The price of storage:
collections of models to decide what, when and how

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Storage

- Storage is less than 4% of total market; total capacity 127 GW; predicted for rate of increase to grow 100 fold over next decade
- What markets can it work in? energy or ancillary (regulation/reserves)
- Can act behind the meter (ie load shifting, supply smoothing)

Economist, March 2012: Yet large-scale deployment of bulk storage systems will require regulatory as well as technical progress. Storage systems do not fit neatly into regulatory frameworks that distinguish between power providers and grid operators, since they can be used by both. Their ability to take power off the grid, store it, and then release it later creates "potential problems for current tariff, billing and metering approaches," notes the EPRI in a recent report. Nor is it clear whether power companies will be allowed to pass on the cost of storage facilities to their customers. But given the technology's potential to make power grids cleaner and more reliable, it seems likely that changes to the rules are in store.

PJM buy/sell model (2009)

- Storage transfers energy over time.
- PJM: given price path p_t, determine charge q_t^+ and discharge q_t^-:

$$\begin{aligned} \max_{s_t, q_t^+, q_t^-} \; & \sum_{t=0}^{T} p_t (q_t^- - q_t^+) \\ \text{s.t. } & \partial s_t = e q_t^+ - q_t^- \\ & 0 \le s_t \le \mathcal{S} \\ & 0 \le q_t^+ \le \mathcal{Q} \\ & 0 \le q_t^- \le \mathcal{Q} \\ & s_0, s_T \text{ fixed} \end{aligned}$$

- what about uncertainties? demand, supply, seasonal, growth
- price shaving, load shifting, transmission line deferral
- no transmission in model - storage location indep., no cycle charges

Characterization of storage

$\mathcal{Q}$	power (discharge) capacity	MW
$\mathcal{S}$	energy capacity (size)	MWh
cycles	measure of duration	
c^0	fixed cost	\$/h
c^1	variable cost	\$/MWh
e	efficiency/energy loss in charging	

- Costs approximate the unit construction and depreciation due to charge and discharge cycles

$$\sum_{t=0}^{T} p_t(q_t^+ - q_t^-) + c^1(q_t^+ + q_t^-) + c^0$$

  $c^1(q_t^+ + q_t^-)$ approximates cost of cycles
- p_t is a stochastic process

Stochastic price paths (day ahead market)

$$
\begin{aligned}
\min_{x,s,q^+,q^-} \; & c^0(x) + \mathbb{E}_\omega \left[\sum_{t=0}^{T} p_{\omega t}(q^+_{\omega t} - q^-_{\omega t}) + c^1(q^+_{\omega t} + q^-_{\omega t}) \right] \\
\text{s.t. } & \partial s_{\omega t} = e q^+_{\omega t} - q^-_{\omega t} \\
& 0 \le s_{\omega t} \le \mathcal{S} x \\
& 0 \le q^+_{\omega t}, q^-_{\omega t} \le \mathcal{Q} x \\
& s_{\omega 0}, s_{\omega T} \text{ fixed}
\end{aligned}
$$

- First stage decision x: amount of storage to deploy.
- Second stage decision: charging strategy in face of uncertainty

GAMS/EMP: Stochastic programming tools

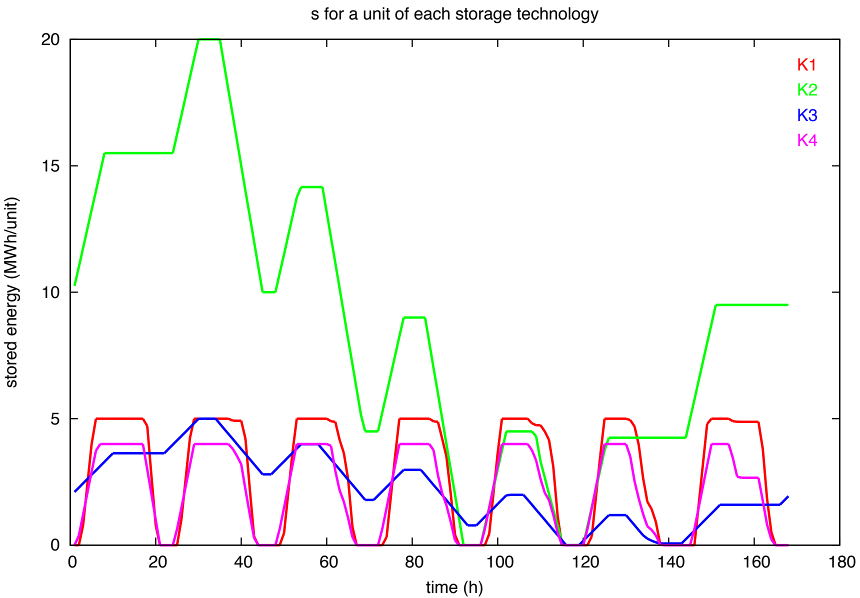
- GAMS has extended mathematical programming tools to build "models of models"
- Given the core model, can annotate parameters as "random variables"
- Automatically solves expected value problem
- Can solve using deterministic equivalent or specialized solvers (including Bender's decomposition, importance sampling (DECIS), etc)
- Also allows for a variety of new constructs (such as risk measures and chance constraints)

$$\mathbb{R}_\omega \left[c^0(x) + \sum_{t=0}^{T} p_{\omega t}(q^+_{\omega t} - q^-_{\omega t}) + c^1(q^+_{\omega t} + q^-_{\omega t}) \right]$$

Four technology example

	k_1	k_2	k_3	k_4
$\mathcal{S}$	5	20	5	4
$\mathcal{Q}$	2	1	0.2	1
e	0.8	0.75	0.85	0.84
c^0	0.9	0.7	0.4	0.75
c^1	0.55	0.6	0.45	1.1

- k_1 and k_4 have only a daily cycle of operation
- k_2 and k_3 display a significant weekly cycle in addition to their daily cycle
- Could enforce $q^+_{\omega kt} = q^+_{kt}$, $q^-_{\omega kt} = q^-_{kt}$, deterministic operating plan


s for a unit of each storage technology
stored energy (MWh/unit)
time (h)
K1
K2
K3
K4
0
5
10
15
20
0
20
40
60
80
100
120
140
160
180

Distribution of (multiple) storage types

Determine storage facilities x_k to build, given distribution of price paths: no entry barriers into market, etc. MOPEC: for all k solve a two stage stochastic program

$$
\begin{aligned}
\forall k : \min_{x_k, s_k, q_k^+, q_k^-} \; & c_k^0(x_k) + \mathbb{E}_\omega \left[\sum_{t=0}^{T} p_{\omega t}(q_{\omega kt}^+ - q_{\omega kt}^-) + c_k^1(q_{\omega kt}^+ + q_{\omega kt}^-) \right] \\
\text{s.t. } & \partial s_{\omega kt} = e q_{\omega kt}^+ - q_{\omega kt}^- \\
& 0 \le s_{\omega kt} \le \mathcal{S} x_k \\
& 0 \le q_{\omega kt}^+, q_{\omega kt}^- \le \mathcal{Q} x_k \\
& s_{\omega k0}, s_{\omega kT} \text{ fixed}
\end{aligned}
$$

and

$$
p_{\omega t} = f\left(\theta, \mathcal{D}_{\omega t} + \sum_k (q_{\omega kt}^+ - q_{\omega kt}^-)\right)
$$

Parametric function (θ) determined by regression. Storage operators react to shift in demand.

Multiple optimization with equilibrium constraint (MOPEC) models

- These problems are not MPEC's, (it's all about who controls what)!

$$\min_{x_a \in X_a} f_a(x_a, x_{-a}, p), \ \forall a \in \{1, \ldots, n\}$$

$$0 \leq H(x, p) \perp p \geq 0$$

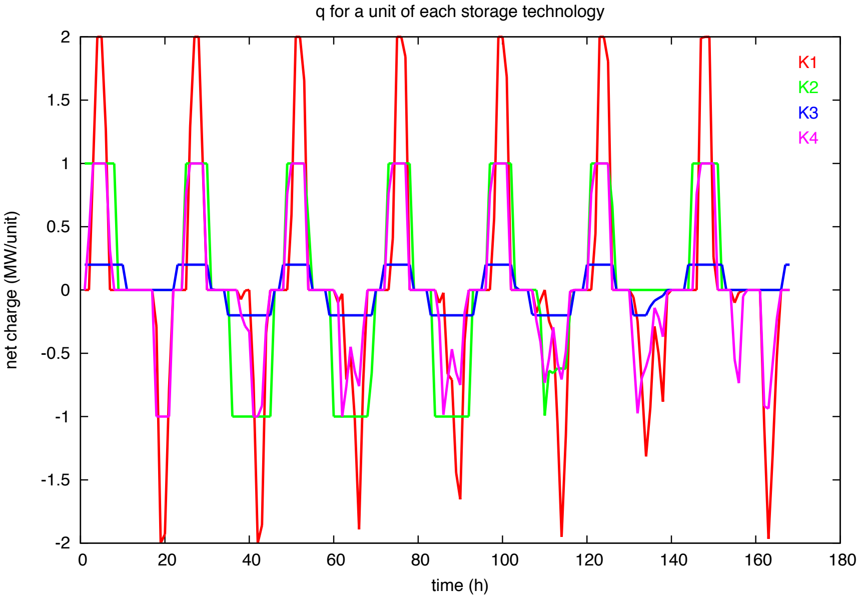
  The complementarity constraints are outside the optimizers control.
- Each agent a solves problem in a Nash sense, with equilibrium constraints modeling market clearing for example
- New contributions to existence, computation of solutions of these models

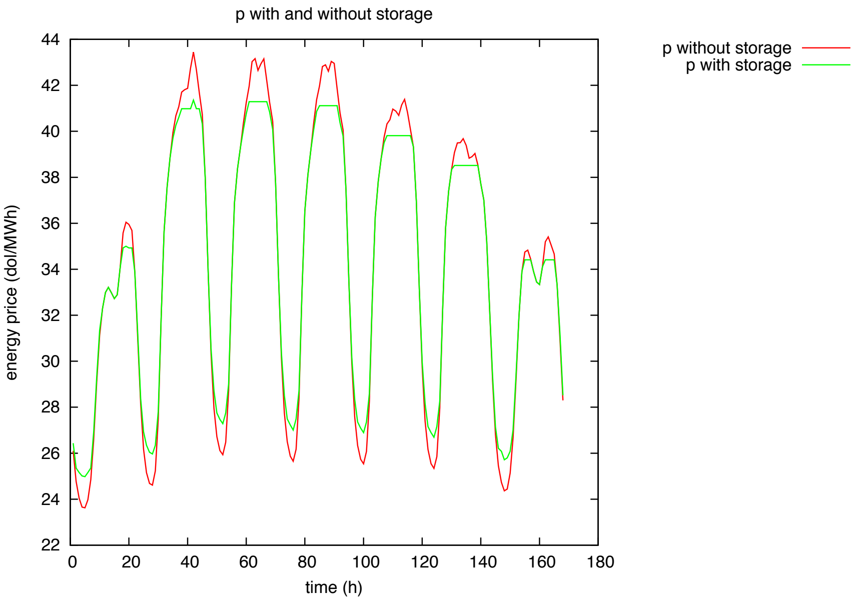
Comparison to expected value solution

Interestingly enough, the resulting equilibria in the two models are quite different. Investment variables in the equilibria:

k	x_k, EV soln	x_k, Stochastic soln
k_1	102.063	143.631
k_2	51.606	621.195
k_3	479.859	0.118
k_4	246.806	85.582

Stochastic programming is not kind to k_3 and k_4. A possible explanation for this is that both these technologies have quite high variable costs relative to their charging capacities, meaning that their recourse actions are expensive.


q for a unit of each storage technology
K1
K2
K3
K4
net charge (MW/unit)
time (h)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
20
40
60
80
100
120
140
160
180


p with and without storage
p without storage
p with storage
energy price (dol/MWh)
time (h)
44
42
40
38
36
34
32
30
28
26
24
22
0
20
40
60
80
100
120
140
160
180

Central plan problem

- Above problem is a Stochastic MOPEC. EMP provides tools to automatically convert into MCP and solve.
- Alternatively:

$$\begin{aligned}
\min_{x,q,s,g} \quad & \mathbb{E} \sum_t \left(C(g_{\omega t}) + \sum_k \left(c_k^0 x_k + c_k^1 |q_{k\omega t}| \right) \right) \\
\text{s.t.} \quad & g = \sum_k q_k + \mathcal{D} \\
& \partial s_k = e q_k^+ - q_k^- \\
& 0 \le q_k^+, q_k^- \le \mathcal{Q} x_k \\
& 0 \le s_k \le \mathcal{S} x_k
\end{aligned}$$

- $g_{\omega t}$ is the generation by conventional resources in scenario ω at time t and the cost function $C(g)$ satisfies $C'(g) = f(g)$.
- Can solve using NLP technology as opposed to MCP technology
- Adding risk-aversion to the MOPEC seems to make the problem nonintegrable, so can only be solved using MCP

Central Zonal Model (nodes i, time t)

Operationally, model involves nodes i and transmission:

$$
\begin{aligned}
\min_{z,\theta,g,q^+,q^-,s} \quad & \sum_{i,t} C_i(g_{i,t}) \\
\text{s.t.} \quad & z = B\mathcal{A}\theta, z \in [-\bar{z}, \bar{z}] \\
& g + q^- - q^+ - \mathcal{A}^T z \geq \mathcal{D} \\
& \underline{g}_i \leq g_{i,t} \leq \bar{g}_i, \\
& \partial s_{i,t} = e q^+_{i,t} - q^-_{i,t}, \\
& 0 \leq q^+_{i,t}, q^-_{i,t} \leq \mathcal{Q}_i, \\
& 0 \leq s_{i,t} \leq \mathcal{S}_i
\end{aligned}
$$

$\mathcal{A}$ is the node-arc incidence matrix

Distributed Model

At a bus i, given the hourly clearing prices $p_{i,t}$, the generator solves:

$$\begin{aligned} \max_{g_i} \quad & \sum_t p_{i,t} g_{i,t} - C_i(g_{i,t}) & \\ \text{s.t.} \quad & \underline{g}_i \le g_{i,t} \le \bar{g}_i, & \forall i, t \end{aligned}$$

and the storage owner solves:

$$\begin{aligned} \max_{q_i^+, q_i^-, s_i} \quad & \sum_t p_{i,t}(q_{i,t}^- - q_{i,t}^+) \\ \text{s.t.} \quad & \partial s_{i,t} = e q_{i,t}^+ - q_{i,t}^-, \\ & 0 \le q_{i,t}^+, q_{i,t}^- \le \mathcal{Q}_i, \\ & 0 \le s_{i,t} \le \mathcal{S}_i \end{aligned}$$

Locational pricing of storage

Given the distributed decisions g, q^+, q^-, s, the ISO maintains the transmission constraints and supply-demand balance, and produces the clearing prices, by enforcing the complementarity constraints:

$$
\begin{aligned}
& z - B\mathcal{A}\theta = 0 && \perp \lambda, \\
& \mathcal{D} \leq g + q^- - q^+ - \mathcal{A}^T z && \perp p \geq 0, \\
& -\lambda + \mathcal{A} p && \perp z \in [-\bar{z}, \bar{z}], \\
& -\mathcal{A}^T B^T \lambda = 0 && \perp \theta
\end{aligned}
$$

Together these optimization problems form a MOPEC and can be solved directly within GAMS.

These models are equivalent to the central model, but exhibit the behaviors of each player in the market.

Approximating transmission

- Generator maximization (given p)
- Storage operation optimization (given p)
- Transmission and market clearing complementarity (given g, q and s)

- Last piece of model (transmission and market clearing) can be replaced by stochastic price process on p (given g, q and s)

$$p_{it} = f\left(\theta, g_{it} + q_{it}^{-} - q_{it}^{+} - \mathcal{D}_{it}\right)$$

- The stochastic process educated by data will model failures and outages but not detailed transmission: complex tradeoff
- Zonal prices give at least 5-9% improvement over average price

Storage in real time market

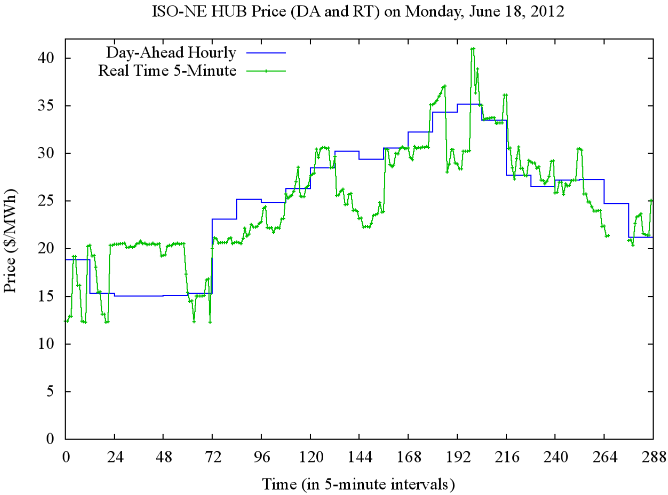
- xcel: cannot make money in real time market
- At time $t = \tau$, determine how to operate right now by solving a forward model:

$$\min_{s_t, q_t^+, q_t^-} \sum_{t=\tau}^{T} p_t(q_t^+ - q_t^-) + \gamma \left\| q^+ - q^- - q^{DA} \right\|_1$$
$$\text{s.t. } \partial s_t = e q_t^+ - q_t^-$$
$$0 \leq s_t \leq \mathcal{S}$$
$$0 \leq q_t^+, q_t^- \leq \mathcal{Q}$$
$$s_\tau, s_T \text{ fixed}$$

- Day ahead solution q^{DA} known
- Penalize deviation from day-ahead storage operation.
- How to estimate p_t for $t \geq \tau$?

Regression, how to do this?

We have day ahead prices for the whole day. We have real time prices up to the last five minutes. We have as much historical data as you want.



K-step Outlook Simple Prediction Model (at the current time τ)

- Forward model: use day ahead prices

$$\hat{p}_t^{\text{RT}} = \begin{cases} p_t^{\text{DA}} + \beta(p_{t-1}^{\text{RT}} - p_t^{\text{DA}}), & t = \tau \\ p_t^{\text{DA}} + \beta(\hat{p}_{t-1}^{\text{RT}} - p_t^{\text{DA}}), & \tau + 1 \leq t \leq \tau + K \\ p_t^{\text{DA}}, & \tau + K + 1 \leq t \leq T \end{cases}$$

- Can solve for β by regression.
- More sophisticated models use GAMS/R interface, e.g.
    - time windows
    - dead-zones
    - LASSO, sparse optimization

Solution process

1. Solve Day Ahead Model
2. Solve prediction model to estimate p_t for $t \geq \tau$
3. Solve operation model for q_t, $t \geq \tau$
4. Implement q_τ
5. Repeat steps 2-4 until $\tau = T$
6. Evaluate implemented solution against real time prices

- day-ahead solution is $240
- day-ahead solution at real time prices: $109
- real-time profit with perfect foresight ($Perfect)
- real-time profit from prediction model ($Predict)
- net benefit for participating in the real-time market

Results for γ * Avg DA Price

γ	0		0.1		0.2	
\$Perfect	300.2		202.0		149.4	
β	\$Predict	\$Net	\$Predict	\$Net	\$Predict	\$Net
0	114.5	5.5	109.0	0	109.0	0
0.3	241.7	132.7	137.0	28.0	109.9	0.9
0.5	245.6	136.6	136.0	27	108.6	-0.4
0.7	249.0	140	150.2	41.2	92.6	-16.4
1	183.1	71.4	117.1	8.1	94.5	-14.5

$$\$\mathrm{Net} = p^{DA}q^{DA} + p^{RT}(q^{RT} - q^{DA}) - \gamma \left\| q^{RT} - q^{DA} \right\|_1$$

Observations

1. The progressive 5-minute decision update can be an effective strategy.
2. Real-time profit depends on how well we can predict the real-time prices, thus a more advanced prediction model is desirable.
3. Penalty for deviation is critical in deciding whether it is possible for a storage unit to make extra profit in participating both day-ahead and real-time market. In other words, a high penalty factor can keep a storage unit from breaking the day-ahead promise.

Conclusions

- Stochastic MOPEC models capture behavioral effects (extended mathematical programming)
- Separate stochastic approximation from optimization
- Tools exist to facilitate easy modeling and solution within GAMS
- Collections of models needed for specific decisions
- Need to understand stochastic processes and their timescales in prediction for optimization
- Policy implications addressable using Stochastic MOPEC
- Can show certain technologies dominate others, some are not viable at all
- Low penalty on deviations from day ahead needed to make real-time market operation feasible