Some Computational Legacies of Lemke and Dantzig in Complementarity

ICCP'05

Michael Ferris
University of Wisconsin, Computer Sciences
ferris@cs.wisc.edu
1940’s – Glenn Miller

- Wartime, innovative, orchestrated new mixtures of instruments/sounds
- Basic feasible solution
- Linear objective and constraints
- Dantzig - Simplex method
- Duality
1950’s – Buddy Holly

- Nash papers
- Lemke – Dual Simplex Method
- Frank-Wolfe – Quadratic Program
- Dantzig et al – Network Programs
- Orchard-Hays – Implementation
1960's – The Beatles

- Dantzig – LP book
- Lemke – Bimatrix games/algorithm
- Cottle – Nonlinear complementarity
- Cottle/Dantzig – Complementary pivoting
- Scarf – computable fixed points
1970's - The Bee Gee's

- Eaves
  - Piecewise linear equations
- Robinson
  - Josephy/Mathiesen - SLCP
- Saunders
  - MINOS
1980’s – Sex Pistols

• Ellipsoid Method
  - Khachiyan

• Interior Point Methods
  - Karmarkar, Kojima et al

• Modeling systems
  - Meeraus, Fourer, Gay
1990’s – U2

- CPLEX
  - Preprocessing
  - Computational Advances Implemented
    - Dual simplex
    - Forrest Tomlin Updates
    - Dual steepest edge

- Stochastic programming
- Nonsmooth/semismoothness
- PATH
What differentiates LP & QP from NCP?

- Succinct problem description
- Functions can be evaluated anywhere
- Merit/objective function
- Embeddable/speed (CPLEX)
- Restartable (Dual simplex)
- Correctness/robustness (trichotomy)
2000’s - The Acrylicks

- MPEC/EPEC
- Dynamic Variational Inequalities
- Generalized games
- Stochastic equilibria
- Data mining/statistics
- Model detail
What is needed?

• VI/MPEC/LCS/DVI require subproblem solution
• Size for model detail
• Nonconvexities/globality for applications
• ROBUSTNESS
  - Computability, exploitation of structure, parametric solution
Equivalent Nonsmooth Map

\[ 0 = F(x_+) + x - x_+ \]

\[ 0 \leq x_+, \ F(x_+) = x_+ - x \geq 0 \]

\[ 0 = F(\pi_C(x)) + x - \pi_C(x) \]

\[ x'_+F(x_+) = x'_+(x_+ - x) = 0 \]
Theorem

- $C$ polyhedral
- $M$ copositive-plus on $\text{rec } C$
  - or $M$ is $L$ on $\text{rec } C$ and invertible on $\text{lin } C$
- Implementable adaptation of Lemke's method terminates
  - at solution of $Mx + q + N_C(x)$
  - or $Mx + q \in (\text{rec } C)^*$ has no solution
Preprocessing

• Partition variables into \((x, y)\)
• Identify skew symmetric structure

\[
0 \in \begin{bmatrix}
F(x) & \ -A^T y \\
Ax & \ -b \\
\end{bmatrix} + \begin{bmatrix}
N_{\mathbb{R}^n_+}(x) \\
N_{\mathbb{R}^m_+}(y)
\end{bmatrix}
\]

• Equivalent polyhedral prob. (Robinson)

\[
0 \in F(x) + N_{\mathbb{R}^n_+} \cap \{z|Az-b \geq 0\}(x)
\]
Separable Structure

- Partition variables into \((x, y)\)
- Identify separable structure

\[
0 \in \begin{bmatrix}
F(x) \\
G(x, y)
\end{bmatrix} + \begin{bmatrix}
N_{\mathbb{R}_+^n}(x) \\
N_{\mathbb{R}_+^m}(y)
\end{bmatrix}
\]

- Reductions possible if

\[
0 \in F(x) + N_{\mathbb{R}_+^n}(x) \text{ uniquely solvable}
\]

\[
0 \in G(x, y) + N_{\mathbb{R}_+^m}(y) \text{ solvable for all } x
\]
Plumbing Improvements

• Preprocess nonlinear function domains
• Enforce (VI) constraints explicitly
• Identify and exploit separable structure
• Modern nonlinear programming techniques
• Must implement known (good!) theory
What can we model via CP?

\[
\min(G(x), H(x)) \leq y \\
\min(F^1(x), F^2(x), \ldots, F^m(x)) = 0 \\
kth\text{-largest}(F^1(x), F^2(x), \ldots, F^m(x)) = 0 \\
\text{Switch off: } g(x)h(x) \leq 0, \ h(x) \geq 0 \\
\text{Variational Inequality: } \text{VI}(F, C)\}
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