Dynamic Risked Equilibria

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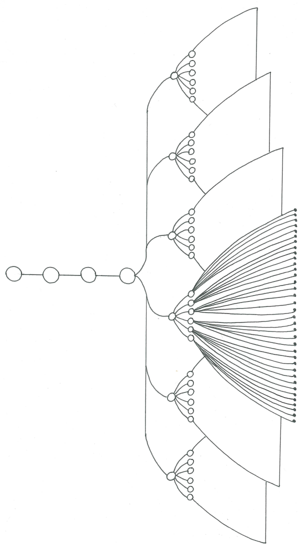
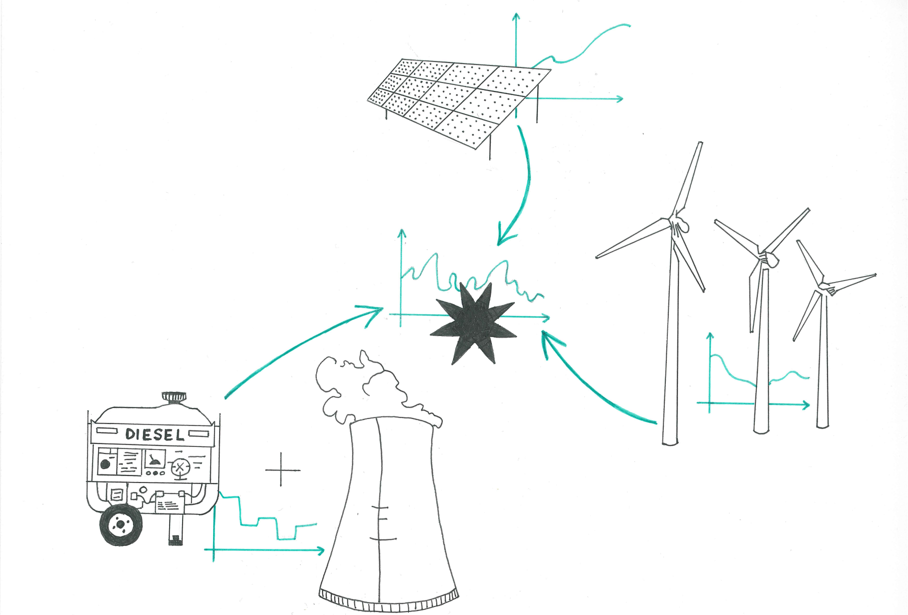
Dynamics and uncertainties (risk neutral)

- Scenario tree is data
- T stages (use 6 here)
- Nodes $n \in \mathcal{N}$, n_+ successors
- Stagewise probabilities $\mu(m)$ to move to next stage $m \in n_+$
- Uncertainties (wind flow, cloud cover, rainfall, demand) $\omega_a(n)$
- Actions u_a for each agent (dispatch, curtail, generate, shed), with costs C_a
- State and shared variables (storage, prices)
- Recursive (nested) definition of expected cost-to-go: $\theta(n) = \sum_{m \in n_+} \mu(m) \left(\sum_{a \in \mathcal{A}} C_a(u_a(m)) + \theta(m)\right)$

$t \in 0, 1, 2, 3, 4, 5, 6$



The setup: agents a =(solar, wind, diesel, consumer)



Model

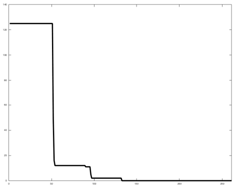
$$\text{SO:} \quad \min_{(\theta,u,x)\in\mathcal{F}(\omega)} \sum_{a\in\mathcal{A}} C_a(u_a(0)) + \theta(0)$$

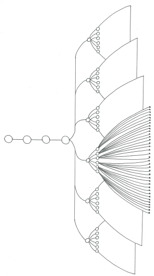
$$\text{s.t.} \;\; \theta(n) \geq \sum_{m\in n_+} \mu(m) \left(\sum_{a\in\mathcal{A}} C_a(u_a(m)) + \theta(m) \right)$$

$$\sum_{a\in\mathcal{A}} g_a(u_a(n)) \geq 0$$

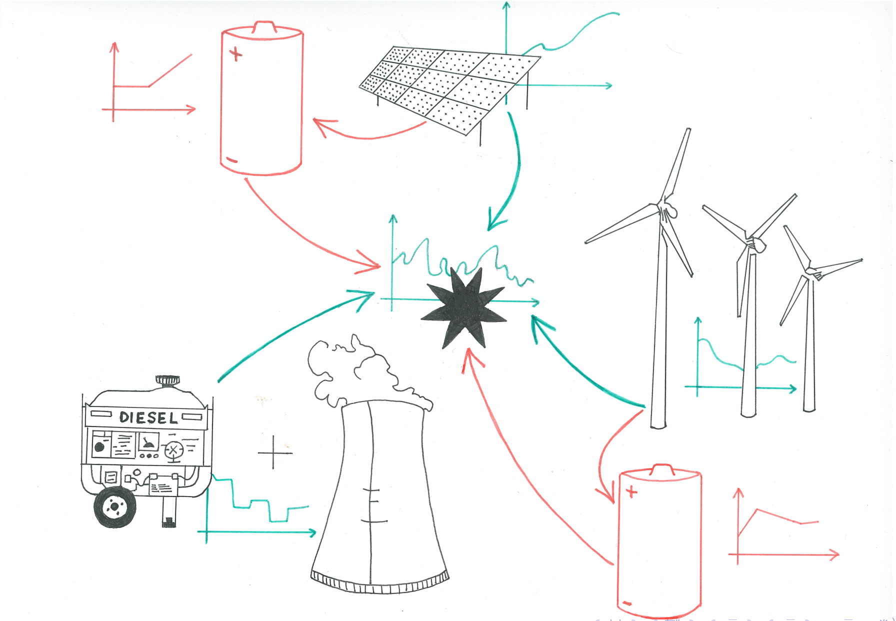
- g_a converts actions into energy.
- Solution (risk neutral, system optimal):
- consumer cost 1,308,201; probability of shortage 19.5%
- No transfer of energy across stages.

Prices π on energy constraint:





Add storage (smoother) to uncertain supply



The Philpott batch problem

Solar panels:

Battery:

Petrol generator:

Pump storage:

Add storage

- Storage allows energy to be moved across stages (batteries, pump, compressed air, etc)
- Solution forcing use of battery consumer cost 1,228,357; probability of shortage 11.5%
- Solution allowing both options consumer cost 207,476; probability of shortage 1.1%

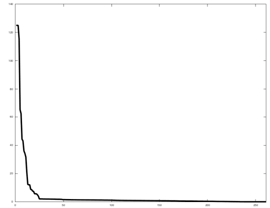
$$\min_{(\theta,u,x)\in\mathcal{F}} \quad \sum_{a\in\mathcal{A}} C_a(u_a(0)) + \theta(0)$$

$$\text{s.t.} \quad x_a(n) = x_a(n_-) - u_a(n) + \omega_a(n)$$

$$\theta(n) \geq \sum_{m\in n_+} \mu(m) \left(\sum_{a\in\mathcal{A}} C_a(u_a(m)) + \theta(m) \right)$$

$$\sum_{a\in\mathcal{A}} g_a(u_a(n)) \geq 0$$

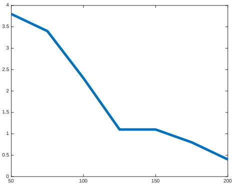
Prices π on energy constraint:



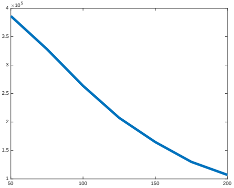
Investment planning: storage/generator capacity

Increasing battery capacity

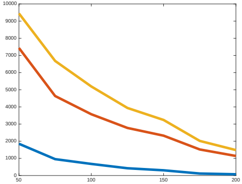
Shortage probability



Consumer cost

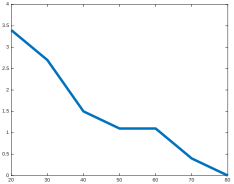


Agent profits

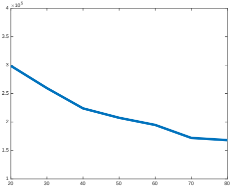


Increasing diesel generator capacity

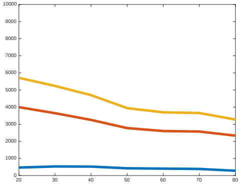
Shortage probability



Consumer cost



Agent profits



MOPEC

$$\min_{x_i} \theta_i(x_i, x_{-i}, \pi) \text{ s.t. } g_i(x_i, x_{-i}, \pi) \leq 0, \forall i$$

$$\pi \text{ solves } \mathrm{VI}(h(x, \cdot), C)$$

```
equilibrium
min theta(1) x(1) g(1)
...
min theta(m) x(m) g(m)
vi h pi cons
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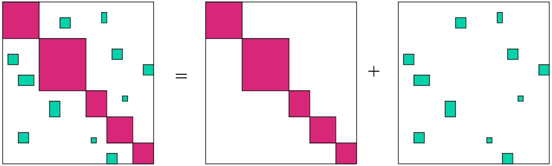
- (Generalized) Nash
- Reformulate optimization problem as first order conditions (complementarity)
- Use nonsmooth Newton methods to solve
- Solve overall problem using "individual optimizations"?



Decomposition (of agents) by prices π

Split up θ into agent contributions θ_a and add weighted constraints into objective:

$$\begin{aligned} \min_{(\theta,u,x)\in\mathcal{F}} \quad & \sum_{a\in\mathcal{A}} C_a(u_a(0)) + \theta_a(0) - \pi^T\left(g_a(u_a(n))\right) \\ \text{s.t. } & x_a(n) = x_a(n_-) - u_a(n) + \omega_a(n) \\ & \theta_a(n) \geq \sum_{m\in n_+} \mu(m)\left(C_a(u_a(m)) + \theta_a(m)\right) \end{aligned}$$

Problem then decouples into multiple optimizations

$$\begin{aligned} \text{RA}(a,\pi)\text{:} \quad \min_{(\theta,u,x)\in\mathcal{F}} \quad & Z_a(0) + \theta_a(0) \\ \text{s.t. } & x_a(n) = x_a(n_-) - u_a(n) + \omega_a(n) \\ & \theta_a(n) \geq \sum_{m\in n_+} \mu(m)(Z_a(m) + \theta_a(m)) \\ & Z_a(n) = C_a(u_a(n)) - \pi(n) g_a(u_a(n)) \end{aligned}$$

SO equivalent to MOPEC (price takers)

- Perfectly competitive (Walrasian) equilibrium is a MOPEC

$$\{(u_a(n), \theta_a(n)), n \in \mathcal{N}\} \in \arg\min \mathrm{RA}(a, \pi)$$

and

$$0 \leq \sum_{a \in \mathcal{A}} g_a(u_a(n)) \perp \pi(n) \geq 0$$

- One optimization per agent, coupled together with solution of complementarity (equilibrium) constraint.
- Overall, this is a Nash Equilibrium problem, solvable as a large scale complementarity problem (replacing all the optimization problems by their KKT conditions) using the PATH solver.
- But in practice there is a gap between SO and MOPEC.
- How to explain?

Perfect competition

$$\max_{x_i} \pi^T x_i - c_i(x_i) \qquad \text{profit}$$

$$\text{s.t. } B_i x_i = b_i, x_i \geq 0 \qquad \text{technical constr}$$

$$0 \leq \pi \perp \sum_i x_i - d(\pi) \geq 0$$

- Assume price taking, no agent can strategically affect π
- Each agent is a price taker
- Two agents, $d(\pi) = 24 - \pi$, $c_1 = 3$, $c_2 = 2$
- KKT(1) + KKT(2) + Market Clearing gives Complementarity Problem
- $x_1 = 0$, $x_2 = 22$, $\pi = 2$

Cournot: two agents (duopoly)

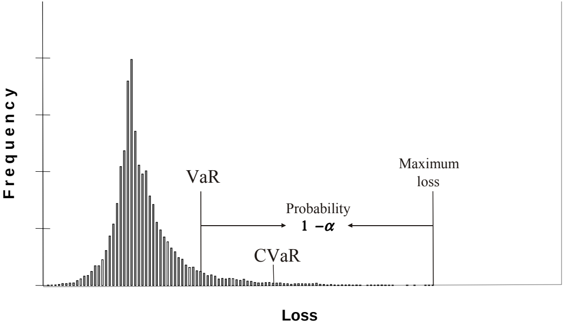
$$\max_{x_i} p(\sum_j x_j)^T x_i - c_i(x_i) \qquad \text{profit}$$

$$\text{s.t. } B_i x_i = b_i, x_i \geq 0 \qquad \text{technical constr}$$

- Cournot: assume each can affect π by choice of x_i
- Inverse demand $p(q)$: $\pi = p(q) \iff q = d(\pi)$
- Two agents, same data
- KKT(1) + KKT(2) gives Complementarity Problem
- $x_1 = 20/3$, $x_2 = 23/3$, $\pi = 29/3$
- Exercise of market power (some price takers, some Cournot, even Stackleberg)

Another explanation: risk

- Modern approach to modeling risk aversion uses concept of risk measures
- $\overline{CVaR}_\alpha$: mean of upper tail beyond α-quantile (e.g. $\alpha = 0.95$)



- Dual representation (of coherent r.m.) in terms of risk sets

$$\rho(Z) = \sup_{\mu \in \mathcal{D}} \mathbb{E}_\mu[Z] = \sup_{\mu \in \mathcal{D}} \mu^T Z$$

- If $\mathcal{D} = \{p\}$ then $\rho(Z) = \mathbb{E}[Z]$
- If $\mathcal{D}_{\alpha,p} = \{\lambda : 0 \leq \lambda_i \leq p_i/(1-\alpha), \sum_i \lambda_i = 1\}$, then

$$\rho(Z) = \overline{CVaR}_\alpha(Z)$$

Risk averse equilibrium

Replace each agents problem by:

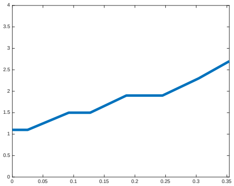
$$\begin{aligned} \text{RA}(a, \pi, \mathcal{D}_a)\text{:} \quad \min_{(\theta,u,x)\in\mathcal{F}} \quad & Z_a(0) + \theta_a(0) \\ \text{s.t.} \quad & x_a(n) = x_a(n_-) - u_a(n) + \omega_a(n) \\ & \theta_a(n) \geq \sum_{m \in n_+} p_a^k(m)(Z_a(m) + \theta_a(m)), \quad k \in K(n) \\ & Z_a(n) = C_a(u_a(n)) - \pi(n) g_a(u_a(n)) \end{aligned}$$

- $p_a^k(m)$ are extreme points of the agents risk set at m
- No longer system optimization
- Must solve using complementarity solver
- Need new techniques to treat stochastic optimization problems within equilibrium

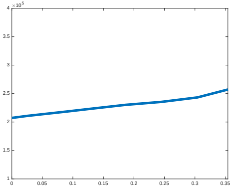
Computational results

Increasing risk aversion

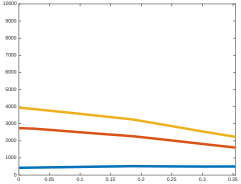
Shortage probability



Consumer cost

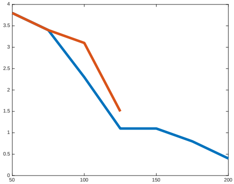


Agent profits

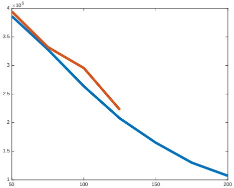


Increasing battery capacity

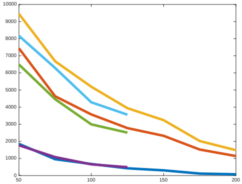
Shortage probability



Consumer cost



Agent profits



Equilibrium or optimization?

Theorem

If (u, θ) *solves* $SO(\mathcal{D}_s)$, *then there is a probability distribution* $(\bar{\mu}(n), n \in \mathcal{N})$ *and prices* $(\pi(n), n \in \mathcal{N})$ *so that defining* $\mathcal{D}_a = \{\bar{\mu}\}$ *for all* $a \in \mathcal{A}$, (u, π) *solves* $RE(\mathcal{D}_{\mathcal{A}})$. *That is, the social plan is decomposable into a risk-neutral multi-stage stochastic optimization problem for each agent, with coupling via complementarity constraints.*

(Observe that each agent must maximize their own expected profit using probabilities $\bar{\mu}$ that are derived from identifying the worst outcomes as measured by SO. These will correspond to the worst outcomes for each agent only under very special circumstances)

- Attempt to construct agreement on what would be the worst-case outcome by trading risk

Contracts in MOPEC (Philpott/F./Wets)

- Can we modify (complete) system to have a social optimum by trading risk?
- How do we design these instruments? How many are needed? What is cost of deficiency?
- Given any node n, an *Arrow-Debreu security* for node $m \in n_+$ is a contract that charges a price $\mu(m)$ in node $n \in \mathcal{N}$, to receive a payment of 1 in node $m \in n_+$.
- Conceptually allows to transfer money from one period to another (provides wealth retention or pricing of ancilliary services in energy market)
- Can investigate new instruments to mitigate risk, or move to system optimal solutions from equilibrium (or market) solutions

Such contracts complete the market (RET)

$$\text{RAT}(a, \pi, \mu, \mathcal{D}_a): \min_{(\theta, Z, x, u, W) \in \mathcal{F}(\omega)} Z_a(0) + \theta_a(0)$$

$$\text{s.t. } \theta_a(n) \geq \sum_{m \in n_+} p_a^k(m)(Z_a(m) + \theta_a(m) - W_a(m)), k \in K(n)$$

$$Z_a(n) = C_a(u_a(n)) - \pi(n) g_a(u_a(n)) + \sum_{m \in n_+} \mu(m) W_a(m)$$

coupled to clearing of energy and clearing of contracts

$$0 \leq -\sum_{a \in \mathcal{A}} W_a(n) \perp \mu(n) \geq 0$$

Theorem

Consider agents $a \in \mathcal{A}$, with risk sets $\mathcal{D}_a(n)$, $n \in \mathcal{N} \setminus \mathcal{L}$. Let (u, θ) solve $SO(\mathcal{D}_s)$ with risk sets $D_s(n) = \bigcap_{a \in \mathcal{A}} \mathcal{D}_a(n)$. There exist prices $(\bar{\pi}(n), n \in \mathcal{N})$ and $(\bar{\mu}(n), n \in \mathcal{N} \setminus \{0\})$ and actions $\bar{u}_a(n), n \in \mathcal{N}$, $\bar{W}_a(n), n \in \mathcal{N} \setminus \{0\}$ that form a multistage risk-trading equilibrium $RET(\mathcal{D}_{\mathcal{A}})$.

Conversely...

Theorem

Consider a set of agents $a \in \mathcal{A}$, each endowed with a polyhedral node-dependent risk set $\mathcal{D}_a(n)$, $n \in \mathcal{N} \setminus \mathcal{L}$. Suppose $(\bar{\pi}(n), n \in \mathcal{N})$ and $(\bar{\mu}(n), n \in \mathcal{N} \setminus \{0\})$ form a multistage risk-trading equilibrium $RET(\mathcal{D}_{\mathcal{A}})$ in which agent a solves $RAT(a, \bar{\pi}, \bar{\mu}, \mathcal{D}_a)$ with a policy defined by $\bar{u}_a(\cdot)$ together with a policy of trading Arrow-Debreu securities defined by $\{\bar{W}_a(n), n \in \mathcal{N} \setminus \{0\}\}$. Then

(i) *$(\bar{u}, \bar{\theta})$ is a solution to $SO(\mathcal{D}_s)$ with $D_s = \{\bar{\mu}\}$,*

(ii) *$\bar{\mu} \in \mathcal{D}_a$ for all $a \in \mathcal{A}$,*

(iii) *$(\bar{u}, \bar{\theta})$ is a solution to $SO(\mathcal{D}_s)$ with risk sets $D_s(n) = \bigcap_{a \in \mathcal{A}} \mathcal{D}_a(n)$,*

where $\bar{\theta}$ is defined recursively (above) with $\mu_\sigma = \bar{\mu}$ and $u_a(n) = \bar{u}_a(n)$.

In battery problem can recover by trading the system optimal solution (and its properties) since the retailer/generator agent is risk neutral

Technical details

- Can prove $\mathrm{SO}(\mathcal{D}_s)$ yields a $\mathrm{RET}(\mathcal{D}_\mathcal{A})$ provided that $\mathcal{D}_s(n) \subseteq \mathrm{int}(\mathbb{R}_+^{|n_+|})$, since in this case a solution (including multipliers) is defined at every node. Establish above result using uniform convergence of solutions arising from a contaminated risk measure:

$$\rho^\nu(Z) = \frac{1}{\nu}\mathbb{E}_{[1/|n_+|]_{n_+}}[Z] + (1 - \frac{1}{\nu}) \max_{\mu \in \mathcal{D}_s(n)} \mathbb{E}_\mu[Z].$$

- Can determine RET solution by solving a system optimization problem and subsequent risk trading optimization problems.

Decomposition (by node and agent)

Each agent a at each node n solves:

$$
\begin{aligned}
\min_{(\theta,u,x)\in\mathcal{F}} \quad & Z_a(0) + \theta_a(0) \\
\text{s.t.} \quad & \theta_a(n) \geq \sum_{m\in n_+} p_a^k(m)(Z_a(m) + \theta_a(m)), \quad k \in K(n) \\
& Z_a(n) = C_a(u_a(n)) - \pi(n) g_a(u_a(n)) \\
& \qquad + \alpha(n)(x_a(n) - x_a(n_-) + u_a(n) - \omega_a(n))
\end{aligned}
$$

coupled to

$$0 \leq \sum_{a\in\mathcal{A}} g_a(u_a(n)) \perp \pi(n) \geq 0$$

and

$$0 = x_a(n) - x_a(n_-) + u_a(n) - \omega_a(n) \perp \alpha(n)$$

Note that decomposition techniques can be naturally extended to this setting and implemented within SELKIE.

Other specializations and extensions

$$\min_{x_i} \theta_i(x_i, x_{-i}, z(x_i, x_{-i}), \pi) \text{ s.t. } g_i(x_i, x_{-i}, z, \pi) \le 0, \forall i, f(x, z, \pi) = 0$$

$$\pi \text{ solves } \mathrm{VI}(h(x, \cdot), C)$$

- NE: Nash equilibrium (no VI coupling constraints, $g_i(x_i)$ only)
- GNE: Generalized Nash Equilibrium (feasible sets of each players problem depends on other players variables)
- Implicit variables: $z(x_i, x_{-i})$ shared
- Shared constraints: f is known to all (many) players
- Force all shared constraints to have same dual variable (VI solution)
- Can use EMP to write all these problems, and convert to MCP form
- Use models to evaluate effects of regulations and their implementation in a competitive environment

Contracts to mitigate risk

- Reserves: set aside operating capacity in future for possible dispatch under certain outcomes
- Contracts of differences and options on these (difference between promise and delivery)
- Contracts for guaranteed delivery of energy in future under certain outcomes (F/Wets)
- Arrow Debreu (pure) financial contracts under certain outcomes - trading risk (Philpott/F/Wets)
- Localized storage as smoothers - transfer energy to future time at a given location (F/Philpott)
- Consider limits on availability, etc

- Need market/equilibrium concept
- Need multiple period dynamic models and risk aversion

Conclusions

- Showed equilibrium problems built from interacting optimization problems
- Equilibrium problems can be formulated naturally and modeler can specify who controls what
- It's available (in GAMS)
- Allows use and control of dual variables / prices
- MOPEC facilitates easy "behavior" description at model level
- Enables modelers to convey simple structures to algorithms and allows algorithms to exploit this
- New decomposition algorithms available to modeler (Gauss Seidel, Randomized Sweeps, Gauss Southwell, Grouping of subproblems)
- Can evaluate effects of regulations and their implementation in a dynamic competitive environment
- Stochastic equilibria - clearing the market in each scenario
- Ability to trade risk using contracts