How to manage households

Michael C. Ferris
(based on collaboration with T. Rutherford and R. Van Nieuwkoop)

Stephen Kleene Professor of Computer Science
Computer Sciences Department and Wisconsin Institute for Discovery, University of Wisconsin, Madison

“FEST”, Judge Business School, Cambridge,

March 17, 2019
Multiple households: why this title?

- Housing prices and location
- How to model his edge?
  - Transportation/economics?
  - Neighbourhood/work/connections?
Total flow and cost (could model as network optimization)

\[ F_{i,j} - \sum_{h,k} X_{h,i,j,k} \perp F_{i,j} \in [0, \bar{F}_{i,j}] \]

\[ \tau_{i,j}^{m} - \alpha_{i,j}^{m} - B_{i,j} \left( \frac{F_{i,j}^{m}}{\kappa_{i,j}} \right)^{4} \perp \tau_{i,j}^{m} \]
Wardrop

\[ \sum_i X_{i,j,k} - \sum_i X_{j,i,k} = N_{j,k} \]

\[ \sum_i X_{h,i,j,k}^m - \sum_i X_{h,j,i,k}^m \geq N_{h,j,k}^m \perp T_{h,j,k}^m \geq 0 \]

\[ T_{i,k} \leq \tau_{i,j} + T_{j,k} \]

\[ T_{h,i,k}^m \leq \tau_{i,j}^m + T_{h,j,k}^m \perp X_{h,i,j}^m \geq 0 \]
Walras: Computable General Equilibria or MOPECs

(C) : \( \max_{x_k \in X_k} U_k(x_k) \) s.t. \( p^T x_k \leq i_k(y, p) \)

(I) : \( i_k(y, p) = p^T \omega_k + \sum_j \alpha_{kj} p^T g_j(y_j) \)

(P) : \( \max_{y_j \in Y_j} p^T g_j(y_j) \)

(M) : \( \max_{p \geq 0} p^T \left( \sum_k x_k - \sum_k \omega_k - \sum_j g_j(y_j) \right) \) s.t. \( \sum_l p_l = 1 \)

This is an example of a MOPEC (multiple optimization problems with equilibrium constraints)
Consumers and producers

- Households (consumers): live $i$, work $j$, skill $h$, transport mode $m$

  \[
  \max_x \text{Utility}(x) \quad \text{s.t.} \quad \lambda_x x \leq I, \ x \in X
  \]

- Markets clear in each component of $x$ (leisure, housing, consumption)

- Production: identical firms at each node using high or low skilled labour and capital inputs to produce $y$

  \[
  \max_{K \geq 0, L \geq 0} p_y y - (p_L L_y + p_K K_y) \quad \text{s.t.} \quad G(K, L) \geq y
  \]
Choice models and links to transportation

- O-D $N_{h,i,k}^m$ is appears in transport and CGE model
- Logit model used to determine $\theta_{h,m}(N_{h,,..}^m, T_{h,,..}^m)$
- Link to transportation via amount of “leisure” $\ell_{h,j,k}^m(T_{h,j,k}^m, \lambda_{h,j}^m)$
- Collect all models (transport and CGE) together into one large MOPEC
Computation: EMP and PATH

- Model has the format:

  Agent o: \( \min_x f(x, y) \)
  
  s.t. \( g(x, y) \leq 0 \) (\( \perp \lambda \geq 0 \))

  Agent v: \( H(x, y, \lambda) = 0 \) (\( \perp y \) free)

- EMP tool (equation annotations) automatically creates an MCP

  \[ \nabla_x f(x, y) + \lambda^T \nabla g(x, y) = 0 \\
  0 \leq -g(x, y) \perp \lambda \geq 0 \\
  H(x, y, \lambda) = 0 \]

- Solve via PATH: [Dirkse, F., Kim, Munson, Ralph]
Extensions: agents solving stochastic models

Replace optimization problems by multistage stochastic programs, and clear markets in each stage.
The model is only as good as the data

- Results for Zurich and Madison [Rutherford, van Nieuwkoop, F.]
- Results for Sydney [Robson, Dixit]
- Results for Eddiefest: