Some Computational Legacies of Lemke and Dantzig in Complementarity

ICCP'05

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1940's - Glenn Miller

- Wartime, innovative, orchestrated new mixtures of instruments/sounds

- Basic feasible solution
- Linear objective and constraints
- Dantzig - Simplex method
- Duality

1950's - Buddy Holly

- Nash papers
- Lemke - Dual Simplex Method
- Frank-Wolfe - Quadratic Program
- Dantzig et al - Network Programs
- Orchard-Hays - Implementation

1960's - The Beatles

- Dantzig - LP book
- Lemke - Bimatrix games/algorithm

- Cottle - Nonlinear complementarity
- Cottle/Dantzig - Complementary pivoting
- Scarf - computable fixed points

1970's - The Bee Gee's

- Eaves
  - Piecewise linear equations
- Robinson
  - Josephy/Mathiesen - SLCP
- Saunders
  - MINOS

1980's - Sex Pistols

- Ellipsoid Method
  - Khachiyan
- Interior Point Methods
  - Karmarkar, Kojima et al
- Modeling systems
  - Meeraus, Fourer, Gay

1990's - U2

- CPLEX
  - Preprocessing
  - Computational Advances Implemented
    - Dual simplex
    - Forrest Tomlin Updates
    - Dual steepest edge
- Stochastic programming
- Nonsmooth/semismoothness
- PATH

What differentiates LP & QP from NCP?

- Succinct problem description
- Functions can be evaluated anywhere
- Merit/objective function

- Embeddable/speed (CPLEX)
- Restartable (Dual simplex)
- Correctness/robustness (trichotomy)

2000's - The Acrylicks

- MPEC/EPEC
- Dynamic Variational Inequalities
- Generalized games
- Stochastic equilibria
- Data mining/statistics
- Model detail

What is needed?

- VI/MPEC/LCS/DVI require subproblem solution
- Size for model detail
- Nonconvexities/globality for applications
- ROBUSTNESS
  - Computability, exploitation of structure, parametric solution

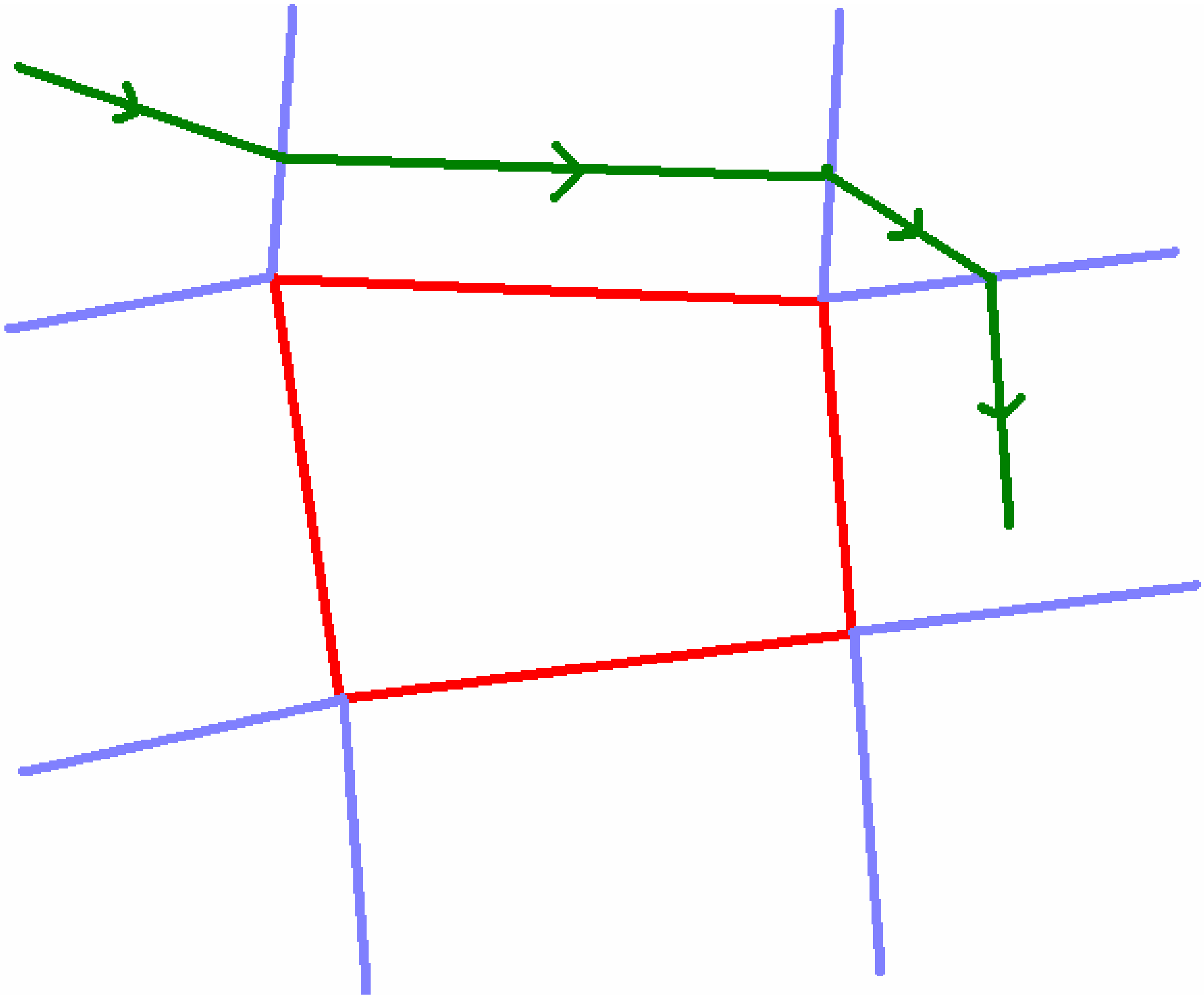
Equivalent Nonsmooth Map

$$0 = F(x_+) + x - x_+$$

$$0 \leq x_+, \; F(x_+) = x_+ - x \geq 0$$

$$0 = F(\pi_C(x)) + x - \pi_C(x)$$

$$x_+' F(x_+) = x_+'(x_+ - x) = 0$$



Theorem

- C polyhedral
- M copositive-plus on rec C
  - or M is L on rec C and invertible on lin C
- Implementable adaptation of Lemke's method terminates
  - at solution of $Mx + q + N_C(x)$
  - or

$$\begin{aligned} Mx + q &\in (\mathrm{rec} C)^* \\ x &\in C \end{aligned}$$

has no solution

Preprocessing

- Partition variables into (x, y)
- Identify skew symmetric structure

$$0 \in \begin{bmatrix} F(x) & -A^T y \\ Ax & -b \end{bmatrix} + \begin{bmatrix} N_{\mathbf{R}^n_+}(x) \\ N_{\mathbf{R}^m_+}(y) \end{bmatrix}$$

- Equivalent polyhedral prob. (Robinson)

$$0 \in F(x) + N_{\mathbf{R}^n_+ \cap \{z | Az - b \geq 0\}}(x)$$

Separable Structure

- Partition variables into (x, y)
- Identify separable structure

$$0 \in \begin{bmatrix} F(x) \\ G(x,y) \end{bmatrix} + \begin{bmatrix} N_{\mathbf{R}^n_+}(x) \\ N_{\mathbf{R}^m_+}(y) \end{bmatrix}$$

- Reductions possible if

$0 \in F(x) + N_{\mathbf{R}^n_+}(x)$ uniquely solvable

$0 \in G(x,y) + N_{\mathbf{R}^m_+}(y)$ solvable for all x

Plumbing Improvements

- Preprocess nonlinear function domains
- Enforce (VI) constraints explicitly
- Identify and exploit separable structure
- Modern nonlinear programming techniques
- Must implement known (good!) theory

What can we model via CP?

$$\min(G(x), H(x)) \leq y$$

$$\min(F^1(x), F^2(x), \ldots, F^m(x)) = 0$$

$$\text{kth-largest}(F^1(x), F^2(x), \ldots, F^m(x)) = 0$$

Switch off: $g(x)h(x) \leq 0, \ h(x) \geq 0$

Variational Inequality: $VI(F, C)\}$

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