# How to manage households

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# **Multiple households:** why this title?





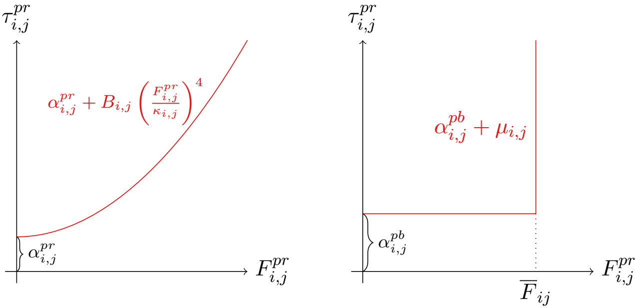



- Housing prices and location
- How to model his edge?
  - Transportation/economics?
  - Neighbourhood/work/connections?

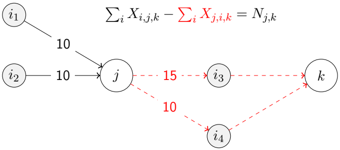
# Total flow and cost (could model as network optimization)

$$F^m_{i,j} - \sum_{h,k} X^m_{h,i,j,k} \perp F^m_{i,j} \in [0, \bar{F}_{i,j}]$$

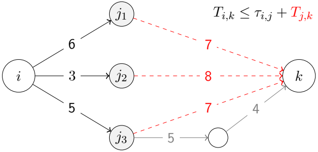
$$\tau^m_{i,j} - \alpha^m_{i,j} - B_{i,j}\left(\frac{F^m_{i,j}}{\kappa_{i,j}}\right)^4 \perp \tau^m_{i,j}$$



# Wardrop




$$\sum_i X^m_{h,i,j,k} - \sum_i X^m_{h,j,i,k} \geq N^m_{h,j,k} \perp T^m_{h,j,k} \geq 0$$




$$T^m_{h,i,k} \leq \tau^m_{i,j} + T^m_{h,j,k} \perp X^m_{h,i,j} \geq 0$$

# Walras: Computable General Equilibria or MOPECs

$$(C) : \max_{x_k \in X_k} U_k(x_k) \text{ s.t. } p^T x_k \leq i_k(y,p)$$

$$(I) : i_k(y,p) = p^T \omega_k + \sum_j \alpha_{kj} p^T g_j(y_j)$$

$$(P) : \max_{y_j \in Y_j} p^T g_j(y_j)$$

$$(M) : \max_{p \geq 0} p^T \left( \sum_k x_k - \sum_k \omega_k - \sum_j g_j(y_j) \right) \text{ s.t. } \sum_l p_l = 1$$

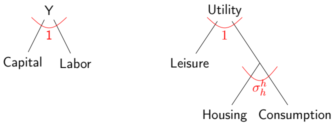
This is an example of a MOPEC (multiple optimization problems with equilibrium constraints)

# Consumers and producers

- Households (consumers): live $i$, work $j$, skill $h$, transport mode $m$

$$\max_{x} Utility(x) \text{ s.t. } \lambda_x x \leq I, x \in X$$

- Markets clear in each component of $x$ (leisure, housing, consumption)




- Production: identical firms at each node using high or low skilled labour and capital inputs to produce $y$

$$\max_{K \geq 0, L \geq 0} p_y y - (p_L L_y + p_K K_y) \text{ s.t. } G(K, L) \geq y$$

# Choice models and links to transportation

- O-D $N^m_{h,i,k}$ is appears in transport and CGE model
- Logit model used to determine $\theta_{h,m}(N^m_{h,\cdot,\cdot}, T^m_{h,\cdot,\cdot})$
- Link to transportation via amount of "leisure" $\ell^m_{h,j,k}(T^m_{h,j,k}, \lambda_{h,j})$
- Collect all models (transport and CGE) together into one large MOPEC

$$\begin{bmatrix} x & x & x & & & & \\ x & x & x & & & & \\ x & x & x & & & & \\ & & x & x & x & x & x \\ & & x & x & x & x & x \\ & & x & x & x & x & x \\ & & x & x & x & x & x \end{bmatrix}$$

# Computation: EMP and PATH

- Model has the format:

$$\begin{array}{lll} \text{Agent o:} & \min\limits_{x} & f(x,y) \\ & \text{s.t.} & g(x,y) \le 0 \quad (\perp \lambda \ge 0) \end{array}$$

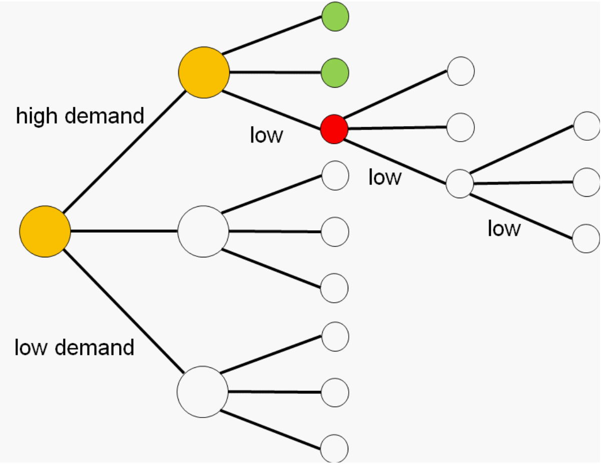
$$\text{Agent v:} \quad H(x,y,\lambda) = 0 \qquad (\perp y \text{ free})$$

- EMP tool (equation annotations) automatically creates an MCP

$$\begin{aligned} \nabla_x f(x,y) + \lambda^T \nabla g(x,y) &= 0 \\ 0 \le -g(x,y) \perp \lambda &\ge 0 \\ H(x,y,\lambda) &= 0 \end{aligned}$$

- Solve via PATH: [Dirkse, F., Kim, Munson, Ralph]

# Extensions: agents solving stochastic models




Replace optimization problems by multistage stochastic programs, and clear markets in each stage

# The model is only as good as the data

- Results for Zurich and Madison [Rutherford, van Nieuwkoop, F.]
- Results for Sydney [Robson, Dixit]
- Results for Eddiefest: