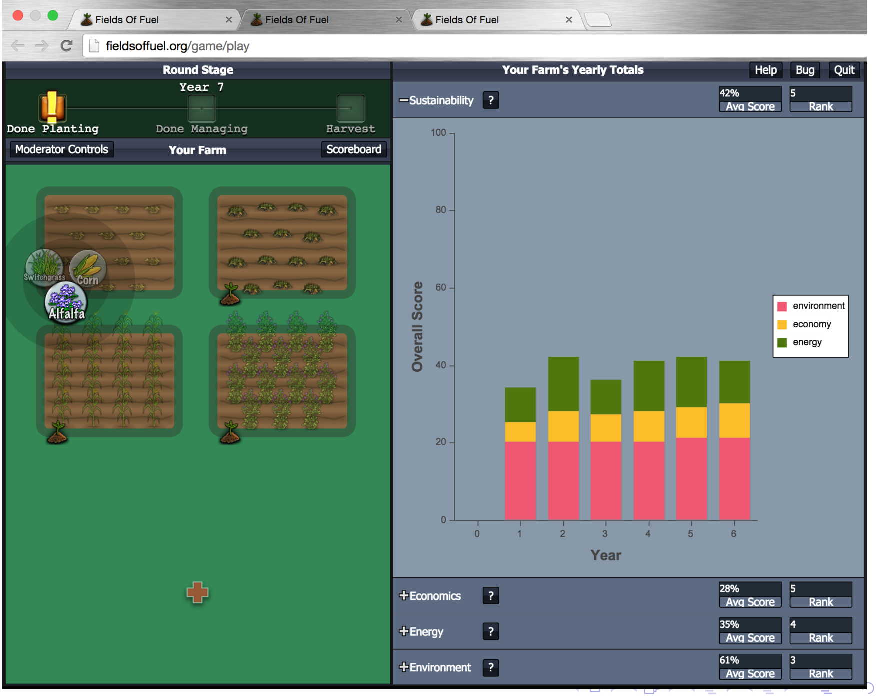
Modeling and Optimization within Interacting Systems

Michael C. Ferris

University of Wisconsin, Madison

Optimization, Sparsity and Adaptive Data Analysis
Chinese Academy of Sciences
March 21, 2015

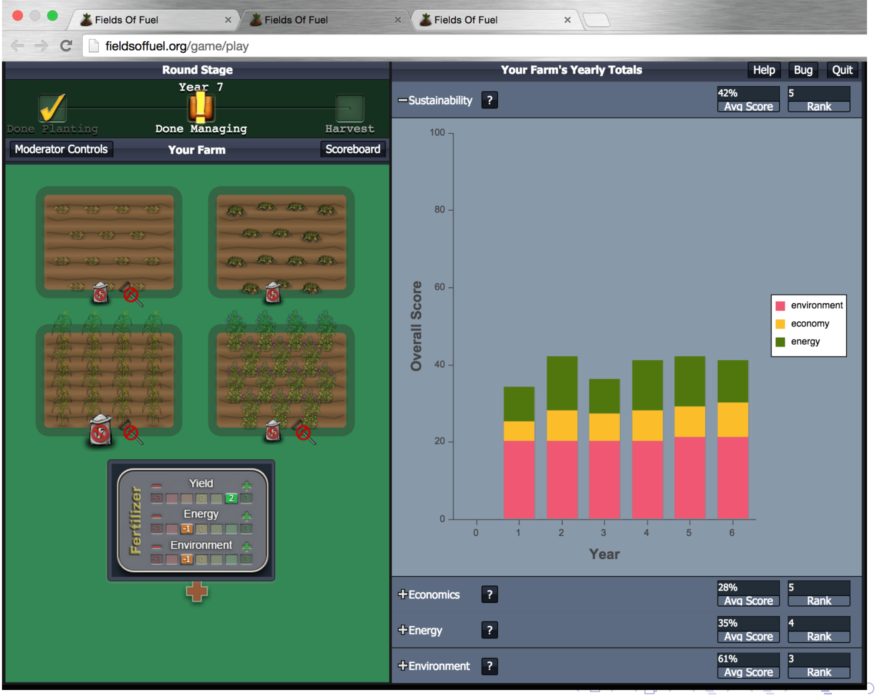


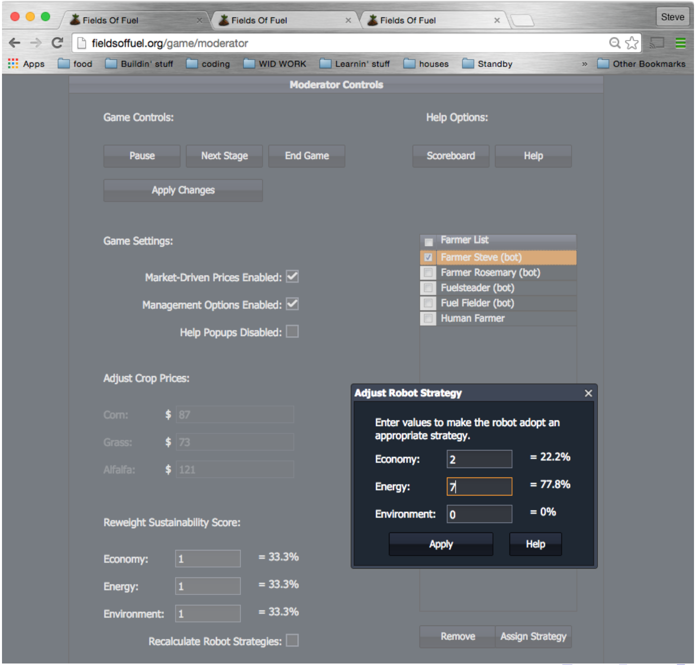
Fields Of Fuel
fieldsoffuel.org/game/play
Round Stage
Year 7
Done Planting
Done Managing
Harvest
Moderator Controls
Your Farm
Scoreboard
Switchgrass
Corn
Alfalfa
Your Farm's Yearly Totals
Help
Bug
Quit
Sustainability
42%
Avg Score
5
Rank
Overall Score
Year
environment
economy
energy
Economics
28%
Avg Score
5
Rank
Energy
35%
Avg Score
4
Rank
Environment
61%
Avg Score
3
Rank


Fields Of Fuel
fieldsoffuel.org/game/play
Round Stage
Year 7
Done Planting
Done Managing
Harvest
Moderator Controls
Your Farm
Scoreboard
Switchgrass
$77 (Crop Price)
$14280 (Revenue)
-$7286 (Costs)
=$6994 (Net Income)
Switchgrass
$77 (Crop Price)
$12283 (Revenue)
-$7286 (Costs)
=$4997 (Net Income)
Corn
$158 (Crop Price)
$43676 (Revenue)
-$18953 (Costs)
=$24722 (Net Income)
Corn
$158 (Crop Price)
$51527 (Revenue)
-$18953 (Costs)
=$32574 (Net Income)
Field Changes Over Time
Economy
Energy
Environment
< Previous Year
6
Next Year >
Your Farm's Yearly Totals
Help
Bug
Quit
Sustainability
42% Avg Score
5 Rank
Overall Score
Year
environment
economy
energy
Economics
28% Avg Score
5 Rank
Prices
Revenue
Feed Market
Fuel Market
Price ($ / Ton)
Year
grass
corn
alfalfa
Energy
35% Avg Score
4 Rank
Environment
61% Avg Score
3 Rank

fieldsoffuel.org
Fields Of Fuel
Round Stage
Year 7
Done Planting
Harvest
Moderator Controls
Your Farm
Scoreboard
21.9 tons/acre
147.7 GJ/acre (Out)
-18.9 GJ/acre (In)
=128.8 GJ/acre (Net)
21.31 tons/acre
143.7 GJ/acre (Out)
-18.6 GJ/acre (In)
=125.1 GJ/acre (Net)
6.63 tons/acre
57.6 GJ/acre (Out)
-10.7 GJ/acre (In)
=46.9 GJ/acre (Net)
7.17 tons/acre
62.3 GJ/acre (Out)
-11.4 GJ/acre (In)
=50.9 GJ/acre (Net)
Field Changes Over Time
Economy
Energy
Environment
< Previous Year
6
Next Year >
Your Farm's Yearly Totals
Help
Bug
Quit
Sustainability
67% Avg Score
1 Rank
Overall Score
Year
environment
economy
energy
Economics
81% Avg Score
3 Rank
Energy
70% Avg Score
1 Rank
Yield
Energy
Energy (GJ)
grass
corn
alfalfa
Environment
45% Avg Score
3 Rank


Fields Of Fuel
fieldsoffuel.org/game/play
Round Stage
Year 7
Done Planting
Done Managing
Harvest
Moderator Controls
Your Farm
Scoreboard
Switchgrass
1590 Soil Fert (N)
61% Beneficial Bugs
560 tons CO2 eq
98% Water Quality
Switchgrass
1070 Soil Fert (N)
61% Beneficial Bugs
569 tons CO2 eq
98% Water Quality
Corn
575 Soil Fert (N)
53% Beneficial Bugs
3041 tons CO2 eq
88% Water Quality
Corn
2146 Soil Fert (N)
43% Beneficial Bugs
3227 tons CO2 eq
85% Water Quality
Field Changes Over Time
Economy
Energy
Environment
< Previous Year
6
Next Year >
Your Farm's Yearly Totals
Help
Bug
Quit
Sustainability
42% Avg Score
5 Rank
Overall Score
Year
environment
economy
energy
Economics
28% Avg Score
5 Rank
Energy
35% Avg Score
4 Rank
Environment
61% Avg Score
3 Rank
Overall
Water
Emissions
Score
Year
soil health
water quality
beneficial bugs
emissions


Fields Of Fuel
fieldsoffuel.org/game/play
Round Stage
Year 7
Done Planting
Done Managing
Harvest
Moderator Controls
Your Farm
Scoreboard
Fertilizer
Yield
Energy
Environment
Your Farm's Yearly Totals
Help
Bug
Quit
Sustainability
42%
Avg Score
5
Rank
Overall Score
Year
environment
economy
energy
Economics
28%
Avg Score
5
Rank
Energy
35%
Avg Score
4
Rank
Environment
61%
Avg Score
3
Rank


Fields Of Fuel
fieldsoffuel.org/game/moderator
Moderator Controls
Game Controls:
Pause
Next Stage
End Game
Apply Changes
Help Options:
Scoreboard
Help
Game Settings:
Market-Driven Prices Enabled:
Management Options Enabled:
Help Popups Disabled:
Adjust Crop Prices:
Corn: $ 87
Grass: $ 73
Alfalfa: $ 121
Reweight Sustainability Score:
Economy: 1 = 33.3%
Energy: 1 = 33.3%
Environment: 1 = 33.3%
Recalculate Robot Strategies:
Farmer List
Farmer Steve (bot)
Farmer Rosemary (bot)
Fuelsteader (bot)
Fuel Fielder (bot)
Human Farmer
Adjust Robot Strategy
Enter values to make the robot adopt an appropriate strategy.
Economy: 2 = 22.2%
Energy: 7 = 77.8%
Environment: 0 = 0%
Apply
Help
Remove
Assign Strategy

Idea and implementation

- Multiple agents interacting independently, along with shared resource
- Farmers (planting and management, leeching, CO2)
- Economy (supply, demand, money), Environment (bug index), Energy
- Use in schools, undergraduate classes and group of Ag/Econ experts

Idea and implementation

- Multiple agents interacting independently, along with shared resource
- Farmers (planting and management, leeching, CO_2)
- Economy (supply, demand, money), Environment (bug index), Energy
- Use in schools, undergraduate classes and group of Ag/Econ experts

- Repeated game
- Single player not interesting - introduce bots
- Implement bots using GAMS
  - Information in: same as a human player
  - Key step: approximate other players actions/response function
  - Different objectives
  - Information out: planting and management decisions

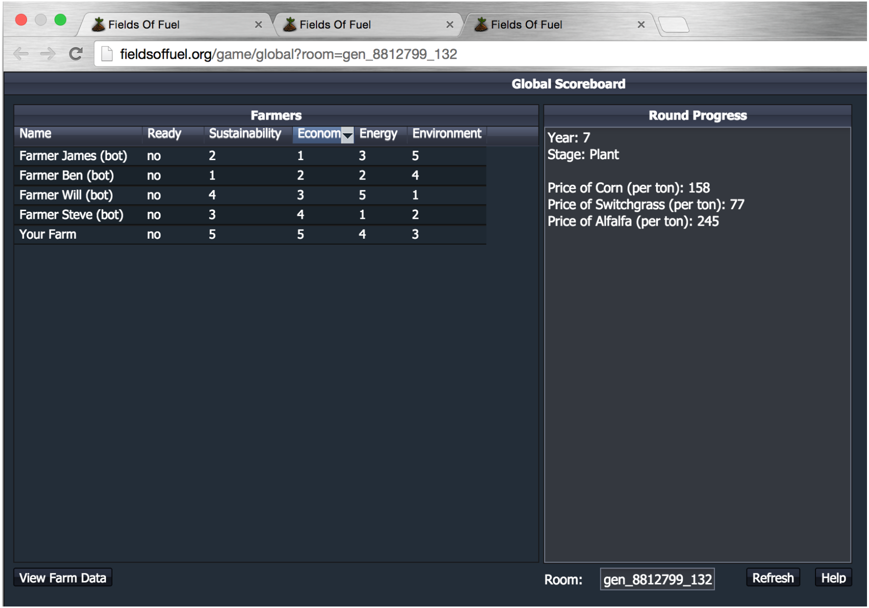
- Point your google chrome browser at: fieldsoffuel.org

Aside: designing bots

- Bots receive same information as human players (see graphs and help)
- Only know own strategy
- Different objectives (economy, energy, environment, combination)

Aside: designing bots

- Bots receive same information as human players (see graphs and help)
- Only know own strategy
- Different objectives (economy, energy, environment, combination)
- Perennials: need history/look-ahead
- Runoff and bug index: need neighbors strategies
- Understand the economy/prices
- Prediction model for next 5 periods
- Solve multistage look-ahead MIP model (in real time)
- Distributed solution, each bot can use multiple cores


Fields Of Fuel
fieldsoffuel.org/game/global?room=gen_8812799_132
Global Scoreboard
Farmers
Name Ready Sustainability Econom Energy Environment
Farmer James (bot) no 2 1 3 5
Farmer Ben (bot) no 1 2 2 4
Farmer Will (bot) no 4 3 5 1
Farmer Steve (bot) no 3 4 1 2
Your Farm no 5 5 4 3
Round Progress
Year: 7
Stage: Plant
Price of Corn (per ton): 158
Price of Switchgrass (per ton): 77
Price of Alfalfa (per ton): 245
View Farm Data
Room: gen_8812799_132
Refresh
Help

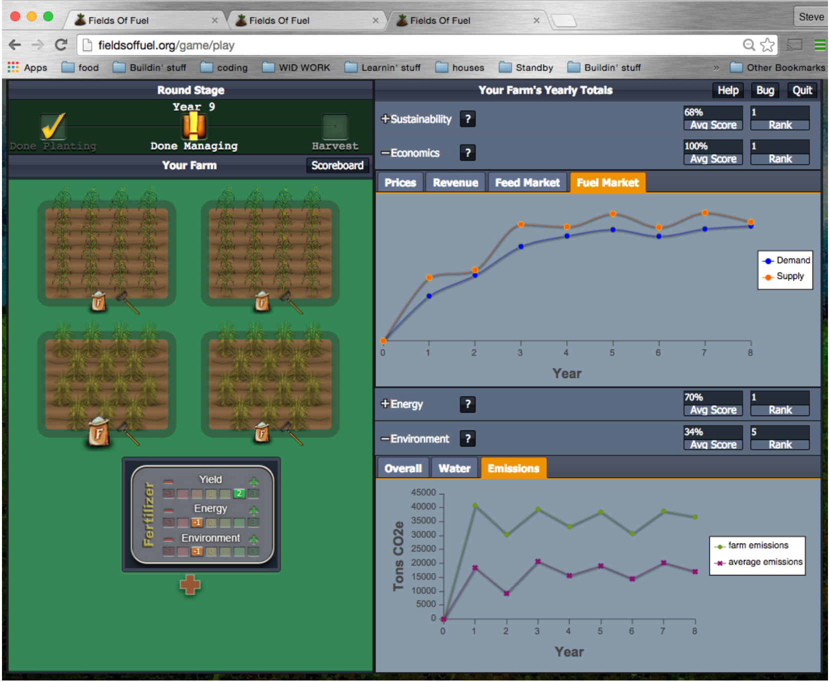
Alternative: the “big data” model

- Collect states, and strategy decisions from real plays over time
- Use “nearest neighbor” to identify a small set of “exemplars”
  - Randomly select an action from a selected exemplar to perform
  - Perform an averaged action from exemplar set (worse performance)
- Test using cross validation and also deploy in real game
- Good CV performance, not used in real game at this time
- Can we use better schemes to exploit this accumulating data?

Alternative: the "big data" model

- Collect states, and strategy decisions from real plays over time
- Use "nearest neighbor" to identify a small set of "exemplars"
  - Randomly select an action from a selected exemplar to perform
  - Perform an averaged action from exemplar set (worse performance)
- Test using cross validation and also deploy in real game
- Good CV performance, not used in real game at this time
- Can we use better schemes to exploit this accumulating data?

- Data can be used to train a program to play like humans so that humans can reason about outcomes of multiple bot-played games
- Question: Can this be used to inform public policy decisions?



Fields Of Fuel
fieldsoffuel.org/game/play
Round Stage
Year 9
Done Planting
Done Managing
Harvest
Your Farm
Scoreboard
Fertilizer
Yield
Energy
Environment
Your Farm's Yearly Totals
Help
Bug
Quit
Sustainability
68%
Avg Score
1
Rank
Economics
100%
Avg Score
1
Rank
Prices
Revenue
Feed Market
Fuel Market
Demand
Supply
Year
Energy
70%
Avg Score
1
Rank
Environment
34%
Avg Score
5
Rank
Overall
Water
Emissions
Tons CO2e
farm emissions
average emissions
Year

(M)OPEC

$$\min_{x} \theta(x, p) \text{ s.t. } g(x, p) \leq 0$$

$$0 \leq p \perp h(x, p) \geq 0$$

```
equilibrium
min theta x g
vi h p
```

- Solved concurrently

(M)OPEC

$$\frac{\min_{x} \theta(x,p) \text{ s.t. } g(x,p) \le 0}{0 \le p \perp h(x,p) \ge 0}$$

$$\begin{aligned} x \perp \nabla_x \theta(x,p) + \lambda^T \nabla_x g(x,p) \\ 0 \le \lambda \perp -g(x,p) \ge 0 \\ 0 \le p \perp h(x,p) \ge 0 \end{aligned}$$

```
equilibrium
min theta x g
vi h p
```

- Solved concurrently
- Requires global solutions of agents problems (or theory to guarantee KKT are equivalent)
- Theory of existence, uniqueness and stability based in variational analysis

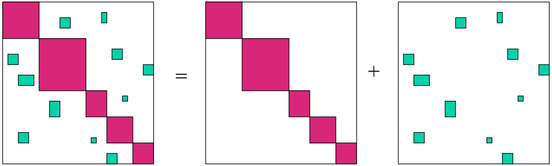
MOPEC

$$\min_{x_i} \theta_i(x_i, x_{-i}, p) \text{ s.t. } g_i(x_i, x_{-i}, p) \leq 0, \forall i$$

$$p \text{ solves VI}(h(x, \cdot), C)$$

```
equilibrium
min theta(1) x(1) g(1)
...
min theta(m) x(m) g(m)
vi h p cons
```

- Reformulate optimization problem as first order conditions (complementarity)
- Use nonsmooth Newton methods to solve complementarity problem
- Solve overall problem using "individual optimizations"?



General Equilibrium models

$$(C): \max_{x_k \in X_k} U_k(x_k) \text{ s.t. } p^T x_k \leq i_k(y,p)$$

$$(P): \max_{y_j \in Y_j} p^T g_j(y_j)$$

$$(M): \max_{p \geq 0} p^T \left(\sum_k x_k - \sum_k \omega_k - \sum_j g_j(y_j) \right) \text{ s.t. } \sum_l p_l = 1$$

$$(I): i_k(y,p) = p^T \omega_k + \sum_j \alpha_{kj} p^T g_j(y_j)$$

This is an example of a MOPEC

Special case: Nash Equilibrium

- Non-cooperative game: collection of players $a \in \mathcal{A}$ whose individual objectives depend not only on the selection of their own strategy $x_a \in C_a = \mathrm{dom}\theta_a(\cdot, x_{-a})$ but also on the strategies selected by the other players $x_{-a} = \{x_a : o \in \mathcal{A} \setminus \{a\}\}$.
- Nash Equilibrium Point:

$$\bar{x}_{\mathcal{A}} = (\bar{x}_a, a \in \mathcal{A}) : \forall a \in \mathcal{A} : \bar{x}_a \in \mathrm{argmin}_{x_a \in C_a} \theta_a(x_a, \bar{x}_{-a}).$$

1. for all $a \in \mathcal{A}$, $\theta_a(\cdot, x_{-a})$ is convex
2. $C = \prod_{a \in \mathcal{A}} C_a$ and for all $a \in \mathcal{A}$, C_a is closed convex.

VI reformulation

Define

$$G : \mathbb{R}^N \mapsto \mathbb{R}^N \text{ by } G_a(x_{\mathcal{A}}) = \partial_a \theta_a(x_a, x_{-a}), a \in \mathcal{A}$$

where ∂_a denotes the subgradient with respect to x_a. Generally, the mapping G is set-valued.

Theorem

Suppose the objectives satisfy (1) and (2), then every solution of the variational inequality

$$x_{\mathcal{A}} \in C \text{ such that } -G(x_{\mathcal{A}}) \in N_C(x_{\mathcal{A}})$$

is a Nash equilibrium point for the game.
Moreover, if C is compact and G is continuous, then the variational inequality has at least one solution that is then also a Nash equilibrium point.

Strongly Convex (Generalized) Nash Equilibria

$$\min_{x_1 \geq 0} \ \frac{1}{2}x_1^2 - \theta x_1 x_2 - 4x_1 \text{ s.t. } x_1 + x_2 \geq 1$$

$$\min_{x_2 \geq 0} \ \frac{1}{2}x_2^2 - x_1 x_2 - 3x_2$$

- No solution for $\theta \geq 1$:

$$x_1(x_2) = (\theta x_2 + 4)_+, \quad x_2(x_1) = (x_1 + 3)_+$$

- Solution $-\frac{4}{3} \leq \theta < 1$: $x_1 = \frac{4+3\theta}{1-\theta}$, $x_2 = x_1 + 3$
- Solution $\theta \leq -\frac{4}{3}$: $x_1 = 0$, $x_2 = 3$
- Jacobi works provided $\theta < 1$, but diagonal dominance theory fails

Recast as a VI

$$M = \left[\begin{array}{cc|c} 1 & -1 & -\theta \\ 1 & & 1 \\ \hline -1 & & 1 \end{array}\right] z = \begin{bmatrix} x_1 \\ \lambda \\ x_2 \end{bmatrix}, q = \begin{bmatrix} -4 \\ -1 \\ -3 \end{bmatrix}$$

$$0 \in Mz + q + N_C(z) \iff 0 \le Mz + q \perp z \ge 0$$

- Problem is not monotone (M not psd), so monotone operator splitting not possible
- New results (F/Rutherford/Wathen) show Jacobi/Gauss Seidel works based on Feingold/Varga (1962)
- M is an L-matrix, so Lemke method (PATH) solves the problem

Key point: models generated correctly solve quickly

Here S is mesh spacing parameter

S	Var	rows	non-zero	dense(%)	Steps	RT (m:s)
20	2400	2568	31536	0.48	5	$0:03$
50	15000	15408	195816	0.08	5	$0:19$
100	60000	60808	781616	0.02	5	$1:16$
200	240000	241608	3123216	0.01	5	$5:12$

Convergence for $S = 200$ (with new basis extensions in PATH)

Iteration	Residual
0	1.56(+4)
1	1.06(+1)
2	1.34
3	2.04(−2)
4	1.74(−5)
5	2.97(−11)

Extension to hierarchical models for policy analysis?

- The latest GTAP database represents global production and trade for 113 country/regions, 57 commodities and 5 primary factors.
- Data characterizes intermediate demand and bilateral trade in 2007, including tax rates on imports/exports and other indirect taxes.
- The core GTAP model is a static, multi-regional model which tracks the production and distribution of goods in the global economy.
- In GTAP the world is divided into regions (typically representing individual countries), and each region's final demand structure is composed of public and private expenditure across goods.

The Model

The GTAP model (MOPEC) may be posed as a system of *nonsmooth equations*:

$$F_+(w, z; t) = 0$$

in which:

- w_r is a vector of regional *welfare* levels
- $z \in \mathbb{R}^N$ represents a vector of endogenous economic variables, e.g. prices and quantities, $z = \begin{pmatrix} P \\ Q \end{pmatrix}$.
- t represents matrices of trade tax instruments – import tariffs (t^M_{irs}) and export taxes (t^X_{irs}) for each commodity i and region r

Optimal Sanctions

Coalition member states strategically choose trade taxes which *minimize* Russian welfare:

$$\min_{t_r : r \in \mathcal{C}} w_{rus}$$

s.t.

$$F_+(w, z; t) = 0$$

$$t_r = \bar{t}_r \quad \forall r \notin \mathcal{C}$$

$$t^M_{i,rus,r} \leq \bar{t}^M_{i,r,rus} \quad \forall r \in \mathcal{C}$$

$$t^X_{i,r,rus} \leq \bar{t}^X_{i,rus,r} \quad \forall r \in \mathcal{C}$$

Optimal Retaliation

Russia choose trade taxes which *maximize* Russian welfare in response to the coalition actions:

$$\max_{t_{rus}} w_{rus}$$

s.t.

$$F_{+}(w, z; t) = 0$$

$$t_r = \begin{cases} \hat{t}_r & r \in \mathcal{C} \\ \bar{t}_r & r \notin \mathcal{C} \end{cases}$$

where $\hat{t}_r$ represents trade taxes for coalition countries ($r \in \mathcal{C}$) from the optimal sanction calculation.

Coalition Member States for Illustrative Calculation

USA	United States
ANZ	Australia and New Zealand
CAN	Canada
FRA	France
DEU	Germany
ITA	Italy
JPN	Japan
GBR	United Kingdom
REU	Rest of the European Union

Welfare Changes (% Hicksian EV)

	sanction	retaliation	tradewar
RUS	-4.4	-3.5	-9.8
$\mathcal{C}$ AVERAGE	0.03	0.05	0.03
CAN	0.021	0.033	0.032
USA	0.007	-0.017	0.032
FRA	0.042	0.020	0.032
DEU	0.119	-0.047	0.032
ITA	0.069	0.050	0.032
GBR	0.045	-0.002	0.032
REU	0.058	0.365	0.032
ANZ	0.011	0.003	0.032
JPN	0.012	-0.020	0.032
CHN	0.115	0.057	0.290
SAU	0.240	1.865	-0.892

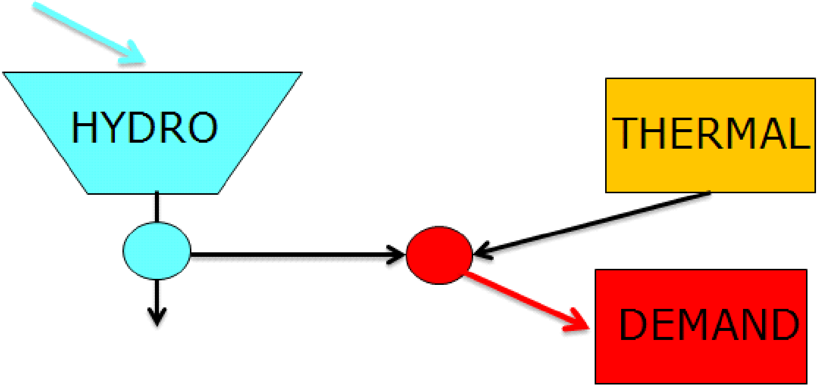
Scenarios and Key Insights

SANCTION If coalition states were to increases tariffs and export taxes on Russia to the same level which is currently applied by Russia on bilateral trade flows with the coalition, Russian welfare could be substantially impacted with no economic cost for any coalition members.

RETALIATION Russia could respond to such sanctions by changing it's own trade taxes, but optimal "retaliation" largely results in a *reduction* rather than an increase in trade taxes on trade flows to and from coalition states. These tariff changes can only partially offset the adverse impact of the sanctions.

TRADEWAR If sanctions and retaliation were to result in an unconstrained trade war, Russia faces a drastic economic cost while the coalition countries could even be slight better off.

Hydro-Thermal System (Philpott/F./Wets)



Simple electricity "system optimization" problem

$$
\begin{aligned}
\text{SO:} \quad \max_{d_k, u_i, v_j, x_i \geq 0} \quad & \sum_{k \in \mathcal{K}} W_k(d_k) - \sum_{j \in \mathcal{T}} C_j(v_j) + \sum_{i \in \mathcal{H}} V_i(x_i) \\
\text{s.t.} \quad & \sum_{i \in \mathcal{H}} U_i\left(u_i\right) + \sum_{j \in \mathcal{T}} v_j \geq \sum_{k \in \mathcal{K}} d_k, \\
& x_i = x_i^0 - u_i + h_i^1, \quad i \in \mathcal{H}
\end{aligned}
$$

- u_i water release of hydro reservoir $i \in \mathcal{H}$
- v_j thermal generation of plant $j \in \mathcal{T}$
- x_i water level in reservoir $i \in \mathcal{H}$
- prod fn U_i (strictly concave) converts water release to energy
- $C_j(v_j)$ denote the cost of generation by thermal plant
- $V_i(x_i)$ future value of terminating with storage x (assumed separable)
- $W_k(d_k)$ utility of consumption d_k

SO equivalent to CE

$$\text{Consumers } k \in \mathcal{K} \text{ solve CP}(k): \max_{d_k \geq 0} \quad W_k(d_k) - p^T d_k$$

$$\text{Thermal plants } j \in \mathcal{T} \text{ solve TP}(j): \max_{v_j \geq 0} \quad p^T v_j - C_j(v_j)$$

$$\text{Hydro plants } i \in \mathcal{H} \text{ solve HP}(i): \max_{u_i, x_i \geq 0} \quad p^T U_i(u_i) + V_i(x_i)$$

$$\text{s.t.} \quad x_i = x_i^0 - u_i + h_i^1$$

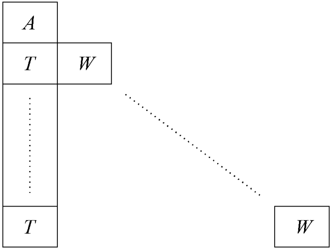
Perfectly competitive (Walrasian) equilibrium is a MOPEC

$$\begin{aligned} \text{CE:} \quad d_k &\in \arg\max \text{CP}(k), & k \in \mathcal{K}, \\ v_j &\in \arg\max \text{TP}(j), & j \in \mathcal{T}, \\ u_i, x_i &\in \arg\max \text{HP}(i), & i \in \mathcal{H}, \\ 0 \leq p \perp \sum_{i \in \mathcal{H}} U_i(u_i) &+ \sum_{j \in \mathcal{T}} v_j \geq \sum_{k \in \mathcal{K}} d_k. \end{aligned}$$

Agents have stochastic recourse?

- Two stage stochastic programming, x^1 is here-and-now decision, recourse decisions x^2 depend on realization of a random variable
- ρ is a risk measure (e.g. expectation, CVaR)

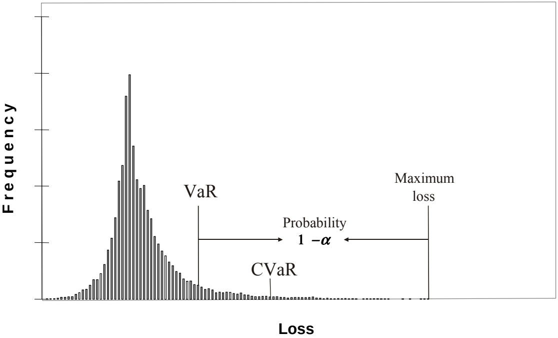
$$\begin{aligned} \text{SP:} \quad \max \quad & c^T x^1 + \rho[q^T x^2] \\ \text{s.t.} \quad & Ax^1 = b, \quad x^1 \geq 0, \\ & T(\omega)x^1 + W(\omega)x^2(\omega) \geq d(\omega), \\ & x^2(\omega) \geq 0, \forall \omega \in \Omega. \end{aligned}$$



EMP/SP extensions to facilitate these models

Risk Measures

- Modern approach to modeling risk aversion uses concept of risk measures
- $\overline{CVaR}_\alpha$: mean of upper tail beyond α-quantile (e.g. $\alpha = 0.95$)



- mean-risk, mean deviations from quantiles, VaR, CVaR
- Much more in mathematical economics and finance literature
- Optimization approaches still valid, different objectives, varying convex/non-convex difficulty

Two stage stochastic MOPEC

$$\text{CP}(k): \max_{d_k^1 \geq 0} \; W_k\left(d_k^1\right) - p^1 d_k^1$$

$$\text{TP}(j): \max_{v_j^1 \geq 0} \; p^1 v_j^1 - C_j(v_j^1)$$

$$\text{HP}(i): \max_{u_i^1, x_i^1 \geq 0} \; p^1 U_i(u_i^1)$$

$$\text{s.t.} \quad x_i^1 = x_i^0 - u_i^1 + h_i^1,$$

$$0 \leq p^1 \perp \sum_{i \in \mathcal{H}} U_i\left(u_i^1\right) + \sum_{j \in \mathcal{T}} v_j^1 \geq \sum_{k \in \mathcal{K}} d_k^1$$

Two stage stochastic MOPEC

$$\text{CP}(k): \max_{d_k^1, d_k^2(\omega) \geq 0} W_k\left(d_k^1\right) - p^1 d_k^1 + \rho[W_k\left(d_k^2(\omega)\right) - p^2(\omega) d_k^2(\omega)]$$

$$\text{TP}(j): \max_{v_j^1, v_j^2(\omega) \geq 0} p^1 v_j^1 - C_j(v_j^1) + \rho[p^2(\omega) v_j^2(\omega) - C_j\left(v_j^2(\omega)\right)]$$

$$\text{HP}(i): \max_{\substack{u_i^1, x_i^1 \geq 0 \\ u_i^2(\omega), x_i^2(\omega) \geq 0}} p^1 U_i(u_i^1) + \rho[p^2(\omega) U_i(u_i^2(\omega)) + V_i(x_i^2(\omega))]$$

$$\text{s.t.} \quad x_i^1 = x_i^0 - u_i^1 + h_i^1,$$

$$x_i^2(\omega) = x_i^1 - u_i^2(\omega) + h_i^2(\omega)$$

$$0 \leq p^1 \perp \sum_{i \in \mathcal{H}} U_i\left(u_i^1\right) + \sum_{j \in \mathcal{T}} v_j^1 \geq \sum_{k \in \mathcal{K}} d_k^1$$

Two stage stochastic MOPEC

$$\text{CP}(k): \quad \max_{d_k^1, d_k^2(\omega) \geq 0} \quad W_k\left(d_k^1\right) - p^1 d_k^1 + \rho[W_k\left(d_k^2(\omega)\right) - p^2(\omega) d_k^2(\omega)]$$

$$\text{TP}(j): \quad \max_{v_j^1, v_j^2(\omega) \geq 0} \quad p^1 v_j^1 - C_j(v_j^1) + \rho[p^2(\omega) v_j^2(\omega) - C_j\left(v_j^2(\omega)\right)]$$

$$\text{HP}(i): \quad \max_{\substack{u_i^1, x_i^1 \geq 0 \\ u_i^2(\omega), x_i^2(\omega) \geq 0}} \quad p^1 U_i(u_i^1) + \rho[p^2(\omega) U_i(u_i^2(\omega)) + V_i(x_i^2(\omega))]$$

$$\text{s.t.} \quad x_i^1 = x_i^0 - u_i^1 + h_i^1,$$

$$x_i^2(\omega) = x_i^1 - u_i^2(\omega) + h_i^2(\omega)$$

$$0 \leq p^1 \perp \sum_{i \in \mathcal{H}} U_i\left(u_i^1\right) + \sum_{j \in \mathcal{T}} v_j^1 \geq \sum_{k \in \mathcal{K}} d_k^1$$

$$0 \leq p^2(\omega) \perp \sum_{i \in \mathcal{H}} U_i\left(u_i^2(\omega)\right) + \sum_{j \in \mathcal{T}} v_j^2(\omega) \geq \sum_{k \in \mathcal{K}} d_k^2(\omega), \forall \omega$$

Equilibrium or optimization?

- Each agent has its own risk measure
- Is there a system risk measure?
- Is there a system optimization problem?

$$\min \sum_i C(x_i^1) + \rho_i \left(C(x_i^2(\omega)) \right) ????$$

- Can we modify (complete) system to have a social optimum by trading risk?
- How do we design these instruments? How many are needed? What is cost of deficiency?
- Can we solve efficiently / distributively?

Example as MOPEC: agents solve a Stochastic Program

Buy y_i contracts in period 1, to deliver $D(\omega)y_i$ in period 2, scenario ω
Each agent i:

$$\begin{aligned} \min \quad & C(x_i^1) + \rho_i\left(C(x_i^2(\omega))\right) & \\ \text{s.t.} \quad & p^1 x_i^1 + v y_i \le p^1 e_i^1 & \text{(budget time 1)} \\ & p^2(\omega) x_i^2(\omega) \le p^2(\omega)(D(\omega) y_i + e_i^2(\omega)) & \text{(budget time 2)} \end{aligned}$$

$$0 \le v \perp -\sum_i y_i \ge 0 \qquad \text{(contract)}$$

$$0 \le p^1 \perp \sum_i \left(e_i^1 - x_i^1\right) \ge 0 \qquad \text{(walras 1)}$$

$$0 \le p^2(\omega) \perp \sum_i \left(D(\omega) y_i + e_i^2(\omega) - x_i^2(\omega)\right) \ge 0 \qquad \text{(walras 2)}$$

Theory and Observations

- agent problems are multistage stochastic optimization models
- perfectly competitive partial equilibrium still corresponds to a social optimum when all agents are risk neutral and share common knowledge of the probability distribution governing future inflows
- situation complicated when agents are risk averse
  - utilize stochastic process over scenario tree
  - under mild conditions a social optimum corresponds to a competitive market equilibrium if agents have time-consistent dynamic coherent risk measures and there are enough traded market instruments (over tree) to hedge inflow uncertainty
- Otherwise, must solve the stochastic equilibrium problem
- Research challenge: develop reliable algorithms for large scale decomposition approaches to MOPEC

What is EMP?

Annotates existing equations/variables/models for modeler to provide/define additional structure

- equilibrium
- vi (agents can solve min/max/vi)
- bilevel (reformulate as MPEC, or as SOCP)
- disjunction (or other constraint logic primitives)
- randvar
- dualvar (use multipliers from one agent as variables for another)
- extended nonlinear programs (library of plq functions)

Currently available within GAMS

Conclusions

- MOPEC problems capture complex interactions between optimizing agents
- Policy implications addressable using MOPEC
- MOPEC available to use within the GAMS modeling system
- Stochastic MOPEC enables modeling dynamic decision processes under uncertainty
- Many new settings available for deployment; need for more theoretic and algorithmic enhancements