MOPEC: multiple optimization problems with equilibrium constraints

Michael C. Ferris

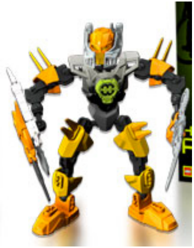
Joint work with: Michael Bussieck, Steven Dirkse, Jan Jagla and Alexander Meeraus

University of Wisconsin, Madison

Optimization Seminar, GERAD & Ecole Polytechnique de Montreal, Montreal

January 19, 2012

Why model?



- to understand (descriptive process, validate principles and/or explore underlying mechanisms)
- to predict (and/or discover new system features)
- to combine (engaging groups in a decision, make decisions, operate/control a system of interacting parts)
- to design (strategic planning, investigate new designs, can they be economical given price of raw materials, production process, etc)
- Must be able to model my problem easily/naturally

Building mathematical models

- How to model: pencil and paper, excel, Matlab, R, python, ...

  - Linear vs nonlinear
  - Deterministic vs probabilistic
  - Static vs dynamic (differential or difference equations)
  - Discrete vs continuous
- Other issues: large scale, tractability, data (rich and sparse)
- Abstract/simplify:
  - Variables: input/output, state, decision, exogenous, random, ...
  - Objective/constraints
  - Black box/white box
  - Subjective information, complexity, training, evaluation
- Just solving a single problem isn't the real value of modeling: e.g. optimization finds "holes" in the model, or couples many models together

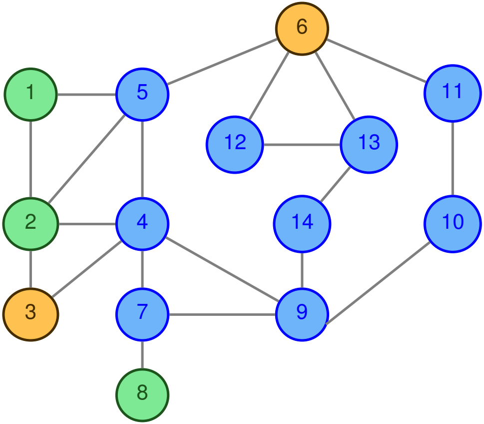
The PIES Model (Hogan)

$$\begin{aligned} \min_x \quad & c^T x \\ \text{s.t.} \quad & Ax = d(p) \\ & Bx = b \\ & x \geq 0 \end{aligned}$$

- Issue is that p is the multiplier on the "balance" constraint of LP
- Extended Mathematical Programming (EMP) facilitates annotations of models to describe additional structure
- empinfo: dualvar p balance
- Can solve the problem by writing down the KKT conditions of this LP, forming an LCP and exposing p to the model
- EMP does this automatically from the annotations

Power Systems: Economic Dispatch

$$\min_{(q,z,\theta)\in\mathcal{F}} \sum_k C(q_k) \text{ s.t. } q_k - \sum_{(l,c)} z_{(k,l,c)} = d_k$$



- Independent System Operator (ISO) determines who generates what
- p_k: Locational marginal price (LMP) at k
- Volatile in "stressed" system
- Can we shed load from consumers to smooth prices?
- FERC (regulator) writes the rules - how to implement?

Understand: demand response and FERC Order No. 745

$$\min_{q,z,\theta,R,p} \sum_k p_k R_k$$

$$\text{s.t.}\ C_1 \geq \sum_k p_k d_k / \sum_k d_k$$

$$C_2 \geq \sum_k (q_k + R_k) p_k / \sum_k (d_k - R_k)$$

$$0 \leq R_k \leq u_k,$$

and (q, z, θ) solves

$$\min_{(q,z,\theta)\in\mathcal{F}} \quad \sum_k C(q_k)$$

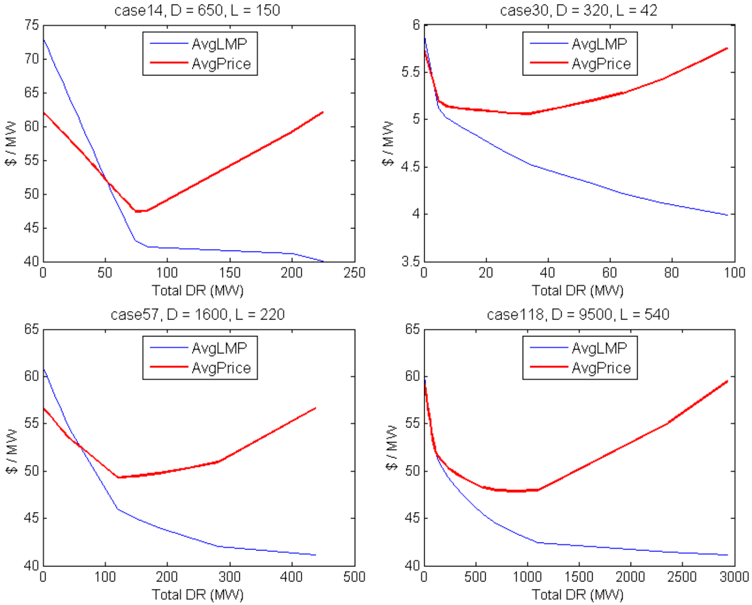
$$\text{s.t.} \quad q_k - \sum_{(l,c)} z_{(k,l,c)} = d_k - R_k \tag{1}$$

where p_k is the multiplier on constraint (1)

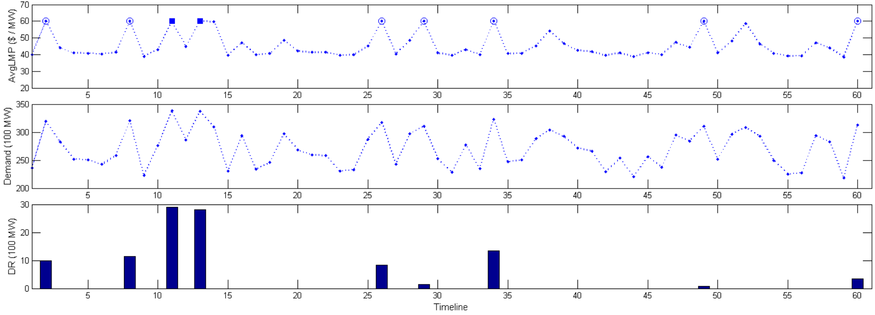
Solution Process (Liu)

- Bilevel program (hierarchical model)
- Upper level objective involves multipliers on lower level constraints
- Extended Mathematical Programming (EMP) annotates model to facilitate communicating structure to solver
  - dualvar p balance
  - bilevel R min cost q z θ balance ...
- Automatic reformulation as an MPEC (single optimization problem with equilibrium constraints)
- Model solved using NLPEC and Conopt
- bilevel $\implies$ MPEC $\implies$ NLP
- Potential for solution of "consumer level" demand response
- Challenge: devise robust algorithms to exploit this structure for fast solution

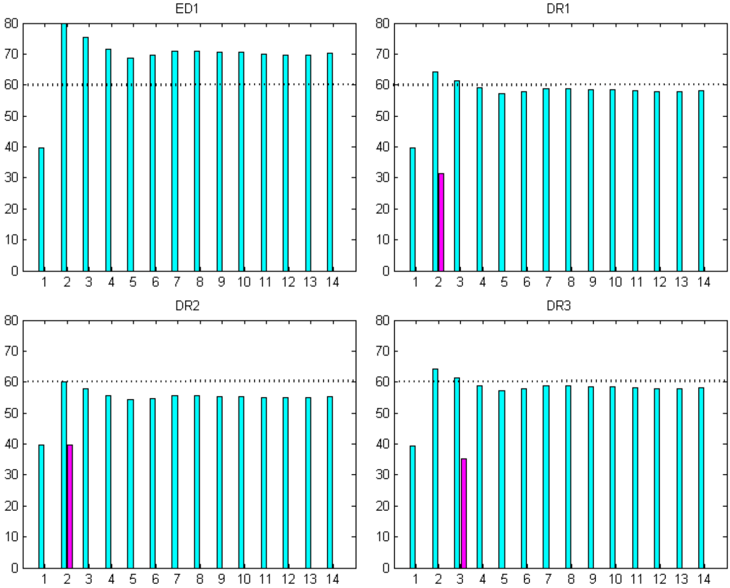
Stability and feasibility



Operational view: LMP, Demand, Response



Alternative models: ED, avg, max, weighted avg



Complementarity Problems in Economics (MCP)

- p represents prices, x represents activity levels
- System model: given prices, (agent) i determines activities x_i

$$G_i(x_i, x_{-i}, p) = 0$$

  x_{-i} are the decisions of other agents.
- Walras Law: market clearing

$$0 \leq S(x,p) - D(x,p) \perp p \geq 0$$

- Key difference: optimization assumes you control the complete system
- Complementarity determines what activities run, and who produces what

Nash Equilibria

- Nash Games: x^* is a Nash Equilibrium if

$$x_i^* \in \arg\min_{x_i \in X_i} \ell_i(x_i, x_{-i}^*, p), \forall i \in \mathcal{I}$$

  x_{-i} are the decisions of other players.

- Prices p given exogenously, or via complementarity:

$$0 \leq H(x,p) \;\perp\; p \geq 0$$

- empinfo: equilibrium
  min loss(i) x(i) cons(i)
  vi H p

- Applications: Discrete-Time Finite-State Stochastic Games. Specifically, the Ericson & Pakes (1995) model of dynamic competition in an oligopolistic industry.

How to combine: Nash Equilibria

- Non-cooperative game: collection of players $a \in \mathcal{A}$ whose individual objectives depend not only on the selection of their own strategy $x_a \in C_a = \mathrm{dom} f_a(\cdot, x_{-a})$ but also on the strategies selected by the other players $x_{-a} = \{x_a : o \in \mathcal{A} \setminus \{a\}\}$.
- Nash Equilibrium Point:

$$\bar{x}_{\mathcal{A}} = (\bar{x}_a, a \in \mathcal{A}) : \forall a \in \mathcal{A} : \bar{x}_a \in \mathrm{argmin}_{x_a \in C_a} f_a(x_a, \bar{x}_{-a}).$$

1. for all $x \in \mathcal{A}$, $f_a(\cdot, x_{-a})$ is convex
2. $C = \prod_{a \in \mathcal{A}} C_a$ and for all $a \in \mathcal{A}$, C_a is closed convex.

VI reformulation

Define

$$G : \mathbb{R}^N \mapsto \mathbb{R}^N \text{ by } G_a(x_{\mathcal{A}}) = \partial_a f_a(x_a, x_{-a}), a \in \mathcal{A}$$

where ∂_a denotes the subgradient with respect to x_a. Generally, the mapping G is set-valued.

Theorem

Suppose the objectives satisfy (1) and (2), then every solution of the variational inequality

$$x_{\mathcal{A}} \in C \text{ such that } -G(x_{\mathcal{A}}) \in N_C(x_{\mathcal{A}})$$

is a Nash equilibrium point for the game.
Moreover, if C is compact and G is continuous, then the variational inequality has at least one solution that is then also a Nash equilibrium point.

Key point: models generated correctly solve quickly

Here S is mesh spacing parameter

S	Var	rows	non-zero	dense(%)	Steps	RT (m:s)
20	2400	2568	31536	0.48	5	$0:03$
50	15000	15408	195816	0.08	5	$0:19$
100	60000	60808	781616	0.02	5	$1:16$
200	240000	241608	3123216	0.01	5	$5:12$

Convergence for $S = 200$ (with new basis extensions in PATH)

Iteration	Residual
0	$1.56(+4)$
1	$1.06(+1)$
2	1.34
3	$2.04(-2)$
4	$1.74(-5)$
5	$2.97(-11)$

General Equilibrium models

$$(C): \max_{x_k \in X_k} U_k(x_k) \text{ s.t. } p^T x_k \leq i_k(y, p)$$

$$(I): i_k(y, p) = p^T \omega_k + \sum_j \alpha_{kj} p^T g_j(y_j)$$

$$(P): \max_{y_j \in Y_j} p^T g_j(y_j)$$

$$(M): \max_{p \geq 0} p^T \left(\sum_k x_k - \sum_k \omega_k - \sum_j g_j(y_j) \right) \text{ s.t. } \sum_l p_l = 1$$

General Equilibrium models

$$(C): \max_{x_k \in X_k} U_k(x_k) \text{ s.t. } p^T x_k \le i_k(y,p)$$

$$(I): i_k(y,p) = p^T \omega_k + \sum_j \alpha_{kj} p^T g_j(y_j)$$

$$(P): \max_{y_j \in Y_j} p^T g_j(y_j)$$

$$(M): \max_{p \ge 0} p^T \left(\sum_k x_k - \sum_k \omega_k - \sum_j g_j(y_j) \right) \text{ s.t. } \sum_l p_l = 1$$

Can reformulate as embedded problem (Ermoliev et al, Rutherford):

$$\max_{x \in X, y \in Y} \quad \sum_k \frac{t_k}{\beta_k} \log U_k(x_k)$$

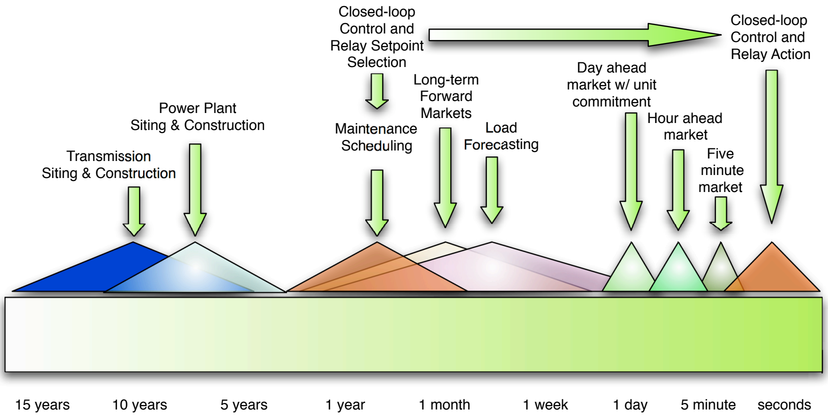
$$\text{s.t. } \sum_k x_k \le \sum_k \omega_k + \sum_j g_j(y_j)$$

$t_k = i_k(y,p)$ where p is multiplier on NLP constraint

Extension: The smart grid

- The next generation electric grid will be more dynamic, flexible, constrained, and more complicated.
- Decision processes (in this environment) are predominantly hierarchical.
- Models to support such decision processes must also be layered or hierachical.
- Optimization and computation facilitate adaptivity, control, treatment of uncertainties and understanding of interaction effects.
- Developing interfaces and exploiting hierarchical structure using computationally tractable algorithms will provide FLEXIBILITY, overall solution speed, understanding of localized effects, and value for the coupling of the system.

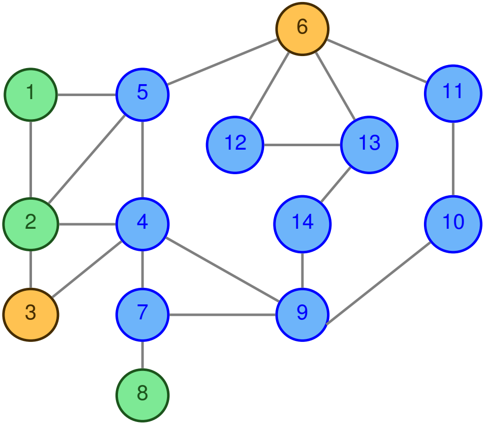
Representative decision-making timescales in electric power systems



A monster model is difficult to validate, inflexible, prone to errors.

Combine: Transmission Line Expansion Model

$$\min_{x \in X} \sum_{\omega} \pi_\omega \sum_{i \in N} d_i^\omega p_i^\omega(x)$$



- Nonlinear system to describe power flows over (large) network
- Multiple time scales
- Dynamics (bidding, failures, ramping, etc)
- Uncertainty (demand, weather, expansion, etc)
- $p_i^\omega(x)$: Price (LMP) at i in scenario ω as a function of x
- Use other models to construct approximation of $p_i^\omega(x)$

Generator Expansion (2): $\forall f \in F$:

$$\min_{y_f} \sum_{\omega} \pi_\omega \sum_{j \in G_f} C_j(y_j, q_j^\omega) - r(h_f - \sum_{j \in G_f} y_j)$$

$$\text{s.t.} \sum_{j \in G_f} y_j \le h_f, y_f \ge 0$$

G_f:	Generators of firm $f \in F$
y_j:	Investment in generator j
q_j^ω:	Power generated at bus j in scenario ω
C_j:	Cost function for generator j
r:	Interest rate

Market Clearing Model (3): $\forall \omega$:

$$\min_{z,\theta,q^\omega} \sum_f \sum_{j \in G_f} C_j(y_j, q_j^\omega) \quad \text{s.t.}$$

$$q_j^\omega - \sum_{i \in I(j)} z_{ij} = d_j^\omega \qquad \forall j \in N(\perp p_j^\omega)$$

$$z_{ij} = \Omega_{ij}(\theta_i - \theta_j) \qquad \forall (i,j) \in A$$

$$-b_{ij}(x) \le z_{ij} \le b_{ij}(x) \qquad \forall (i,j) \in A$$

$$\underline{u}_j(y_j) \le q_j^\omega \le \overline{u}_j(y_j)$$

z_{ij}:	Real power flowing along line ij
q_j^ω:	Real power generated at bus j in scenario ω
θ_i:	Voltage phase angle at bus i
Ω_{ij}:	Susceptance of line ij
$b_{ij}(x)$:	Line capacity as a function of x
$\underline{u}_j(y)$, $\overline{u}_j(y)$:	Generator j limits as a function of y

Solution approach

- Use derivative free method for the upper level problem (1)
- Requires $p_i^\omega(x)$
- Construct these as multipliers on demand equation (per scenario) in an Economic Dispatch (market clearing) model
- But transmission line capacity expansion typically leads to generator expansion, which interacts directly with market clearing
- Interface blue and black models using Nash Equilibria (as EMP):

empinfo: equilibrium
forall f: min expcost(f) y(f) budget(f)
forall ω: min scencost(ω) q(ω) ...

Feasibility

$$\text{KKT of } \min_{y_f \in Y} \sum_{\omega} \pi_\omega \sum_{j \in G_f} C_j(y_j, q_j^\omega) - r(h_f - \sum_{j \in G_f} y_j) \quad \forall f \in F \quad (2)$$

$$\text{KKT of } \min_{(z,\theta,q^\omega) \in Z(x,y)} \sum_{f} \sum_{j \in G_f} C_j(y_j, q_j^\omega) \quad \forall \omega \quad (3)$$

- Models (2) and (3) form a complementarity problem (CP via EMP)
- Solve (3) as NLP using global solver (actual $C_j(y_j, q_j^\omega)$ are not convex), per scenario (SNLP) this provides starting point for CP
- Solve (KKT(2) + KKT(3)) using EMP and PATH, then repeat
- Identifies CP solution whose components solve the scenario NLP's (3) to global optimality

Scenario	ω_1	ω_2
Probability	0.5	0.5
Demand Multiplier	8	5.5

SNLP (1):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	3.05	4.25	3.93	4.34	3.39
ω_2		4.41	4.07	4.55	

Scenario	ω_1	ω_2
Probability	0.5	0.5
Demand Multiplier	8	5.5

SNLP (1):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	3.05	4.25	3.93	4.34	3.39
ω_2		4.41	4.07	4.55	

EMP (1):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	2.86	4.60	4.00	4.12	3.38
ω_2		4.70	4.09	4.24	

Firm	y_1	y_2	y_3	y_6	y_8
f_1	167.83	565.31			266.86
f_2			292.11	207.89	

Scenario	ω_1	ω_2
Probability	0.5	0.5
Demand Multiplier	8	5.5

SNLP (2):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	0.00	5.35	4.66	5.04	3.91
ω_2		4.70	4.09	4.24	

Scenario	ω_1	ω_2
Probability	0.5	0.5
Demand Multiplier	8	5.5

SNLP (2):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	0.00	5.35	4.66	5.04	3.91
ω_2		4.70	4.09	4.24	

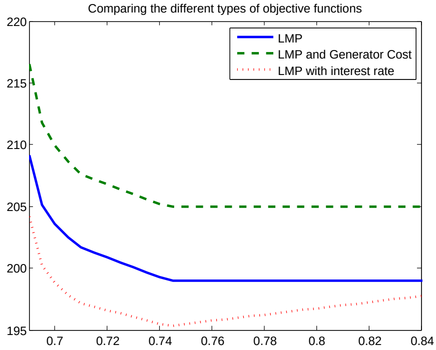
EMP (2):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	0.00	5.34	4.62	5.01	3.99
ω_2		4.71	4.07	4.25	

Firm	y_1	y_2	y_3	y_6	y_8
f_1	0.00	622.02			377.98
f_2			283.22	216.79	

Observations

- But this is simply one function evaluation for the outer "transmission capacity expansion" problem
- Number of critical arcs typically very small
- But in this case, p_j^ω are very volatile
- Outer problem is small scale, objectives are open to debate, possibly ill conditioned



- Economic dispatch should use AC power flow model
- Structure of market open to debate
- Types of "generator expansion" also subject to debate
- Suite of tools is very effective in such situations

What is EMP?

Annotates existing equations/variables/models for modeler to provide/define additional structure

- equilibrium
- vi (agents can solve min/max/vi)
- bilevel (reformulate as MPEC, or as SOCP)
- disjunction (or other constraint logic primitives)
- randvar
- dualvar (use multipliers from one agent as variables for another)
- extended nonlinear programs (library of plq functions)

Currently available within GAMS

Conclusions

- Modern optimization within applications requires multiple model formats, computational tools and sophisticated solvers
- EMP model type is clear and extensible, additional structure available to solver
- Extended Mathematical Programming available within the GAMS modeling system
- Able to pass additional (structure) information to solvers
- Embedded optimization models automatically reformulated for appropriate solution engine
- Exploit structure in solvers
- Extend application usage further