Can have the "master" generating waves of "worker" jobs to run in parallel, but
    - each worker job must be scheduled anew in the Condor pool
    - the master application has to handle the details
- Switch to a different grid engine! Or …
- Master-Worker (MW) addresses these issues!

MW-GAMS

- Back-end grid engine is switchable
  - Usecondor=mw,lnx,win,glow,sun
- Data common to all tasks is sent to workers only once
- (Try to) retain workers until the whole computation is complete—don't release them after a single task
- Modeler just flips a switch!

Conclusions

- Massive parallel and distributed computing environments are available (e.g. Condor, IBM, SUN)
- Grid computing capability available for optimizers in convenient environment via simple language extensions to modeling languages
- Today's modeling languages are well suited to experiment with coarse grain parallel approaches for solving difficult problems

Future extensions

- Real-time problem solution (as opposed to high throughput)
- Re-optimization (model updating)
- Global optimization
- Commercial use
- Saving intermediate solution results
- Further application deployment