Sampling Issues for Optimization in Radiotherapy

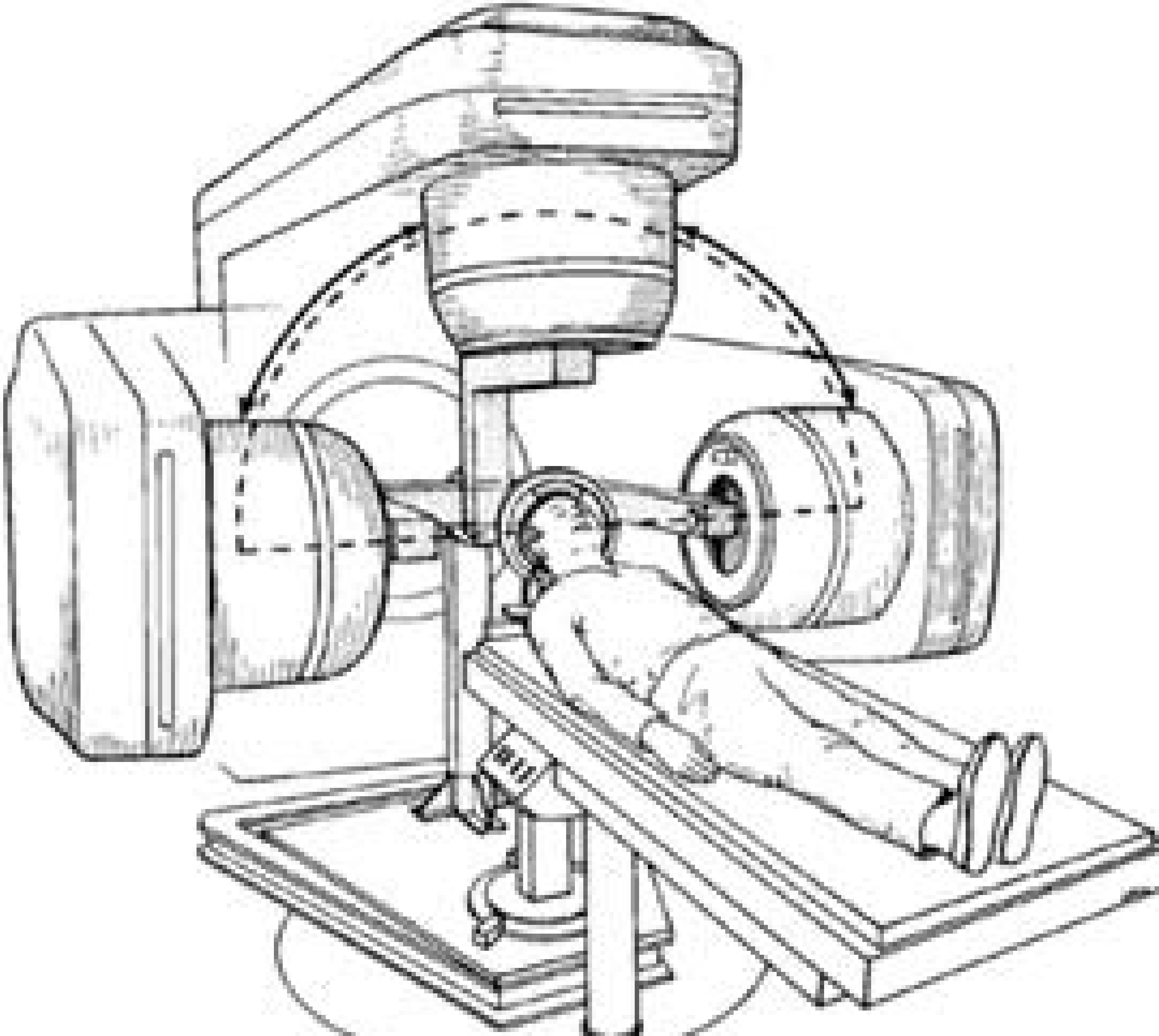
Michael C. Ferris
R. Einarsson
Z. Jiang
D. Shepard

Radiation Treatment Planning

- Cancer is the 2nd leading cause of death in U.S.
  - Only heart disease kills more
- Expected this year in the U.S. (American Cancer Society)
  - New cancer cases = 1.33 million (> 3,600/day)
  - Deaths from cancer = 556,500 (> 1,500/day)
  - New brain/nerv. sys. cancer cases > 18,300 (> 50/day)
- Cancer treatments: surgery, radiation therapy, chemotherapy, hormones, and immunotherapy

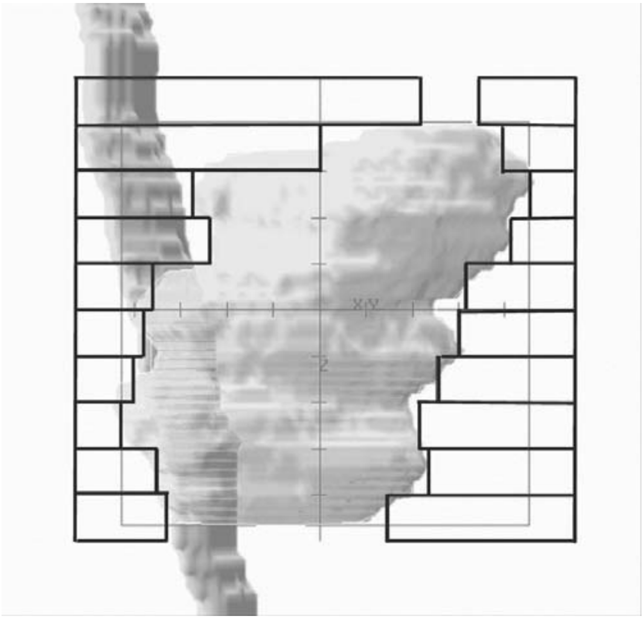
Radiation As Cancer Treatment

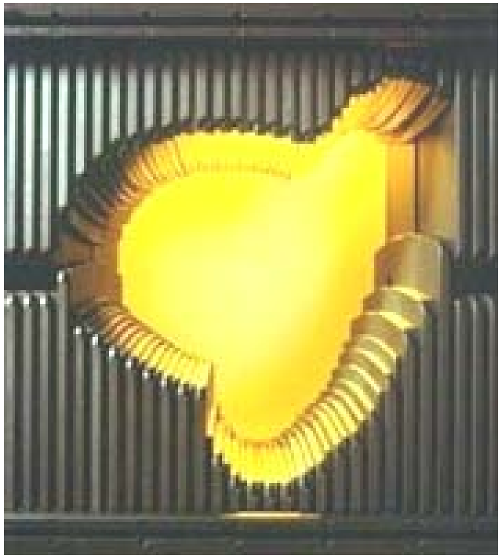
- Interferes with growth of cancerous cells
- Also damages healthy cells, but these are more able to recover
- Goal: deliver specified dose to tumor while avoiding excess dose to healthy tissue and at-risk regions (organs)



Beam's eye view

- Beam's eye view at a given angle is determined based upon the beam source that intersects the tumor
- The view is constructed using a multi-leaf collimator





Dose Distribution

- Experts determine an ideal dose distribution for a particular target
  - Covers target (tumor)
  - Limits radiation to healthy/at-risk regions
- Delivery plan = optimization problem

Delivery Plan

$$
\begin{array}{ll}
\text{min} & f(Dose) \\
\text{subject to} & Dose(i) = \sum_A w_A D_A(i) \\
& Dose(Sens(k)) \leq U(k) \\
& L \leq Dose(Target) \\
& w_A \geq 0
\end{array}
$$

plus some integrality constraints

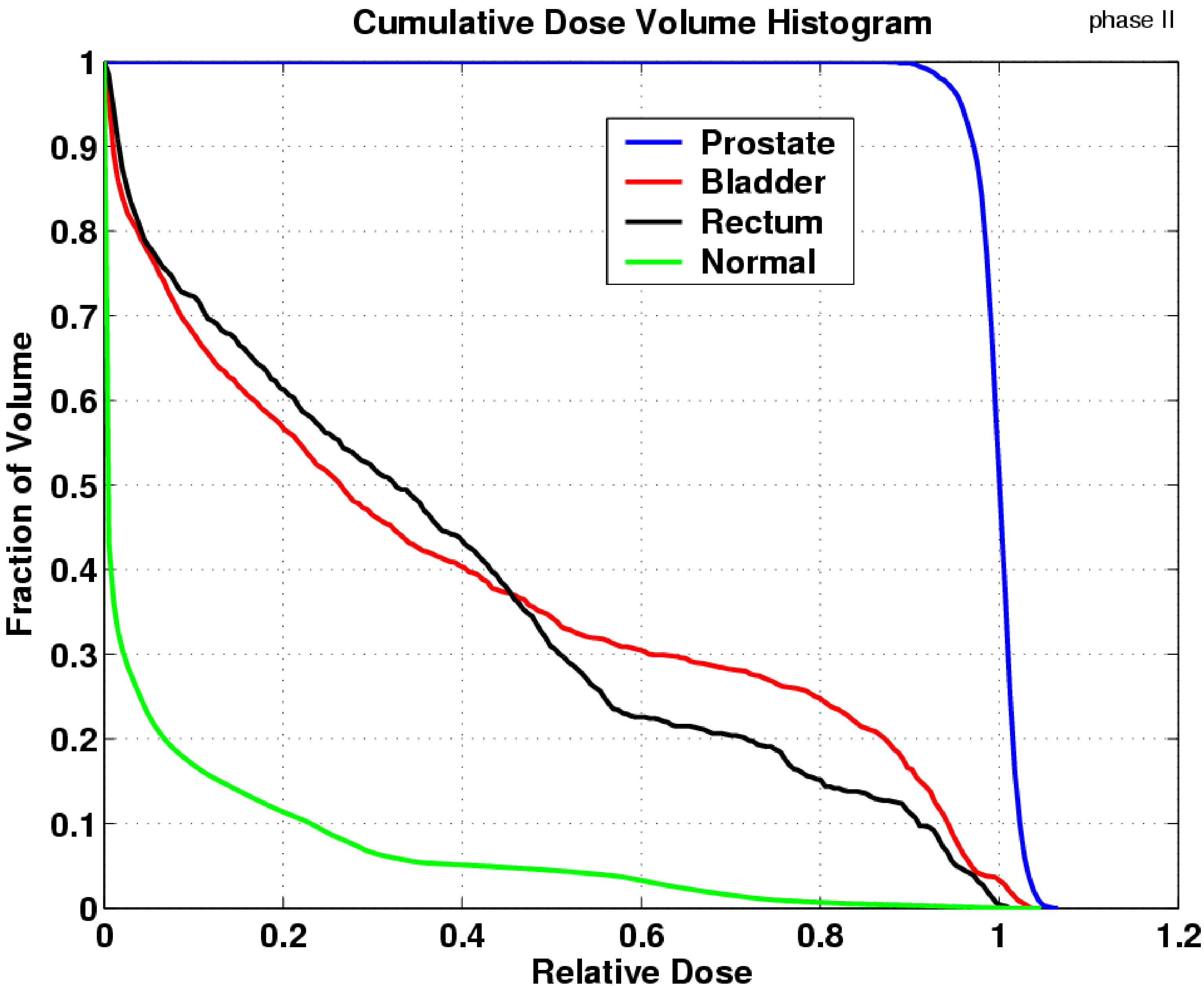
Mixed Integer Approach

$$\psi_A = \begin{cases} 1 & \text{if use angle A} \\ 0 & \text{else} \end{cases}$$

$$0 \le w_A \le W\psi_A$$

$$\sum_A \psi_A \le K$$

$$Dose(i) := \sum_A w_A D_A(i)$$



Cumulative Dose Volume Histogram
phase II
Prostate
Bladder
Rectum
Normal
Fraction of Volume
Relative Dose
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1
1.2

One form of objective

$$f(Dose) = \sum_{O_j} \sum_{i \in O_j} \lambda_j \max\{Dose_i - \theta_j, 0\}$$

Dose/Volume Constraints

- e.g. (Langer) no more than 5% of region R can receive more than U Gy

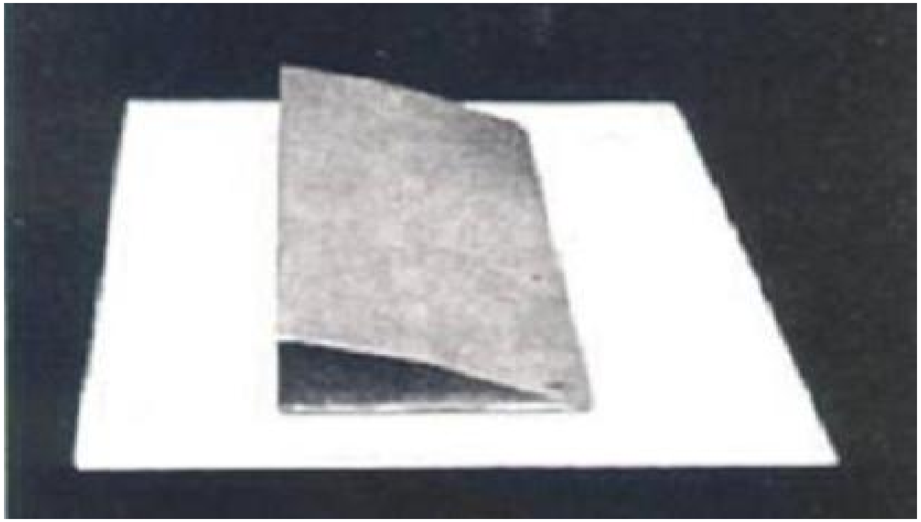
$$(\bar{U} - U)Viol(i) \geq Dose(i) - U$$

$$\sum_R Viol(i) \leq \frac{5|R|}{100}$$

$$Viol(i) \in \{0, 1\}$$

Wedges

- A metallic wedge filter can be attached in front of the collimator.
- It attenuates the intensity of radiation in a linear fashion from one side to other.
- Particularly useful for a curved patient surface



- 5 positions considered: Open, North, East, South, and West.

Mixed Integer Approach

$$\phi_{A,F} = \begin{cases} 1 & \text{if use angle A, field F} \\ 0 & \text{else} \end{cases}$$

$$\phi_{A,F} \leq \psi_A$$

$$0 \leq w_{A,F} \leq W \phi_{A,F}$$

$$\sum_A \psi_A \leq K$$

$$Dose(i) := \sum_{A,F} w_{A,F} D(A, F, i)$$

Conformal Therapy

- Conventional treatment
- Beam's eye view (collimator shaping)
- Multiple angles (choose subset)
- Wedges (modify intensity over field)
- Non-coplanar beams (choose which planes)
- Avoidance (upper bounds)
- Homogeneity, conformality
- Dose/volume constraints

Assumptions/Setting

- Dose calculation via Monte Carlo
- Objective is "truth"; we really do want to minimize it
- Limit discussion to beam angle selection; ideas are perfectly generalizable
- Limit "planning tool" to 3DCRT via MIP (most widely used)

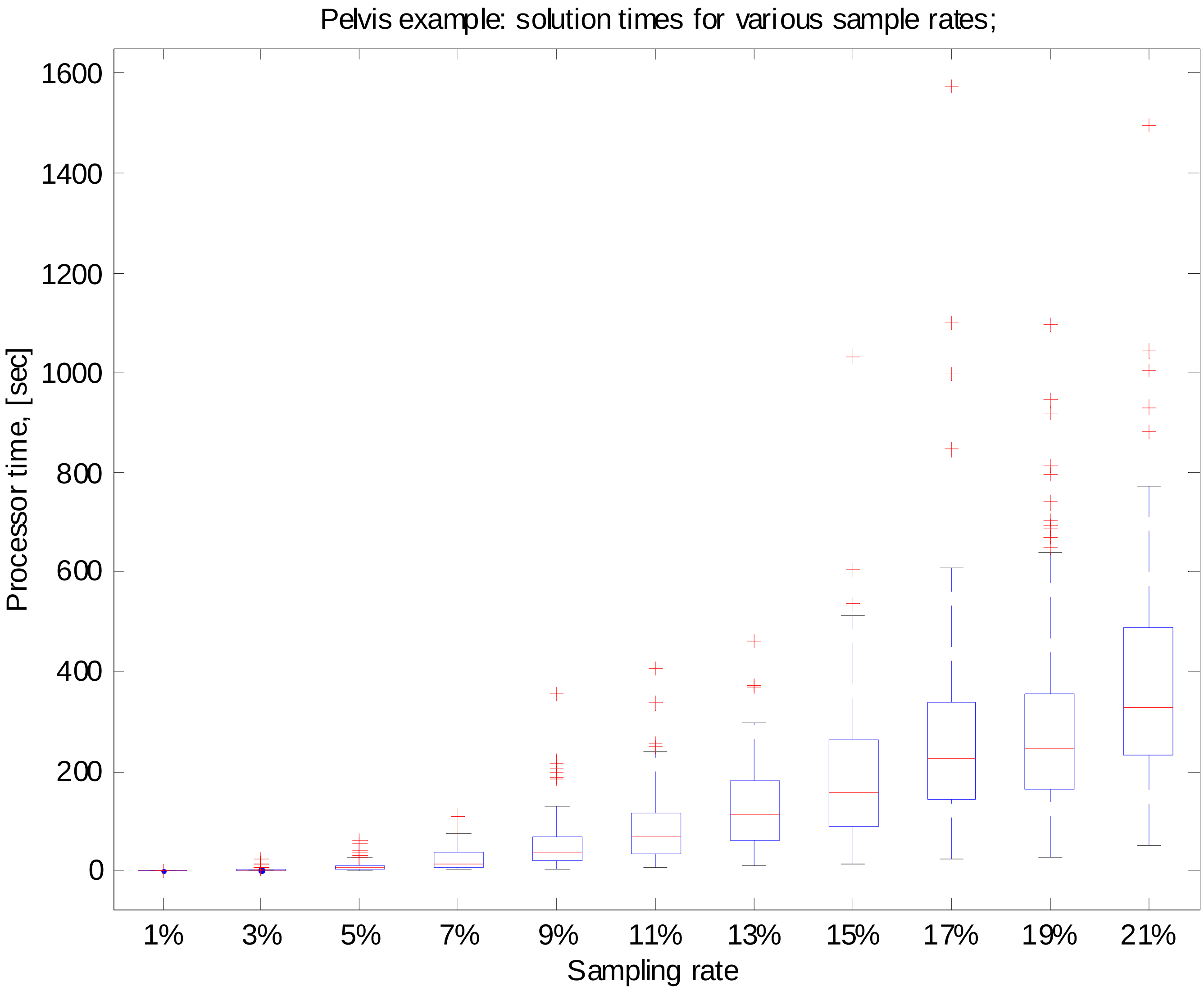
Remarks

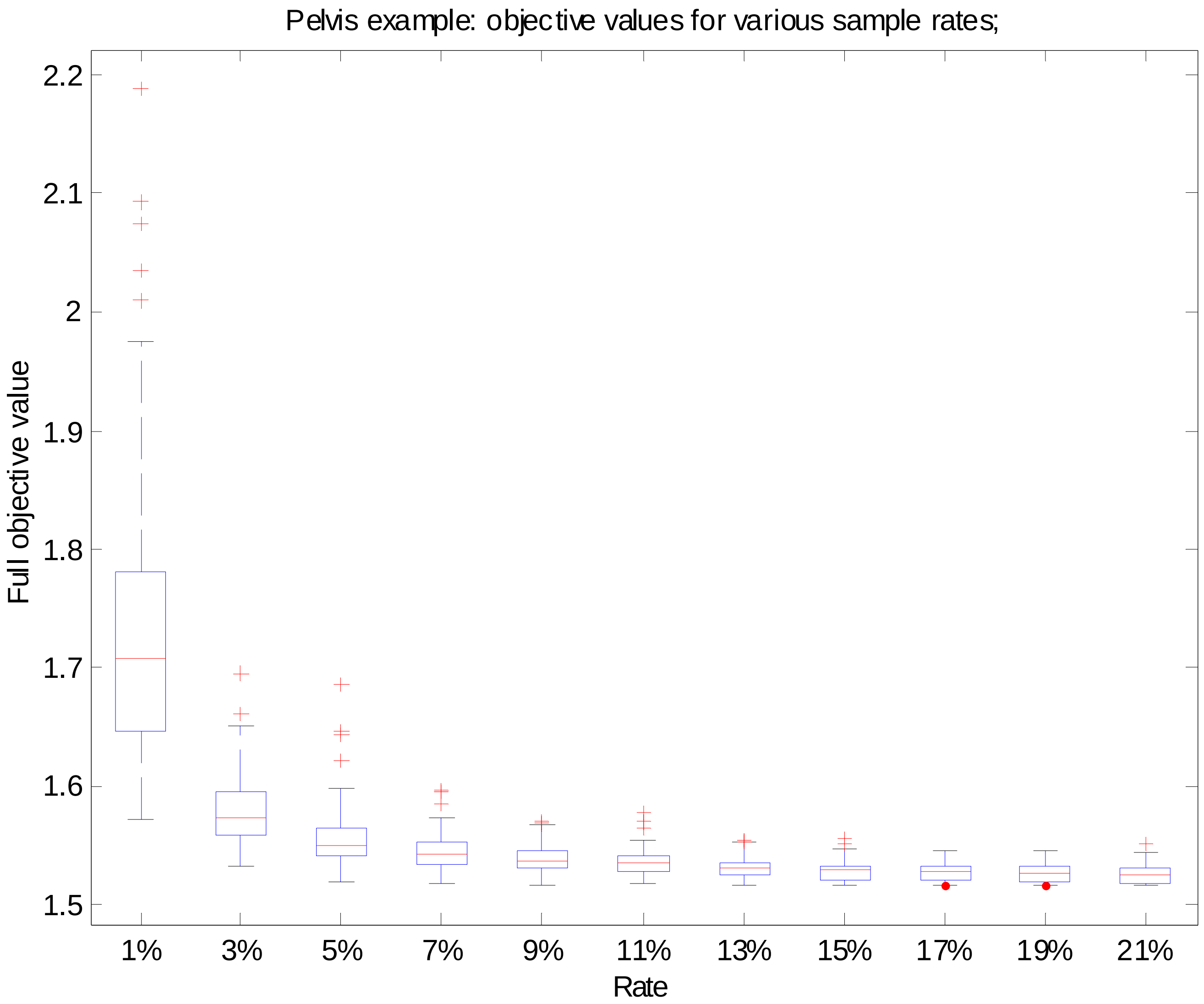
- CPLEX 9.0 used, tight tolerances
- Branch/Bound/Cut code
- LP relaxation solved using dual simplex (small samples) and barrier method (large samples)
- Terma may add sparsity, CPLEX removes dense columns in factor

Problems

- Large computational times
- Large variance in computing times
  - 5000-12500 sec (for 60,000 voxel case)
- Ineffective restarts (what if trials?)
- Large amounts of data

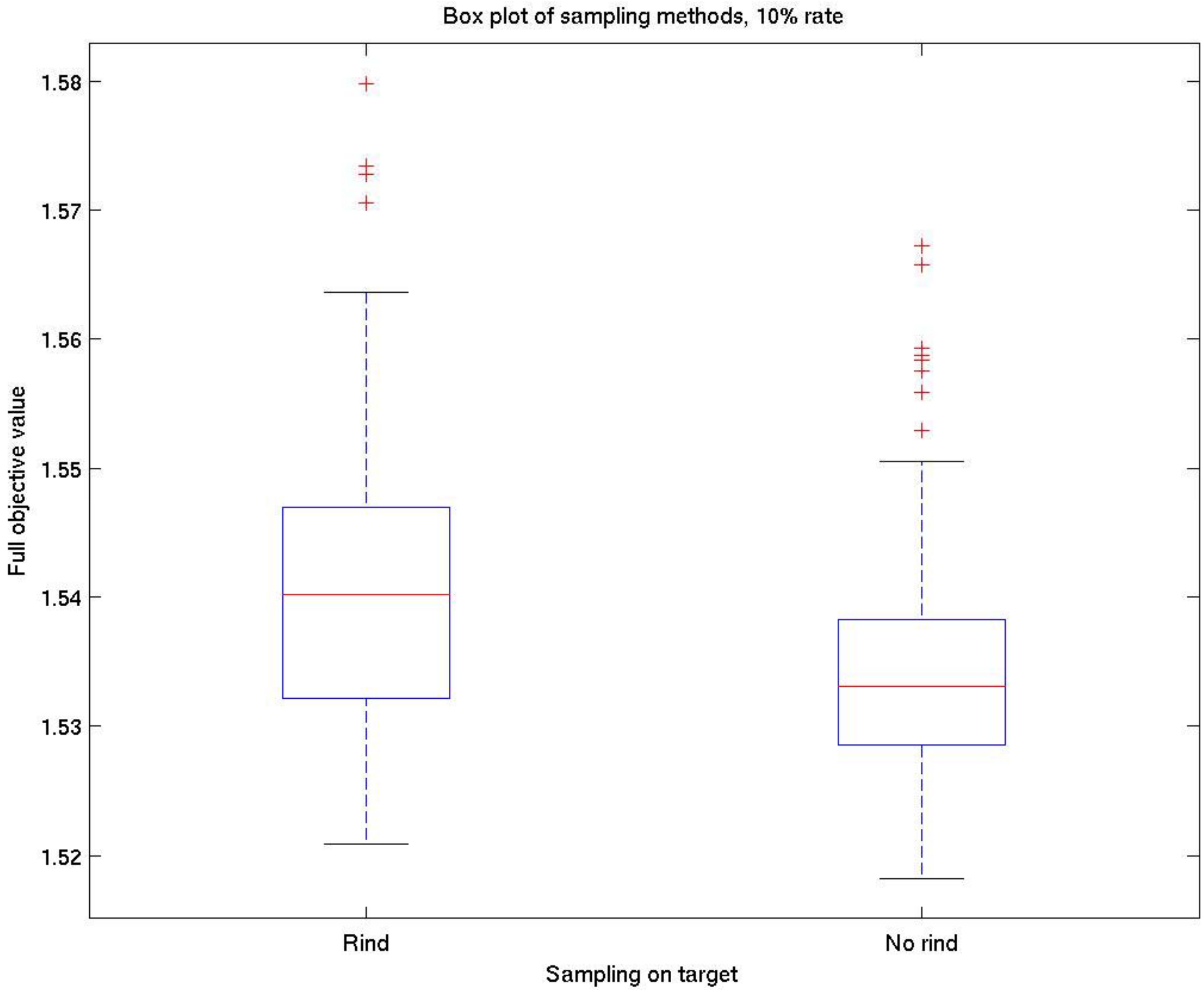
- Try sampling of voxels (carefully)

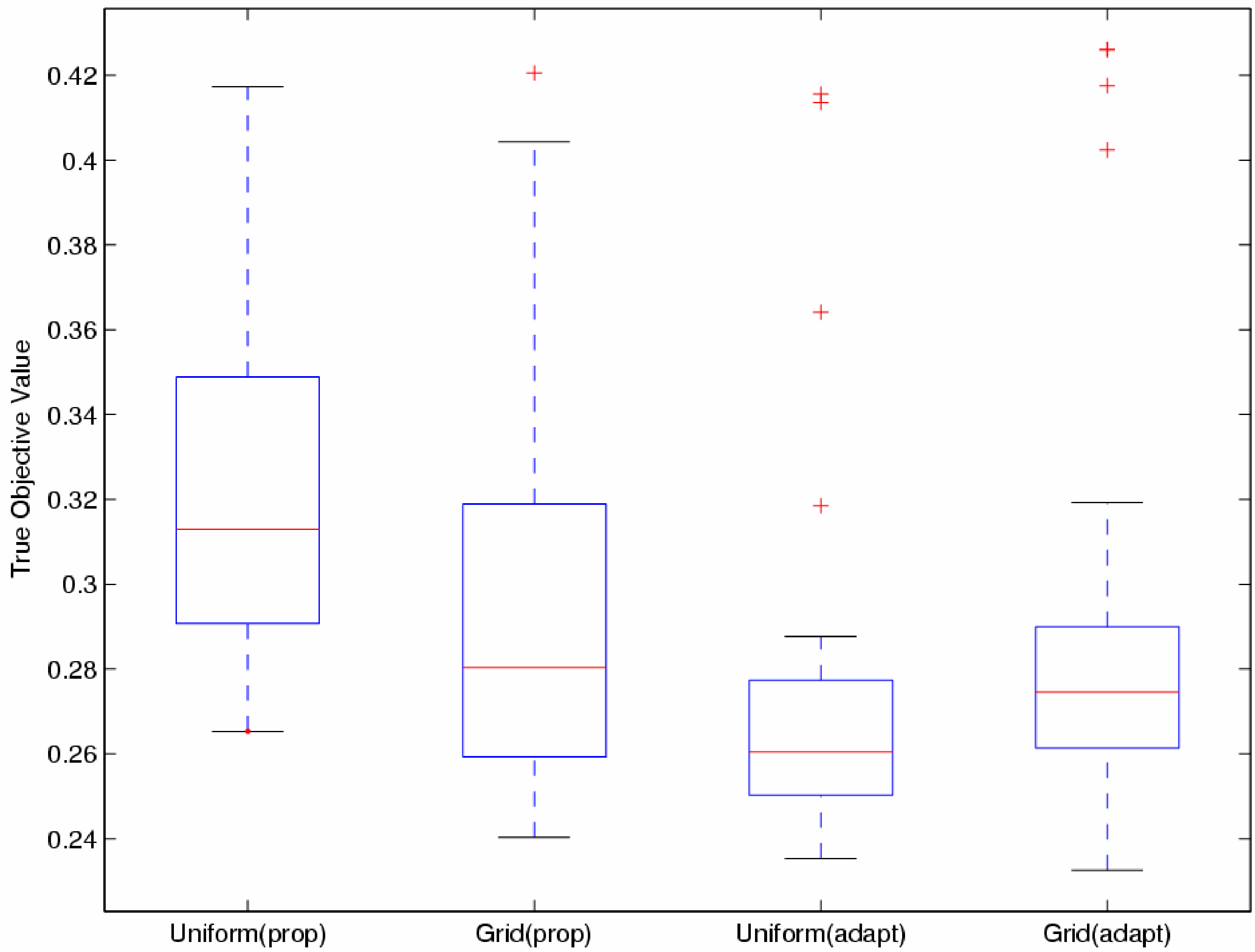

Pelvis example: solution times for various sample rates;
Processor time, [sec]
Sampling rate
0
200
400
600
800
1000
1200
1400
1600
1%
3%
5%
7%
9%
11%
13%
15%
17%
19%
21%


Pelvis example: objective values for various sample rates;
Full objective value
Rate
2.2
2.1
2
1.9
1.8
1.7
1.6
1.5
1%
3%
5%
7%
9%
11%
13%
15%
17%
19%
21%

Naïve sampling fails

- Normal tissue
  - Many more voxels available
  - Streaking effects
  - Use 5x sample on 2nd largest structure
- Small structures
  - Minimum sample size
- Homogeneity/min/max on PTV
  - 2x sample on PTV, rind sampling
- Large gradients on OAR's
  - 2x sample on OAR's
- Need adaptive mechanism


Box plot of sampling methods, 10% rate
1.58
1.57
1.56
1.55
1.54
1.53
1.52
Full objective value
Rind
No rind
Sampling on target



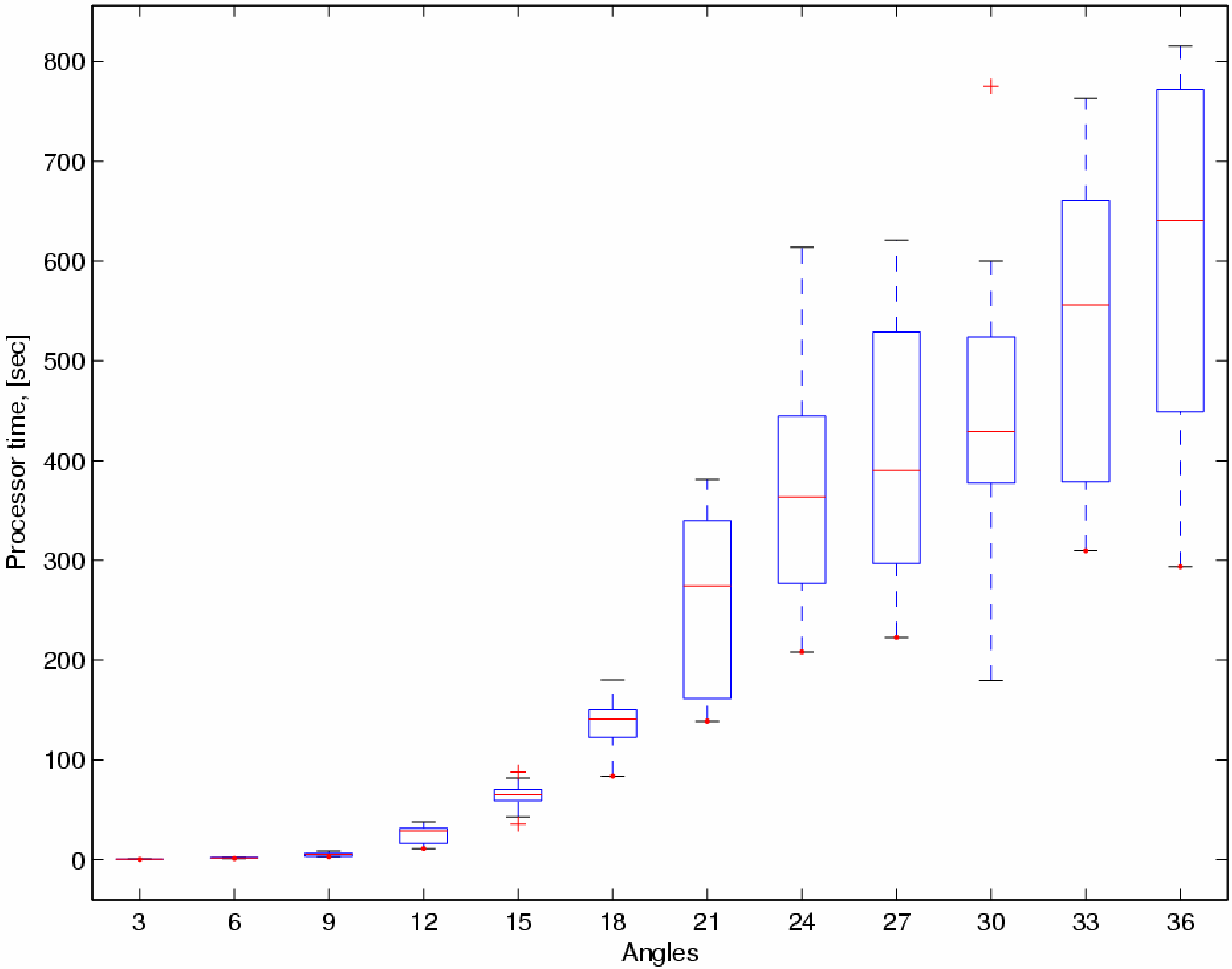
True Objective Value
0.42
0.4
0.38
0.36
0.34
0.32
0.3
0.28
0.26
0.24
Uniform(prop)
Grid(prop)
Uniform(adapt)
Grid(adapt)

Time/quality tradeoff

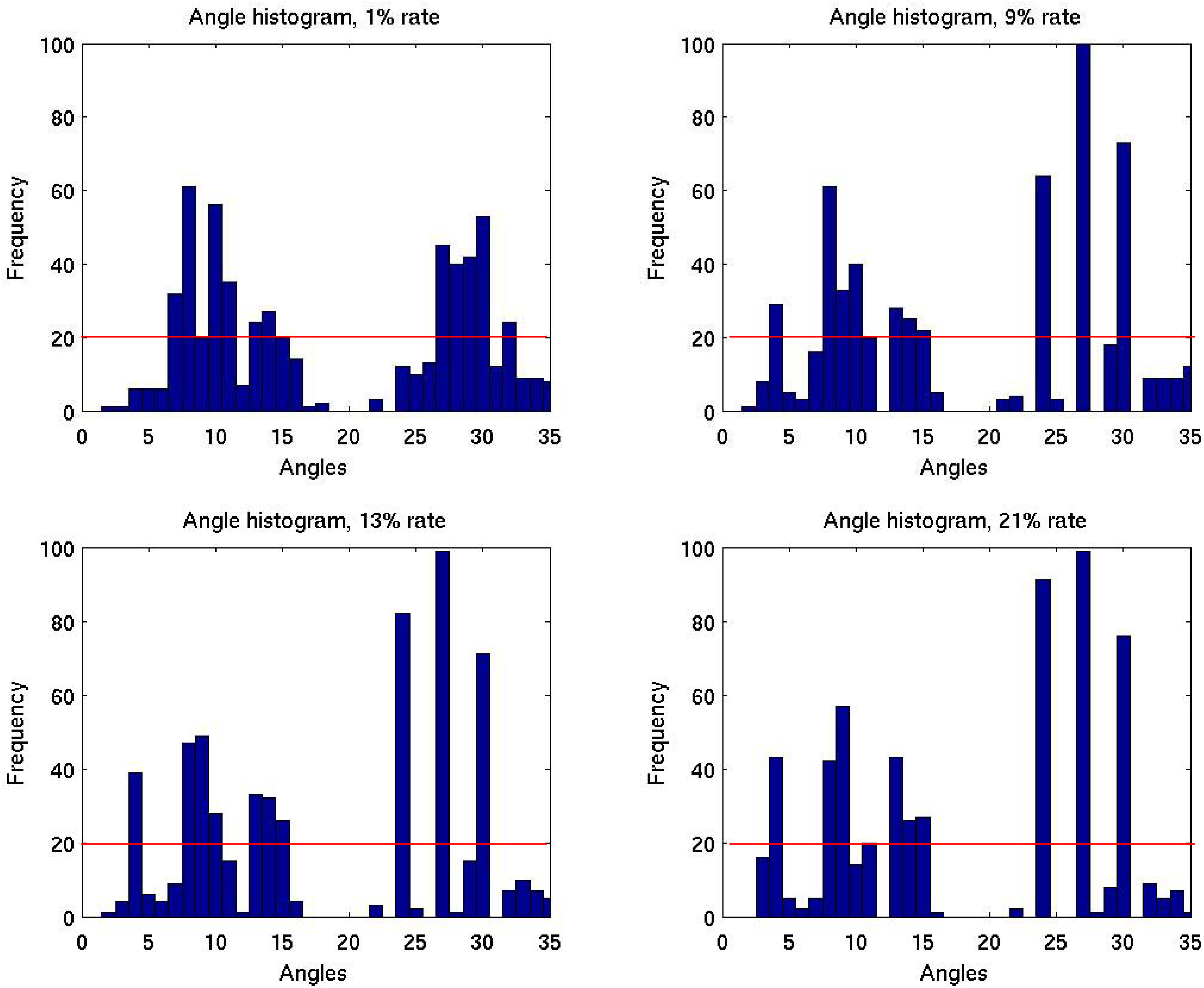
- Not really satisfactory
- Split up problem into two phases
  - Find a reduced set of angles at coarse sampling
  - Optimize with reduced set of angles with finer sample
- Reduced angle problem much faster

$$\binom{36}{6} >> \binom{10}{6}$$

- But doesn't identify angle set well...



Processor time, [sec]
800
700
600
500
400
300
200
100
0
3
6
9
12
15
18
21
24
27
30
33
36
Angles

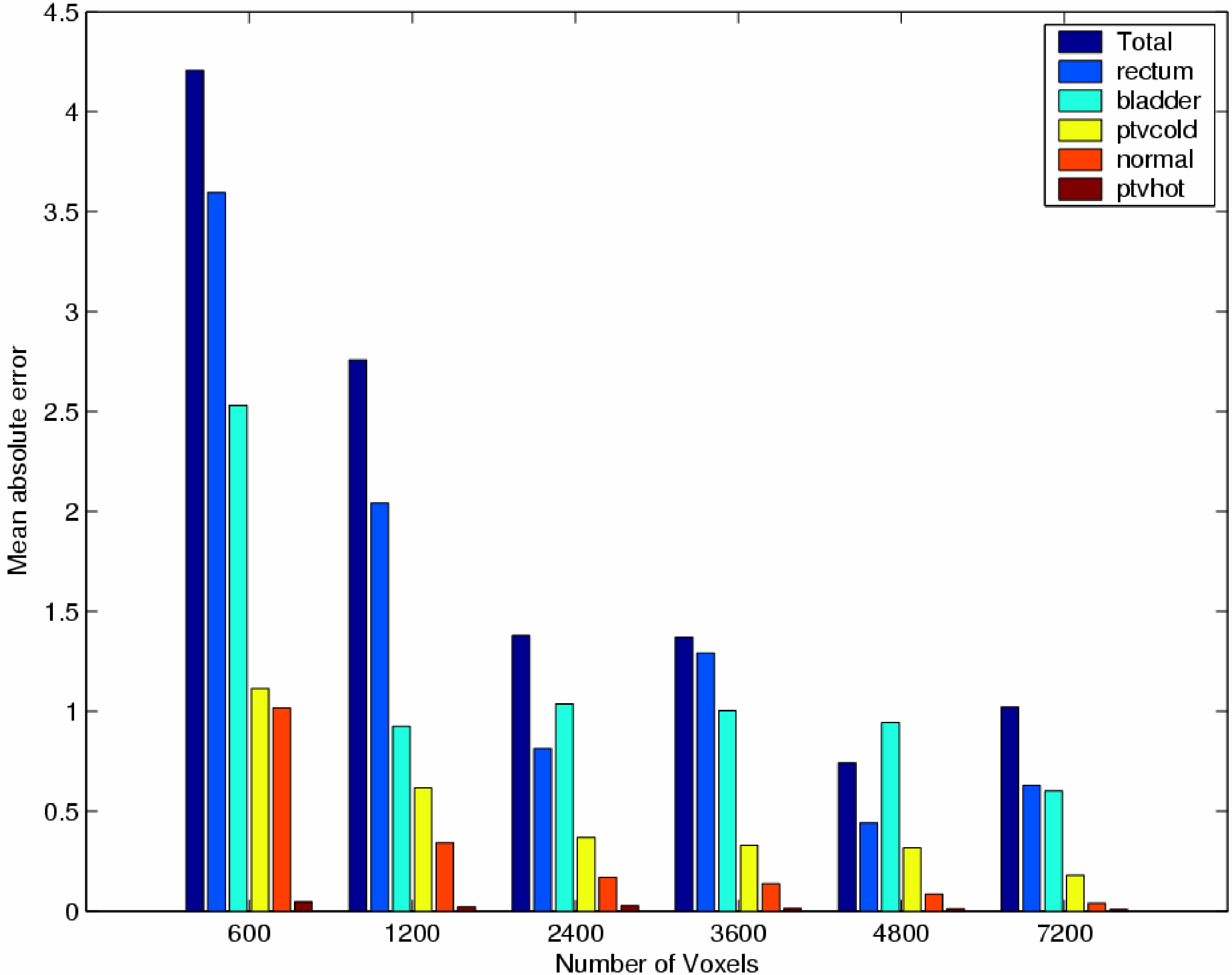

Angle histogram, 1% rate
Angle histogram, 9% rate
Angle histogram, 13% rate
Angle histogram, 21% rate
Frequency
Angles
0
20
40
60
80
100
0
5
10
15
20
25
30
35

Multiple samples

- Generate K instances at very coarse sampling rate
- Use histogram information to suggest promising angles
- How many? (e.g. K=10)
- How to select promising angles? (frequency > 20%)

Full Objective Value

- >20% scheme may lose best solution
- Can calculate the objective function with complete sample cheaply from solution of sampled problem
- Use extra information in 2 ways:
  1. Select only those angles that appear in the best “full value” solutions
  2. Refine samples in organs where discrepancies are greatest



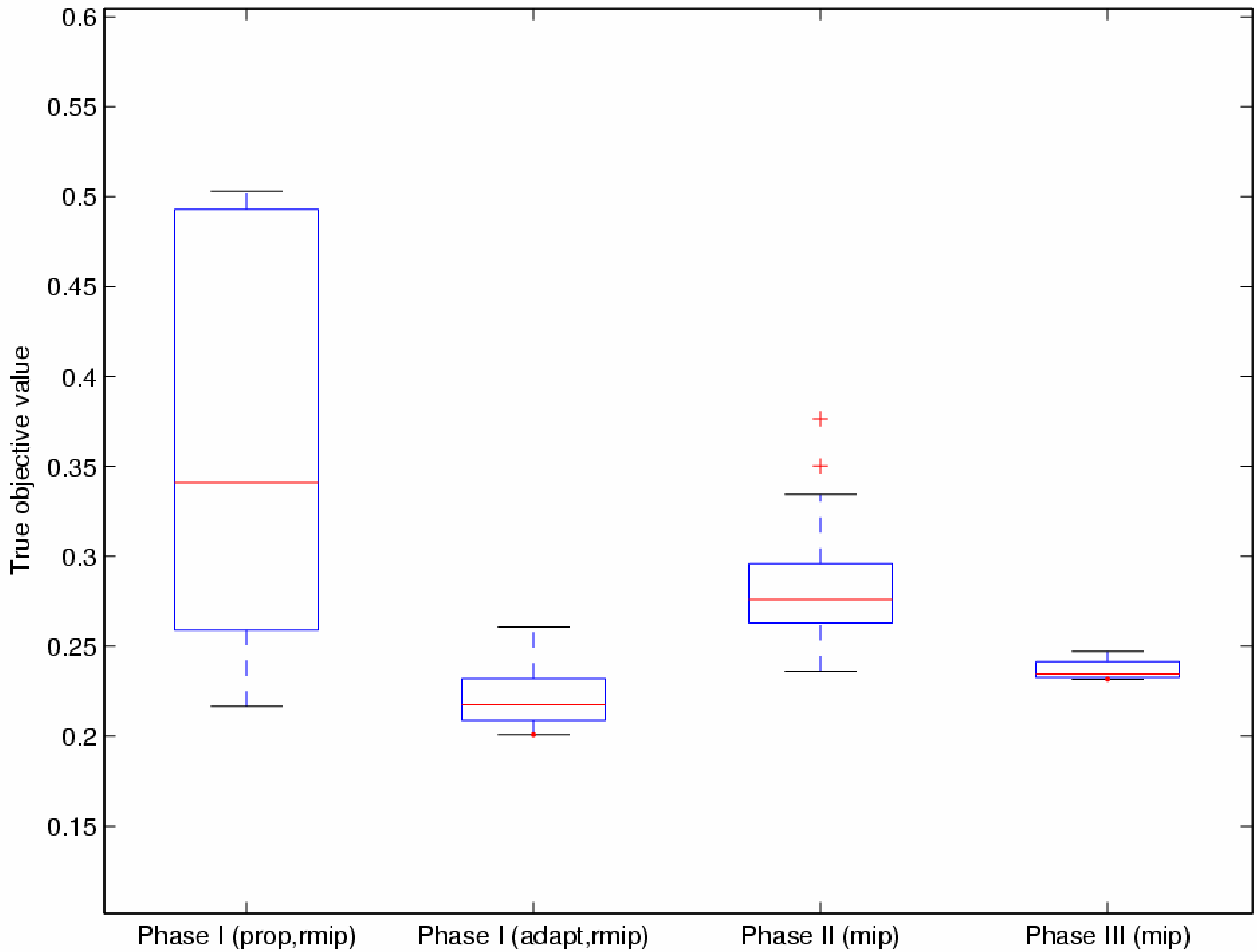
Total
rectum
bladder
ptvcold
normal
ptvhot
Mean absolute error
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
600
1200
2400
3600
4800
7200
Number of Voxels

Phase I

- Must be very fast to be useful
- LP relaxation much quicker (allowing larger sample rates) and time variance much smaller
- But too many angles suggested…
- Utilize this procedure to do "gross reduction", followed by Phase II to refine angles further
- Also adapt the sample (where sampled objective values are poor approximations)

Sampling Process

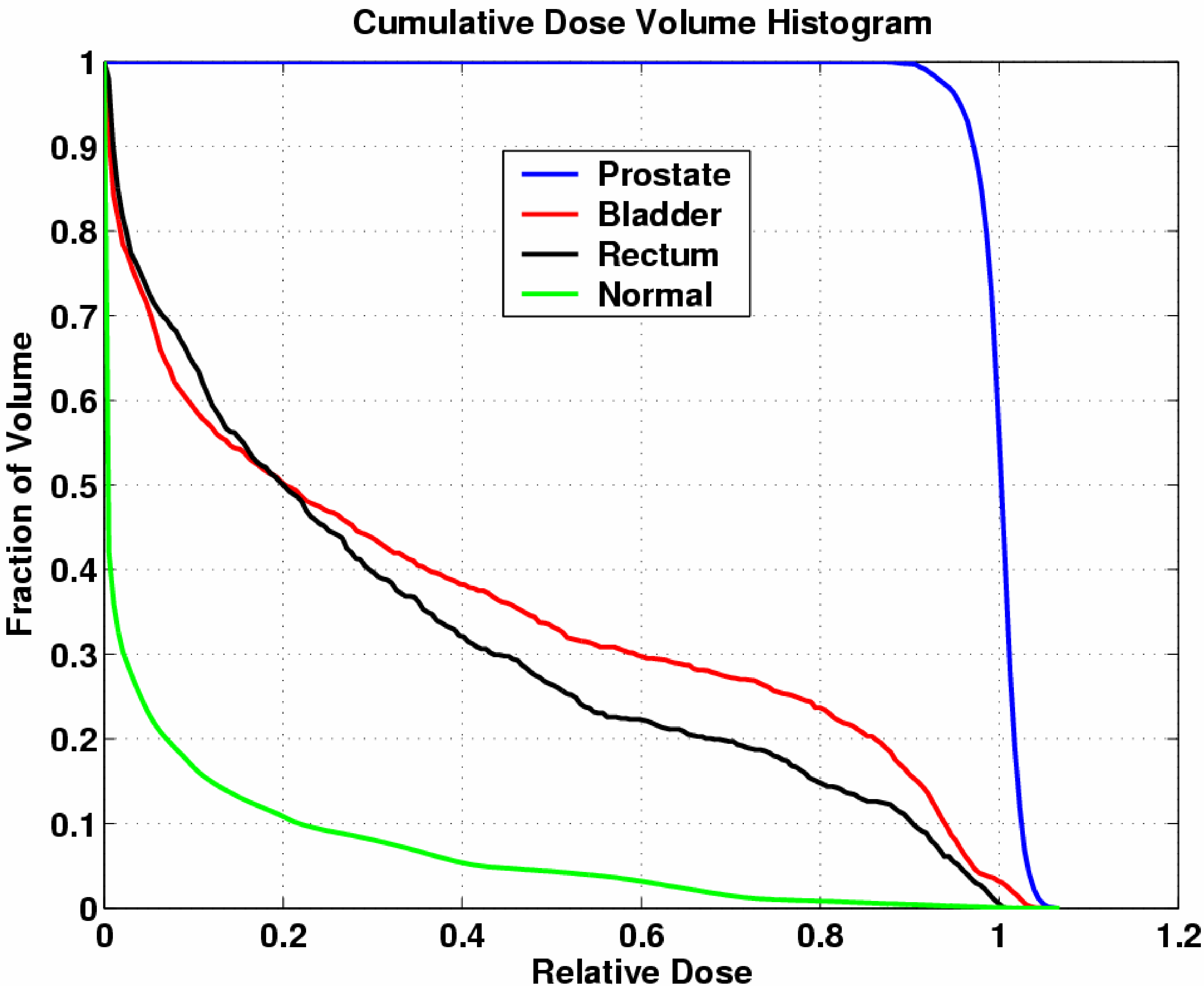
- Determine initial sample size
- Phase I: use all angles
  - 10 sample LP's solutions determine $\mathcal{A}_{\mathcal{I}}$
- Phase II: use reduced set of angles
  - 10 sample MIP's determine $\mathcal{A}_{\mathcal{II}}$
- Phase III: use further reduced set
  - Increase sample rate, solve single MIP



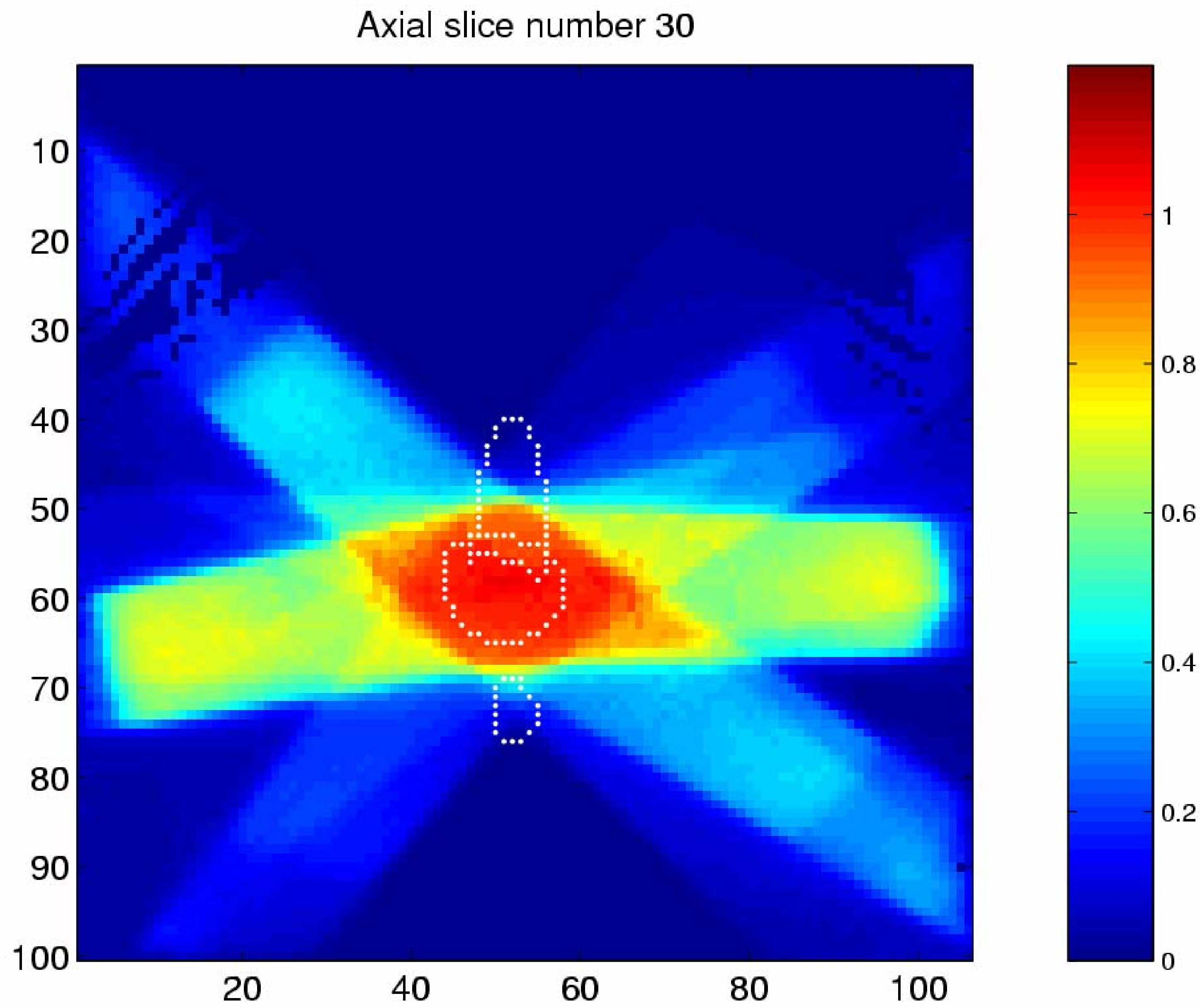
True objective value
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
Phase I (prop,rmip)
Phase I (adapt,rmip)
Phase II (mip)
Phase III (mip)

Pelvis case

- 3K prostate, 1.5K bladder, 1K rectum, 557K normal
- Time for "full problem": 12.5K secs
- Time Phase I: 32 secs
- Time Phase II: 18 secs
- Time Phase III: 147 secs
- Solution: 80, 110, 130, 240, 270, 320



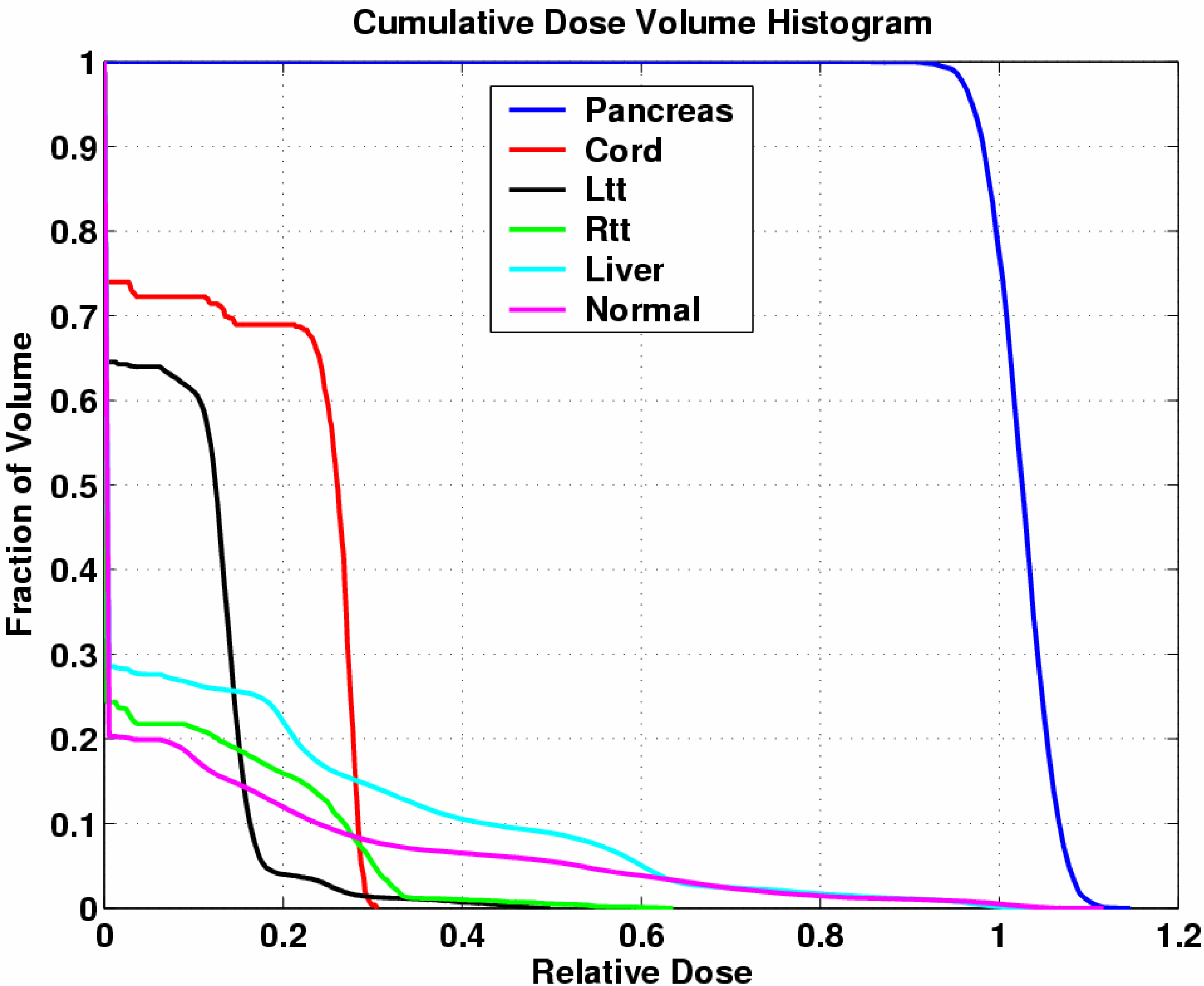
Cumulative Dose Volume Histogram
Prostate
Bladder
Rectum
Normal
Fraction of Volume
Relative Dose
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1
1.2



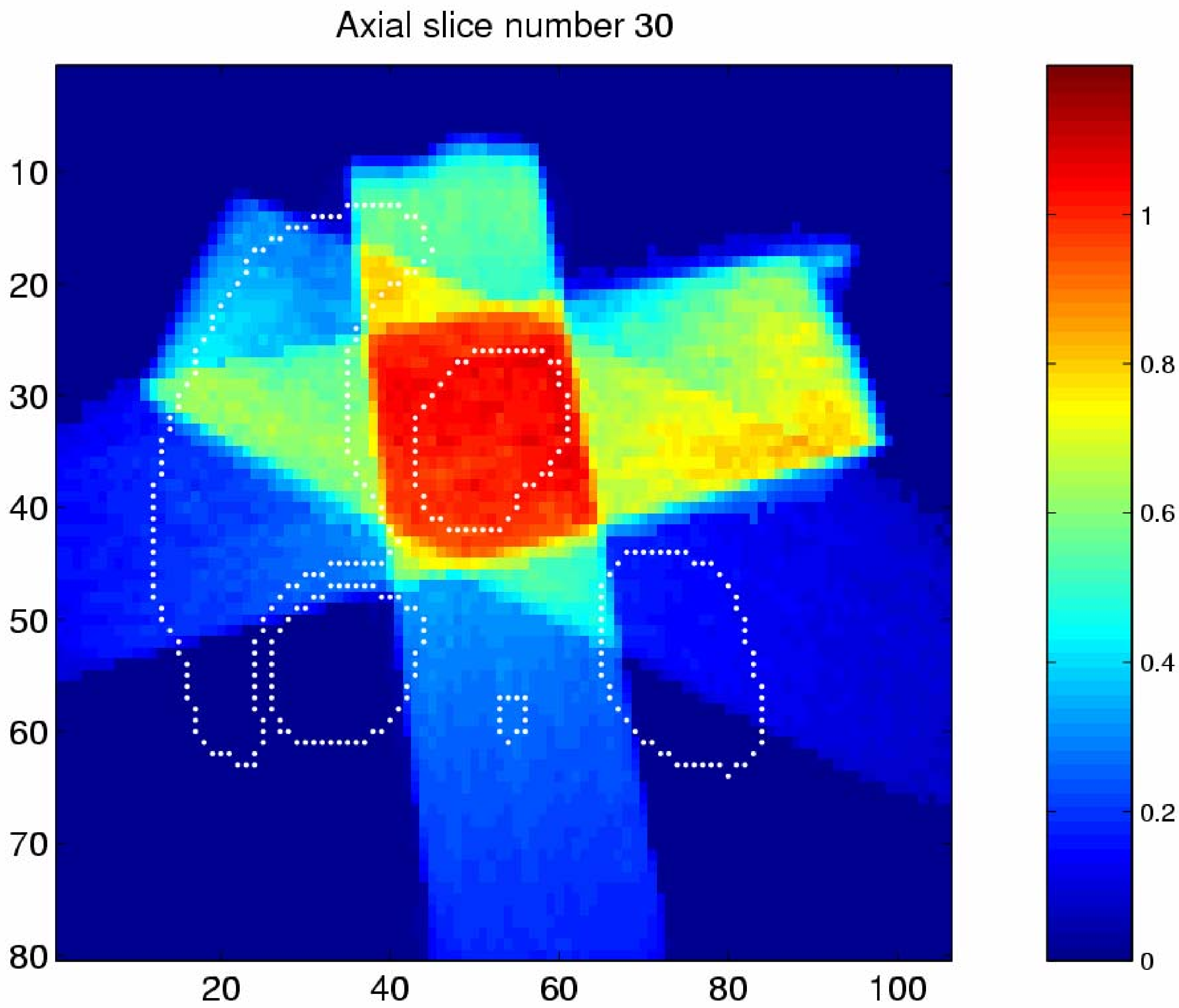
Axial slice number 30
10
20
30
40
50
60
70
80
90
100
20
40
60
80
100
1
0.8
0.6
0.4
0.2
0

Pancreas case

- 6K pancreas, 515 cord, 9K ltt, 6K rtt, 54K liver, 502K normal
- Time for “full problem”: 1200 secs
- Time Phase I: 5 secs
- Time Phase II: 229 secs
- Time Phase III: 56 secs
- Solution: 80, 290, 350 (+ wedges)

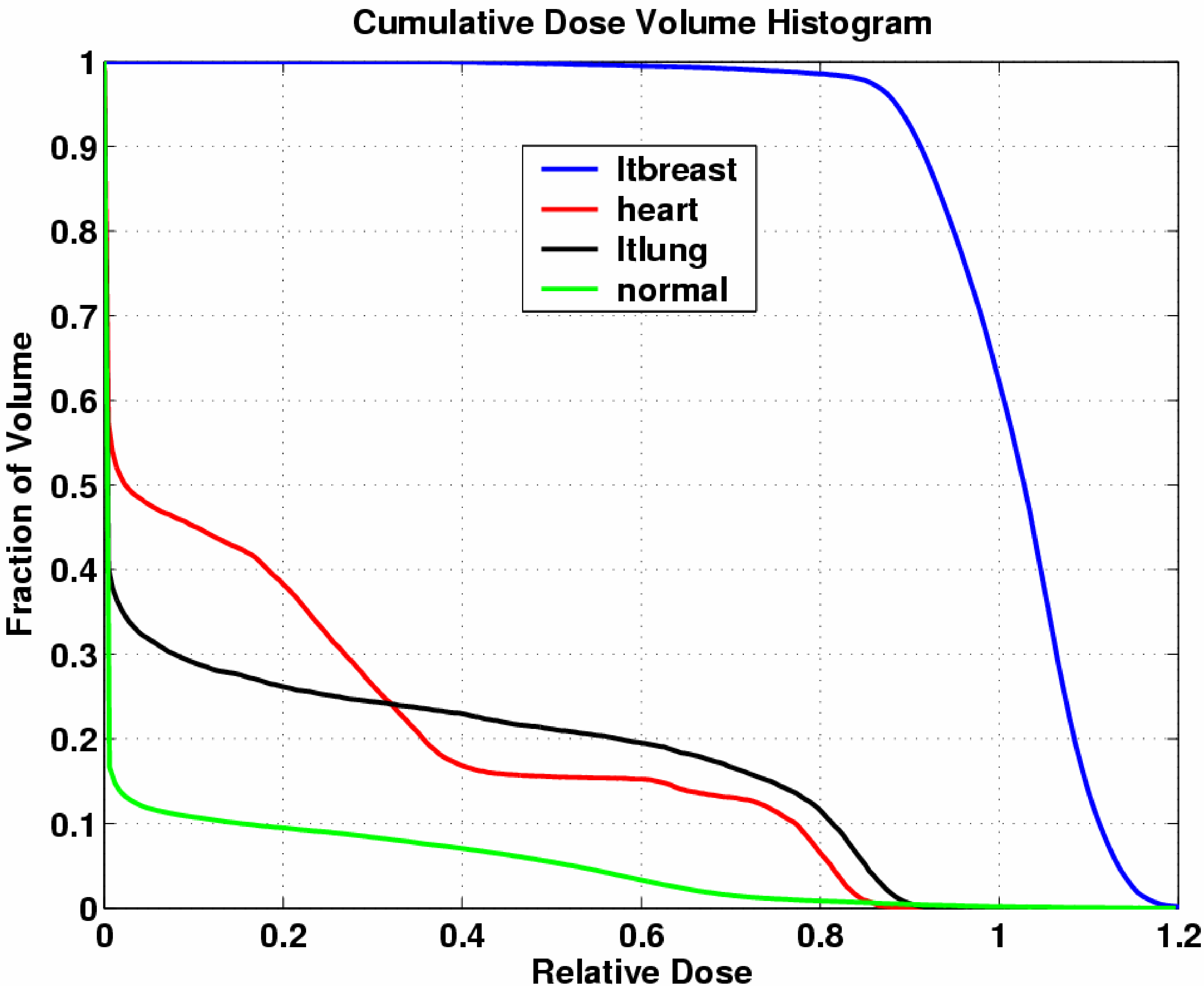


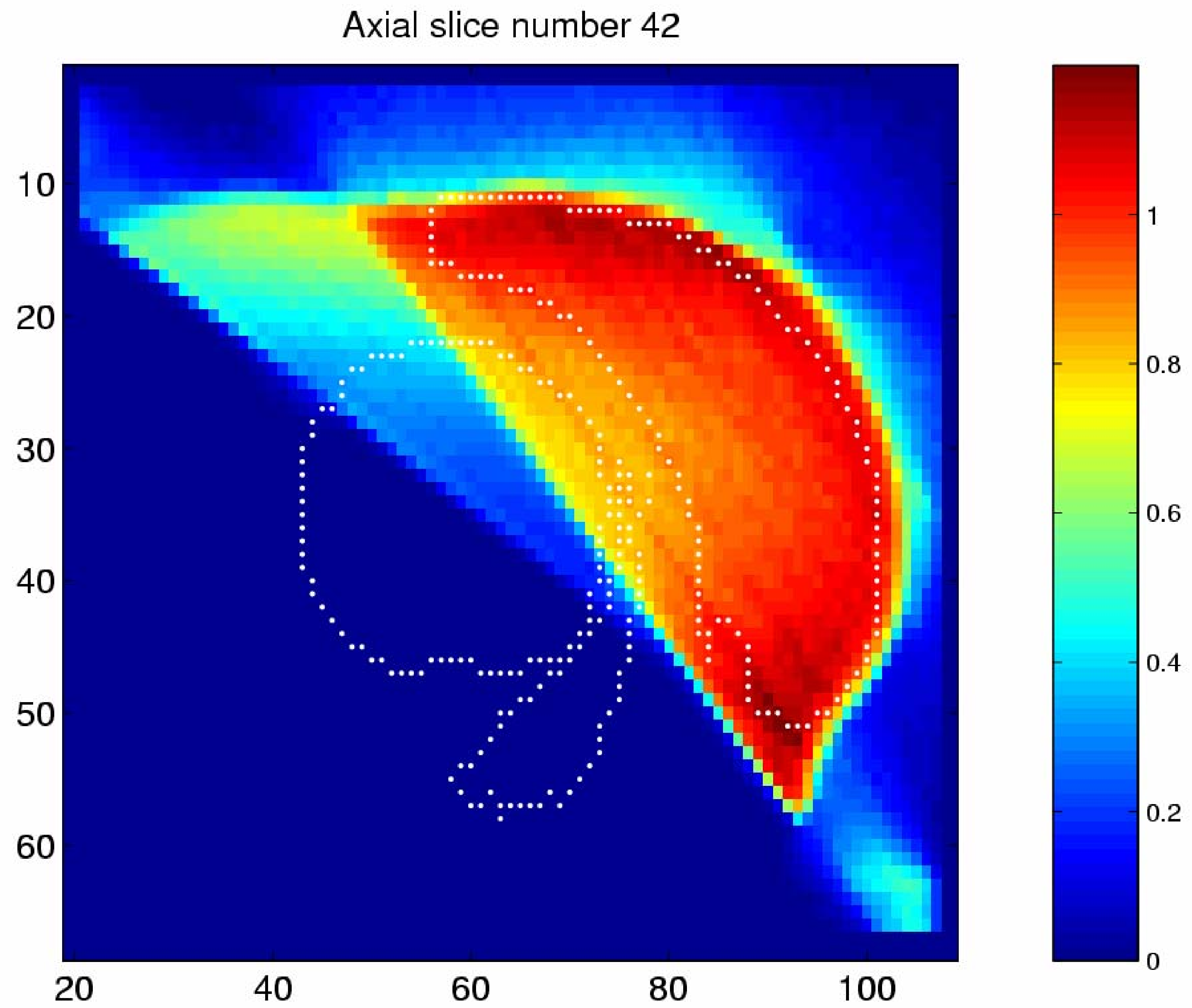
Cumulative Dose Volume Histogram
Pancreas
Cord
Ltt
Rtt
Liver
Normal
Fraction of Volume
Relative Dose
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1
1.2


Axial slice number 30
10
20
30
40
50
60
70
80
20
40
60
80
100
1
0.8
0.6
0.4
0.2
0

Breast case

- 39K ptv, 13K heart, 11K rind breast, 71K normal
- Time Phase I: 29 secs
- Time Phase II: 56 secs
- Time Phase III: 3 secs
- Solution: 130, 290 (+ wedges)

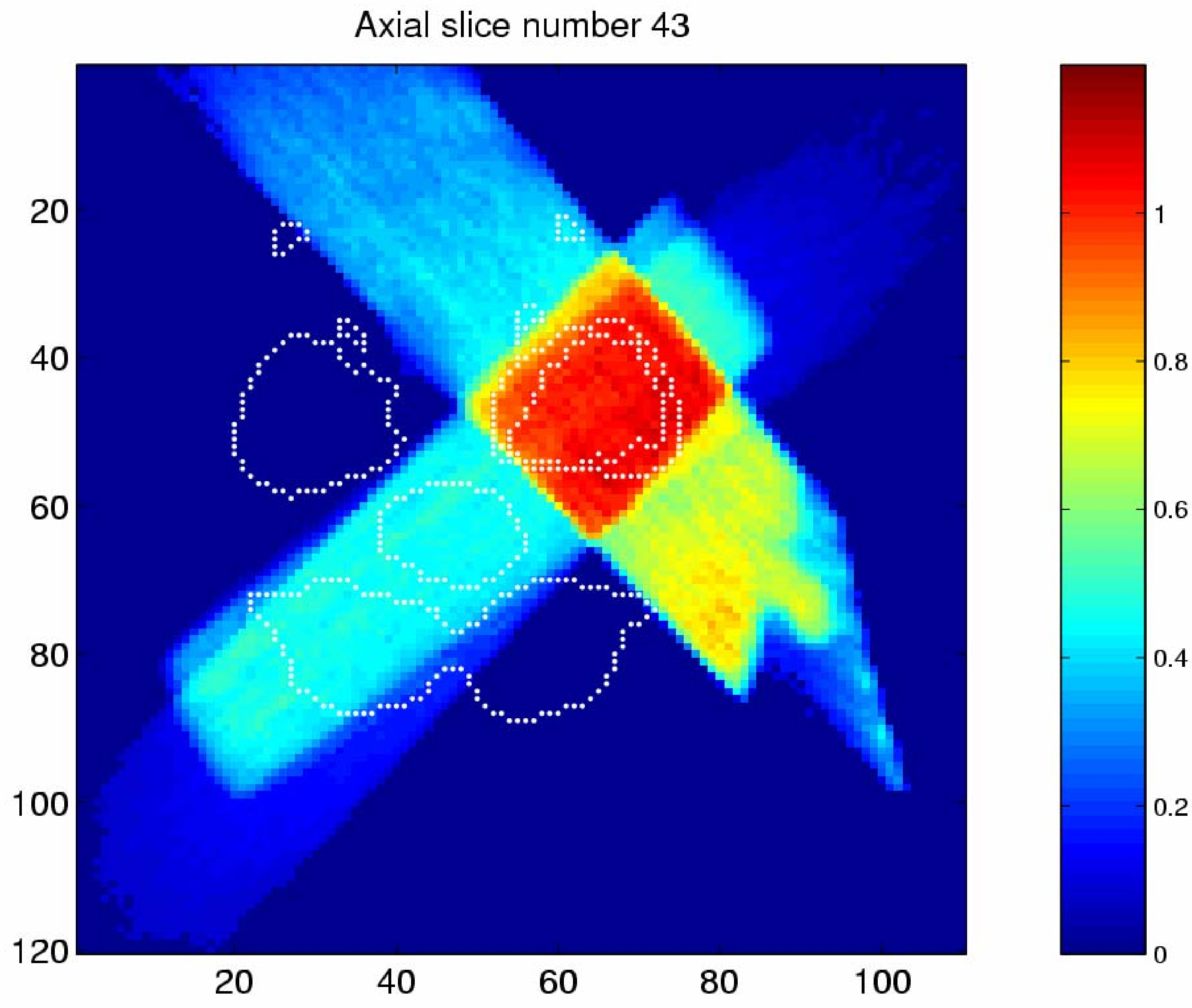

Cumulative Dose Volume Histogram
ltbreast
heart
ltlung
normal
Fraction of Volume
Relative Dose
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1
1.2



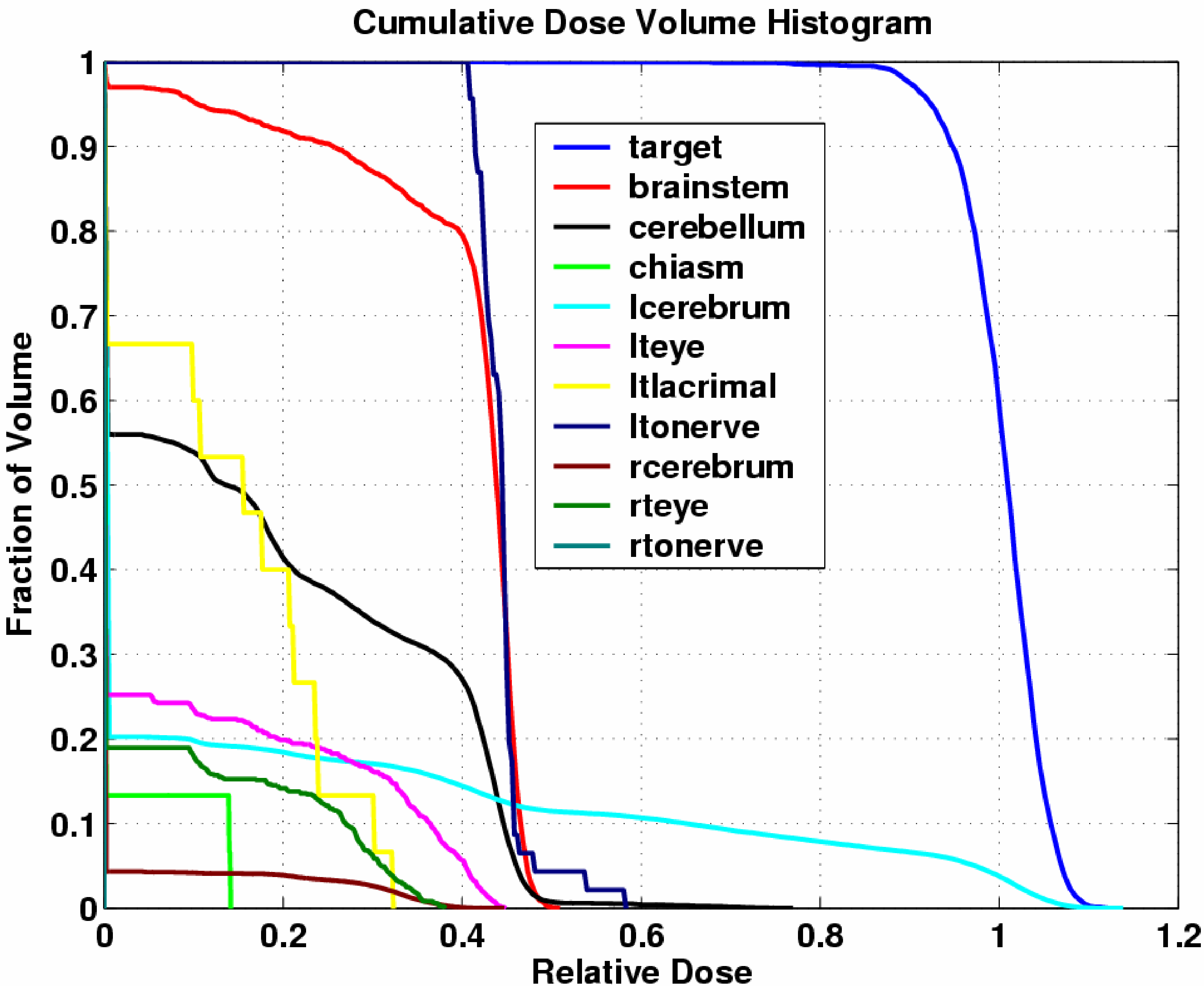
Axial slice number 42
10
20
30
40
50
60
20
40
60
80
100
1
0.8
0.6
0.4
0.2
0

Head/Neck case

- 2K ptv, 51K l/rcerebrum, 2K brainstm, 14K cerebellum, others (15-833)
- Time for “full problem”: 2542 secs
- Time Phase I: 5 secs
- Time Phase II: 53 secs
- Time Phase III: 10 secs
- Solution: 40, 140, 230 (+ wedges)



Axial slice number 43
20
40
60
80
100
120
20
40
60
80
100
1
0.8
0.6
0.4
0.2
0



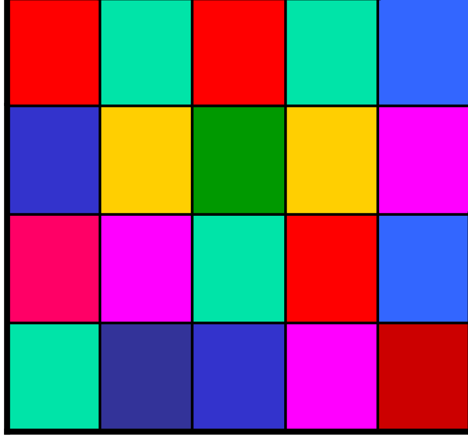
Cumulative Dose Volume Histogram
Fraction of Volume
Relative Dose
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1
1.2
target
brainstem
cerebellum
chiasm
lcerebrum
lteye
ltlacrimal
ltonerve
rcerebrum
rteye
rtonerve

Extensions

- Within 3DCRT
  - Wedges, energy levels, non-coplanar beams all optimized concurrently
- Tomotherapy
- IMAT
- IMRT
- Larger and more complex cases

Conclusions

- Optimization improves treatment planning
- Adaptive sampling is effective tool for solution time reduction
- Future work needed for more complex delivery devices and for adaptive radiotherapy



IMRT

- Beam = collection of pencils
- Intensity maps -> deliverable shapes -> intensities of shapes
- Limit number of deliveries

- Optimize intensities and shapes concurrently
- Arc based delivery

Commonalities

- Target (tumor)
- Regions at risk
- Maximize kill, minimize damage
- Homogeneity, conformality constraints
- Amount of data, or model complexity
- Mechanism to deliver dose